Efficiency Limit of Long-distance Magneto-Inductive Power Transfer

R.R.A.Syms, T.Floume

EEE Dept., Imperial College London, Exhibition Road, London, SW7 2AZ, UK
FAX +44-207-594-6308 Email r.syms@imperial.ac.uk

Abstract

The problem of optimising long-distance power delivery via magneto-inductive waveguides is considered, and limits to efficiency are derived. Arrangements for impedance matching are proposed, and it is shown that efficiency can be improved by operation off-resonance, when losses are at a minimum, using broadband matching transformers.

1. Introduction

Magneto-inductive (MI) power transfer has long been of interest for biomedical implants [1, 2]. More recently, applications involving moving equipment [3], electric vehicles [4] and general loads [5] have been considered. However, MI power transfer over long distances has largely been ignored, although its lack of DC connections may be useful in safety-critical applications. In this case, intermediate resonators are required as 'stepping stones', so the arrangement is a waveguide [6], recently demonstrated in cable form [7]. Here, we consider the limits of transfer efficiency and highlight strategies for improvement.

2. Magneto-inductive waves

At mains frequency, the wavelength is so large that the product of propagation constant k and distance d is small, even when d is of the order of 100 km. The line impedance can be ignored, and the focus placed on low current, high voltage systems that improve efficiency in the presence of loss. In contrast, efficient MI power transfer must combine power system and RF principles. Fig. 1 shows a MI waveguide formed from a set of L-C resonators coupled by mutual inductance M. The first element contains a source V_S with real impedance Z_S and reactive elements L_S, C_S and M_S. The last contains a load Z_L and reactive elements L_L, C_L and M_L. Source, line and load losses are represented by resistors R_S, R and R_L.

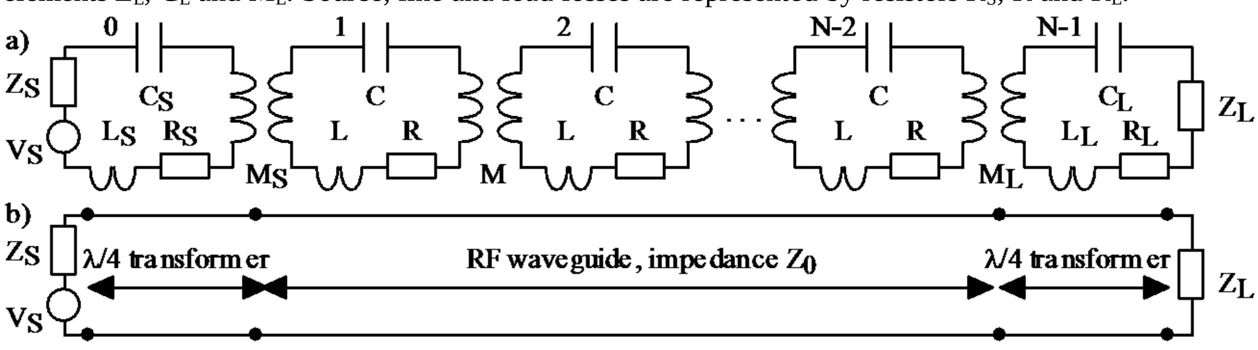

Figure 1. a) Impedance matched MI waveguide and b) RF analogue.

Assuming nearest-neighbour coupling, the current I_n in the n^{th} line element at angular frequency ω satisfies the recurrence equation [6]:

$$(R + j\omega L + 1/j\omega C)I_n + j\omega M(I_{n-1} + I_{n+1}) = 0 \tag{1}$$

Assumption of the wave solution $I_n = I_0 \exp(-jnka)$, where k is the propagation constant and a is the period, then yields the dispersion relation:

$$1 - \omega_0^2/\omega^2 - j\omega_0/\omega Q_0 + \kappa \cos(ka) = 0 \tag{2}$$

Here $\omega_0 = 1/(LC)^{1/2}$ is the angular resonant frequency, $\kappa = 2M/L$ the coupling coefficient and $Q_0 = \omega_0 L/R$ the Q-factor. Eqn. 2 may be solved for the complex propagation constant $k = k' - jk''$. In the lossless case, propagation is limited to the band $1/(1 + |\kappa|)^{1/2} \leq \omega/\omega_0 \leq 1/(1 - |\kappa|)^{1/2}$, whose centre lies at $\omega = \omega_0$, when $k'a = \pi/2$. The characteristic impedance is $Z_0 = j\omega M \exp(-jk'a)$, which has the real value $Z_{0M} = \omega_0 M$ at resonance [6]. For low loss, $k''a \approx 1/\{\kappa Q \sin(k'a)\}$ and has the minimum $k''a \approx 1/\kappa Q_0 \approx R/2Z_{0M}$.

