Metamaterial composites and super-resolution

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Abstract

We consider the optical properties of composites containing metamaterial inclusions in a normal material matrix, with the scale size of inclusions much finer than the wavelength. We consider the case where these inclusions have sharp corners, and use analytic results of Hetherington and Thorpe to argue that it is then possible to deduce the shape of the corner (its included angle) by spectral measurements on absorptance of such composites, irrespective of the size of the inclusions with respect to the wavelength. We support these analytic arguments by highly accurate numerical results for the effective permittivity function of such composites, as a function of the permittivity ratio of inclusions to matrix, which show that this function has a continuous spectral component with limits independent of area fraction of inclusions, and the same for square and staggered square lattices.

1. Introduction

This paper links themes evoked in two classic papers, one in mathematics [1] and the other in physics [2]. The first of these poses the question as to whether the spectral content of the radiation from a source can reveal its shape. The second shows that the use of negatively-refracting metamaterials in a plane slab can lead to a super-resolving perfect lens, also known as a superlens. We will consider a two-dimensional composite material, composed of polygonal inclusions made of a metamaterial (by which we mean an artificial material with a dielectric constant which has a negative real part and a very small imaginary part) and placed in a positive dielectric matrix material. We will show that, in the spirit of Pendry, the metamaterial makes possible resolution of an important structural feature of the inclusions, irrespective of how much smaller than the wavelength they are. We will also show that, in the spirit of Kac, this feature relates to the shape of the inclusion, being in fact the corner angle of the polygon, and

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that it is deduced from spectral measurements on the composite. The fact that a spectral feature could be determined by corner shape, independent of (say) the area fraction of inclusions, was first suggested by Hetherington and Thorpe [3], on the basis of an elegant argument and numerical evidence for dilute composites.

We will base our demonstration firstly on analytic results relating to the spectrum of the effective dielectric permittivity function of the composite material, and secondly on remarkably accurate numerical results for this spectrum obtained using a new technique. The numerical results for the singularity spectrum of the effective dielectric permittivity function reveal that it has a continuous part which runs between upper and lower limits of the permittivity ratio of the two components of the metamaterial composite which do not vary at all with the area fraction of the composite. It is complemented by a discrete spectrum of poles which does evolve with area fraction.

The results we give here are interesting in the insights they give into the connection between metamaterials and super-resolution. They are also important in furthering our understanding of the connection between inclusion shape, geometrical arrangement and spectral properties of the effective permittivity function. This connection helps in the design of structures having enhanced absorption over a wide wavelength range for applications in photothermal or photovoltaic captors [4, 5, 6, 7], or offering strongly enhanced local fields for applications like sensing or nonlinear optical elements.

2. Numerical results

The method we describe provides highly accurate solutions for electrostatic or magnetostatic fields in composites with a complex and close-to-negative ratio of permittivity or permeability between that of a periodic set of polygonal inclusions and the matrix material which separates them. The difficulty of finding such solutions resides in the singularities of fields which occur in the neighbourhoods of the sharp corners of the inclusions. These fields, as well as having very large magnitudes near corners, may also oscillate very rapidly there.

To overcome this difficulty, we apply an integral equation approach and use a novel numerical method called *recursive compressed inverse preconditioning* [8, 9, 10, 11] . Conceptually this is a local multilevel technique which makes a change of basis and expresses the non-smooth solution to the single-layer integral equation in terms of a piecewise smooth transformed layer density which can be cheaply resolved by polynomials.

To illustrate the accuracy and stability of the method, we have constructed animations showing the evolution of the effective permittivity of the square and staggered square arrays of square cylinders, as the area fraction of the inclusions varies from zero until the inclusions almost touch. Figure 1 shows a frame of one of our animations. It will be noticed that the inclusions have come very close to touching, but the numerical results are still stable and accurate. They show that there is a branch-cut of the effective permittivity function in the interval [-3,-1/3] of the permittivity ratio; in this interval, the effective permittivity can have a one-sided complex limit as the imaginary part of the permittivity ratio tends down to zero. Away from the branch-cut, the effective permittivity function has a sequence of poles which become more and more dense as the squares come close to touching. Exactly at the touching area fraction, these poles become dense, with the result that the branch-cut fills the entire negative real axis of the permittivity ratio, as required by analytic results [12].

3. Conclusion

The results we have presented are important in several ways. Firstly, they show that, by measuring the optical absorption spectrum of a composite containing polygonal inclusions with a permittivity ratio to the matrix negative or close to it, one can determine the corner angle of the polygons, irrespective of

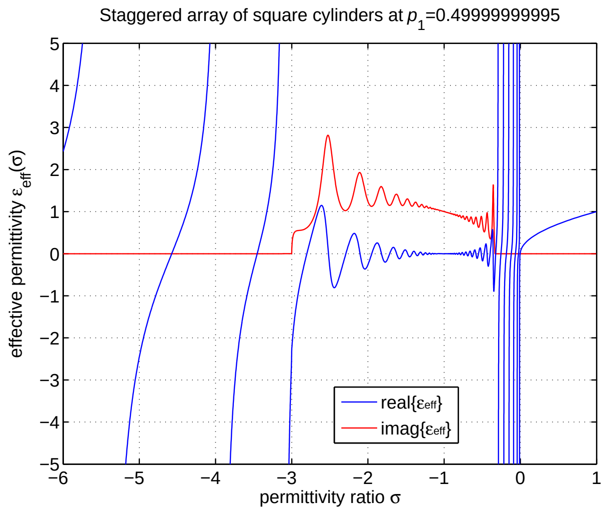

Fig. 1: The effective permittivity of a staggered square array very close to touching, as a function of permittivity ratio.

how much smaller than the wavelength the inclusions are. Secondly, they show that such composites can offer a new type of metamaterial for designers: one offering not only discrete resonances such as those employed hitherto, but also a continuous band of resonances. These composites may then be an ideal building block for metamaterials designed to operate over a wide wavelength range. Thirdly, we have exhibited the results of uniquely powerful and accurate numerical technique, capable of showing in exquisite detail the spectral behaviour of structured metamaterials in which there are local regions of strongly concentrated fields.

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