Analytical model for periodic arrays of slits/strips printed on dielectric slabs: TE and TM polarizations

R. Rodríguez-Berral¹, F. Mesa¹, F. Medina²

¹Department of Applied Physics 1, ETSII, University of Seville
Av. Reina Mercedes s/n, 41012-Seville, Spain
Fax: + 34 954239434; email: rrberral@us.es; mesa@us.es
²Department of Electronics and Electromagnetism, Physics Faculty, University of Seville
Av. Reina Mercedes s/n, 41012-Seville, Spain
Fax: + 34 954239434; email: medina@us.es

Abstract

This paper provides a systematic procedure to extract an equivalent circuit model to characterize the electromagnetic transmission and reflection properties of periodic arrays of slits or strips sandwiched between a pair of dielectric slabs. The method is valid for TE and TM polarization and for normal and oblique incidence.

1. Introduction

The analysis of the scattering of planar uniform waves by planar periodic arrays of scatterers or apertures in opaque screens has become very popular since the discovery of extraordinary optical transmission (EOT) [1]. A couple of excellent and comprehensive reviews on the topic can be found in [2, 3]. In a first stage, most of the papers dealt with free standing metallic structures wihout consideration of the dielectric substrates that give mechanical support to such structures. However, it is obvious that the presence of dielectric slabs not only modifies the transmission spectrum but also adds flexibility to the design of devices based on this phenomenology. Due to this reason, a number of authors have stressed the influence of the presence of dielectric slabs [4–7]. Simultaneously, some of the authors of this paper have developed an analytical circuit-like model explaining extraordinary transmission in a number of situations [8–10]. These latter papers did not consider the presence of dielectric layers adjacent to the structured metal surface because it was not trivial to include this fact in the model in an efficient and accurate manner. An heuristic strategy combined with a semi-numerical approach has been recently proposed [11] to deal with narrow slit systems sandwiched between a couple of dielectric slabs. This model was restricted to normal incidence for a TM-polarized uniform wave. A minimal model for anomalous extraordinary transmission in 2-D periodic structures based on similar circuit methodology has recently been demonstrated in [12]. In the present work we report on a systematic strategy to obtain the circuit parameters of of the same class of structures studied in [11]. The model is now extended with respect to [11] in three directions: a) slot-like and strip-like structures are considered; b) TM and TE polarizations are included in the analysis, and c) oblique incidence is accounted for. We also include losses by means of a phenomenological model. The numerical full-wave results have been generated using a mode matching method and the analytical results show an excellent agreement with the modematching data in a wide range of situations. The availability of a quasi-analytical model has obvious advantages over numerical approaches commonly used by other authors.

2. Statement of the problem

An example structure of the type analyzed in this work is shown in Fig. 1. A 1-D array of relatively narrow slits sandwiched between two dielectric slabs are illuminated by linearly polarized uniform waves (TM and TE cases are considered). In the presentation, using analytical arguments, it will be demonstrated that some circuit models exactly match the numerical results in the very narrow slit limit. For non-zero width slits, the model remains valid provided that the slit width is electrically small (subwavelength slits). The quasi-complementary structure (printed strips instead of etched slits) can also be



Fig. 1: Example of structure under consideration under normal TM and TE illumination. Narrow strips and oblique incidence have also been considered.

easily modelled by using the appropriate (but very different) circuit model. Some of the phenomenology reported in previous papers [4, 7] for 2-D periodic structures also appears in the case of 1-D periodic systems, especifically that linked to the presence of slab modes. For instance, anomalous extraordinary transmission peaks (those associated with TE polarization) are shown to be possible if dielectric loading is present, as previously discussed in [5]. In the frame of our model, this result is self-evident from circuit theory reasoning. Some of the circuit parameters of the model are known in closed form. However, some of them have to be extracted from numerically obtained information at a few frequency points. With a proper choice of these points and a physically sound circuit, the number of sampling points is kept very low (no more than 4 or 5 points are typically required). Once the parameters have been obtained, analytical and full-wave numerical data are indistinguishable even for extremely complex spectra. This is an important advantage of using our proposed modeling technique.