3. Impedance matched power transfer

These results imply that the best performance is obtained at resonance, when loss is minimum and Z_0 is real. Standard theory then suggests that the source and load should match each other to optimise power transfer (so $Z_S = Z_L$, and by symmetry $L_S = L_L$, $C_S = C_L$, $M_S = M_L$ and $R_S = R_L$). To eliminate reflections, the load should also match the line impedance. However, if $Z_L \neq Z_{0M}$, matching can still be achieved if the mutual inductance M_L satisfies $\omega_0 M_L = \sqrt{(Z_{0M} Z_L)}$. Inductance scaling laws then suggest that $L_L = L\,(Z_L/Z_{0M})$, $C_L = 1/(\omega_0^2 L_L)$ and $R_L = R\,(Z_L/Z_{0M})$. In this case the input and output elements act as quarter-wave transformers, leading to the RF analogue in Fig. 1b.

Power transfer may then be modelled by writing the governing equations in the form $\underline{V} = \underline{Z}\underline{I}$, where $\underline{V}$ is an N-element vector of impressed voltages, $\underline{Z}$ is an N x N impedance matrix and $\underline{I}$ is an N-element current vector, so that $\underline{I} = \underline{Z}^{-1}\underline{V}$. Fig. 2a shows example current variations for 50-element lines with $R/Z_{0M} = 0.01$. For $Z_L = Z_{0M}$, there is simple decay of the current modulus. For $Z_L = 0.01\ Z_{0M}$, there is an additional ten-fold step-down in current at the input, followed by a ten-fold step-up at the output. However, in each case there are minimal reflections and the start and end currents are the same.

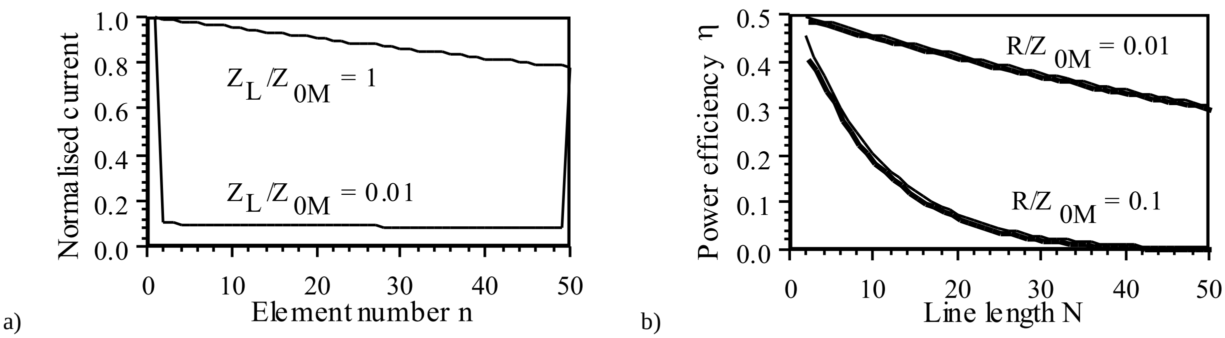

Figure 2. Example variations of a) current modulus along lossy MI waveguides, and b) efficiency with line length.

4. Power efficiency

The efficiency η is the power delivered to the load divided by the total power dissipated, or:

$$\eta = |I_{N-1}|^2 Z_L / \{|I_0|^2(Z_L + R_L) + (|I_1|^2 + |I_2|^2 \ldots |I_{N-2}|^2)R + (|I_{N-1}|^2(Z_L + R_L)\} \tag{3}$$

If $Z_L = Z_{0M}$, Eqn. 3 may be evaluated analytically by assuming a given initial current I_0, followed by a current decay in the form $|I_n| = |I_0| \exp(-R/2Z_{0M})$. The result is:

$$\eta \approx \exp[-(N-1)R/Z_{0M}] / \{2 + (R/Z_{0M}) \exp[-(N-1)R/Z_{0M}]\} \approx 1/2 \exp[-(N-1)R/Z_{0M}] \tag{4}$$

As expected, η has a maximum of 1/2, and is reduced from this value by losses. Fig. 2 compares this result (thin lines) with an exact calculation by the $\underline{V} = \underline{Z}\underline{I}$ method (thick lines), for different R/Z_{0M}. The agreement is excellent. A similar result is obtained if $Z_L \neq Z_{0M}$; this follows from the need to maintain matching, and implies that (in contrast to AC power) stepping down the current does not improve matters.

Efficiency can, however, be improved by noting that the assumption of minimum loss at resonance follows from an approximation. Fig. 3a shows the exact solution of Eqn. 2, for $Q_0 = 100$ and different values of κ. As κ rises, the minimum in $k''a$ actually lies at frequencies increasingly above ω_0.

ISBN 978-952-67611-0-7

Unfortunately, Z_0 is no longer real at this point, so multiple reflections and standing waves arise with standard terminations. The thin lines in Fig. 3b show detailed calculations of efficiency for 50-element lines with similar parameters when the arrangement of Fig. 1 is used. Clearly, the oscillations prevent full advantage being taken of any potential improvement in η.