3. Numerical results

In Fig. 2 we plot the transmission curves for an example structure consisting of slits of width w periodically distributed with a period a and sandwiched between two dielectric layers of relative permittivities ε_1 and ε_2 with thicknesses d_1 and d_2 . Both TM and TE polarizations are considered. The agreement between numerical and analytical results is very good. Analytical model captures even the most fine details of the transmission spectra. Similar curves can be obtained for narrow strips and for oblique incidence. This procedure does not only provides a fast method to generate the transmission/reflection spectra, but a theoretical frame to explain their nature.

4. Conclusion

Circuit models having a few parameters to be determined are proposed for lamellar diffraction gratings consisting of 1-D periodic arrays of slits or strips sandwiched between two dielectric slabs. The models are valid for both TM and TE polarization and for oblique incidence. From a few numerically-generated



Fig. 2: Transmission spectra for a structure with parameters: $a = 5.0 \text{ mm}, d_1 = 2.0 \text{ mm}, d_2 = 3.0 \text{ mm}, d_2$ $\varepsilon_1 = 2.2, \, \varepsilon_2 = 3.0, \, w = 0.5 \, \text{mm.}$

scattering coefficients (mode matching is used as numerical technique) the whole spectrum can be obtained.

Acknowledgements

This work has been funded by the Spanish Ministry of Science and Innovation with European Union FEDER funds (projects TEC2010-16948 and CSD2008-00066) and by the Spanish Junta de Andalucía (project TIC-4595).

References

- [1] T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff, Extraordinary optical transmission through sub-wavelength hole arrays, Nature, vol. 391, pp. 667-669, 1998.
- [2] F. J. García-de-Abajo, Colloquium: Light scattering by particle and hole arrays, Rev. Modern Phys., vol. 79, pp. 1267-1290, 2007.
- [3] F. J. García-Vidal, L. Martín-Moreno, T. W. Ebbesen, and L. Kuipers, Light passing through subwavelength apertures, Rev. Mod. Phys., vol. 82, pp. 729-787, 2010.
- [4] V. Lomakin, E. Michielssen, Enhanced transmission through metallic plates perforated by arrays of subwavelength holes and sandwiched between dielectric slabs, Phys. Rev. B, vol. 71, p. 235117, 2005.
- [5] E. Moreno, L. Martín-Moreno, F. J. García-Vidal, Extraordinary optical transmission without plasmons: the
- s-polarization case, *J. Opt. A: Pure Appl. Opt.*, vol. 8, p. S94-S97, 2006. [6] S. A. Kuznetsov, M. Navarro-Cía, V. V. Kubarev, A. V. Gelfand, M. Beruete, I. Campillo, M. Sorolla, Regular and anomalous Extraordinary Optical Transmission at the THz-gap, Opt. Express, vol. 17, no. 14, pp. 11730-11738, July 2009.
- [7] R. Ortuno, C. García-Meca, F. J. Rodríguez-Fortuno, J. Martí, A. Martínez, Multiple extraordinary optical transmission peaks from evanescent coupling in perforated metal plates surrounded by dielectrics, Opt. Express, vol. 18, no. 8, pp. 7893-7898, April 2010.
- [8] F. Medina, F. Mesa, and R. Marqués, "Extraordinary transmission through arrays of electrically small holes from a circuit theory perspective," IEEE Trans. Microwave Theory Tech., vol. 56, no. 12, pp. 3108-3120, Dec. 2008.
- [9] F. Medina, F. Mesa, D. C. Skigin, Extraordinary Transmission Through Arrays of Slits: a Circuit Theory Model, IEEE Trans. Microwave Theory Tech., vol. 58, no. 1, pp. 105-115, Jan. 2010.
- [10] F. Medina, F. Mesa, J. A. Ruíz-Cruz, Jesús M. Rebollar, and J. R. Montejo-Garai, "Study of extraordinary transmission in a circular waveguide system," IEEE Trans. Microw. Theory Tech., vol. 58, no. 6, pp. 1532-1542, June 2010.
- [11] R. Rodríguez-Berral, F. Medina, F. Mesa, Circuit model for a periodic array of slits sandwiched between two dielectric slabs, Appl. Phys. Lett., vol. 96, p. 161104, 2010.
- [12] M. Beruete, M. Navarro-Cía, S. A. Kuznetsov, M. Sorolla, Circuit approach to the minimal configuration of terahertz anomalous extraordinary transmission, Appl. Phys. Lett., vol. 98, p. 014106, 2011.