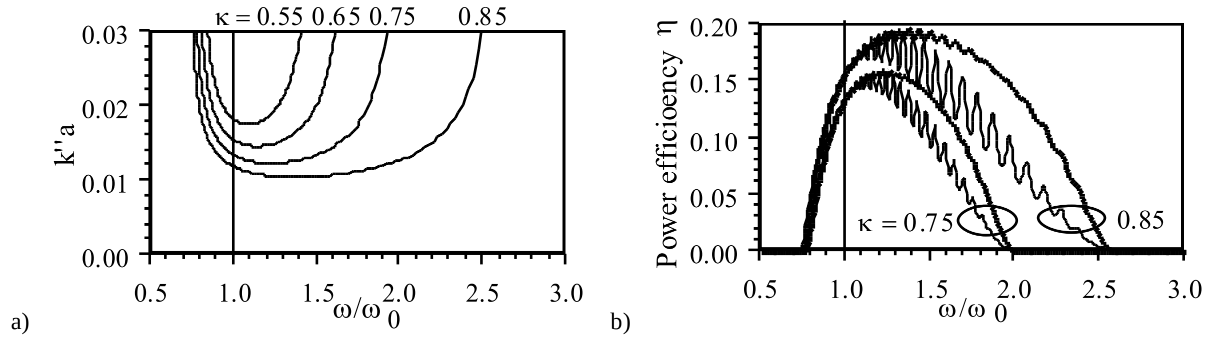

Figure 3. Example variation of a) k''a and b) power efficiency η with frequency, for Q_0 = 100 and different values of κ.

Performance may be improved using an alternative transducer, which allows exact impedance matching at two frequencies, ω_0 and $\omega_0/\sqrt{(1 - \kappa^2)}$, with low reflectivity in the important range between [8]. All that is required is to halve the inductance and double the capacitance in the terminating elements, as in Fig. 4. The bold lines in Fig. 3b show the improvement in efficiency (for κ = 0.85, around 25%) at frequencies above ω_0. The arrangement may be combined with current scaling, allowing high-impedance MI waveguides to be matched to low-impedance loads.

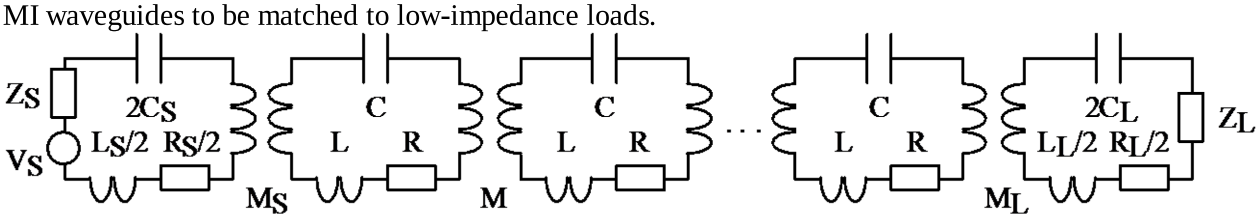

Figure 4. MI waveguide with broadband transducers for improved impedance matching.

5. Conclusions

Limits to the efficiency of MI power systems have been identified, and operation off-resonance using broadband transducers has been shown to provide a clear improvement in performance.

References

[1] Kadefors R., Kaiser E., Petersén I. "Energizing implantable transmitters by means of coupled inductance coils" *IEEE Trans. Biomedical Engng.* Vol. BME-16, pp. 177-183, 1969

[2] Schuder J.C., Gold J.H., Stephenson H.E. "An inductively coupled RF system for the transmission of 1 kW of power through the skin" *IEEE Trans. Biomed. Engng.* Vol. BME-18, pp. 265-273, 1971

[3] Klotz K.W., Divan D.M., Novotny D.W., Lorenz R.D. "Contactless power delivery system for mining applications" *IEEE Trans. Industry Apps.* Vol. 31, pp. 27-35, 1995

[4] Hirai J., Kim T.W., Kawamura A. "Study on intelligent battery charging using inductive transmission of power and information" *IEEE Trans. Power Elect.* Vol. 15, pp. 335-345, 2000

[5] Kurs A., Karalis A., Moffatt R., Joannopoulos J.D., Fisher P., Solijacic M. "Wireless power transfer via strongly coupled magnetic resonances" *Science* Vol. 317, pp. 83-86, 2007

[6] Shamonina E., Kalinin V.A., Ringhofer K.H. Solymar L. "Magnetoinductive waveguide" *Elect. Lett.* Vol. 38, pp. 371-373, 2002

[7] Syms R.R.A., Solymar L., Young I.R., Floume T. "Thin-film magneto-inductive cables" *J. Phys. D. Appl. Phys.* Vol. 43, 055102, 2010

[8] Syms R.R.A., Solymar L., Young I.R. "Broad-band coupling transducers for magneto-inductive cable" *J. Phys. D. Appl. Phys.* Vol. 43, 285003, 2010

