Closed-form analysis of electrically thick diffraction gratings with internal structure

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Abstract

This contribution reports on a very accurate analytical model for 1–D arrays of slits in thick screens having internal structure. The model is based on circuit-theory arguments. Full-wave numerical data and analytical results show a very good agreement up to the onset of the first diffraction order.

1. Introduction

The scattering of planar uniform waves by sub-wavelength apertures in metal screens has been a research topic for decades but the discovery of extraordinary optical transmission (EOT) [1–3] has brought up a flourishing attention on this matter. Although most attention has focused on 2-D arrays of apertures, the 1-D diffraction grating problem has also been revisited in the context of EOT. For instance, one of the first theoretical papers reporting on a qualitative and quantitative theory for the experimental discovery of Ebbesen [1] analyzed the transmission through electrically thick 1-D diffraction gratings [4]. The role of dielectric slabs loading arrays of apertures has also been investigated within this frame [5]. Other theoretical [6] and experimental [7, 8] works studied the *phase resonances* arising from the interaction between closely spaced slits grouped to give place to *compound* transmission gratings. It is worth mentioning that many of the results in the cited papers about slits were reported decades ago [9–11], but the repercussion of those pioneering works has been very limited on recent researches due to difficult access to the original sources. The geometry of the slits considered in the above mentioned papers was uniform. Apart from a Fano-like resonance appearing close to the frequency for which the period of the structure is one wavelength, the transmission resonances are of Fabry-Pérot type and the resonance frequencies are integer multiples of a fundamental harmonic. This restriction can be removed by using slits with internal geometry, such as the stepped slits numerically and experimentally studied in [12]. Some of the authors of this paper have been working during the last few years on the development of analytical models for EOT phenomena. The most relevant results have been obtained for 2-D arrays of holes in thick metal screens [13] and 1-D arrays of slits (diffraction gratings) in thick metal screens [14]. The purpose of this contribution is to extend those analytical models to stepped slits. It will be shown how the analytical model (which is a circuit model) predictions perfectly agree with full-wave numerical data obtained with mode matching or finite element codes.

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2. Statement of the problem

The slits grating with internal structure considered in this contribution is shown un Fig. 1(a). For normal incidence the problem can be reduced to the cascading of several parallel plate transmission line step discontinuities, as shown in Fig. 1(b) and (c). The equivalent circuit for this situation is depicted in Fig. 1(d). All the paramaters of the circuit are known in closed form: the characteristic admittances of the parallel-plate transmission lines are trivial. The edge capacitances (C_{ij}) can be obtained using a conformal mapping scheme for the low frequency limit and the the arguments in [14] to account for the frequency dependence. Thus, the scattering parameters of the original diffraction grating can be very easily computed using conventional distributed circuits theory. As it will be shown in the next section, the analytical results are surprisingly accurate when compared with numerically intensive computations.

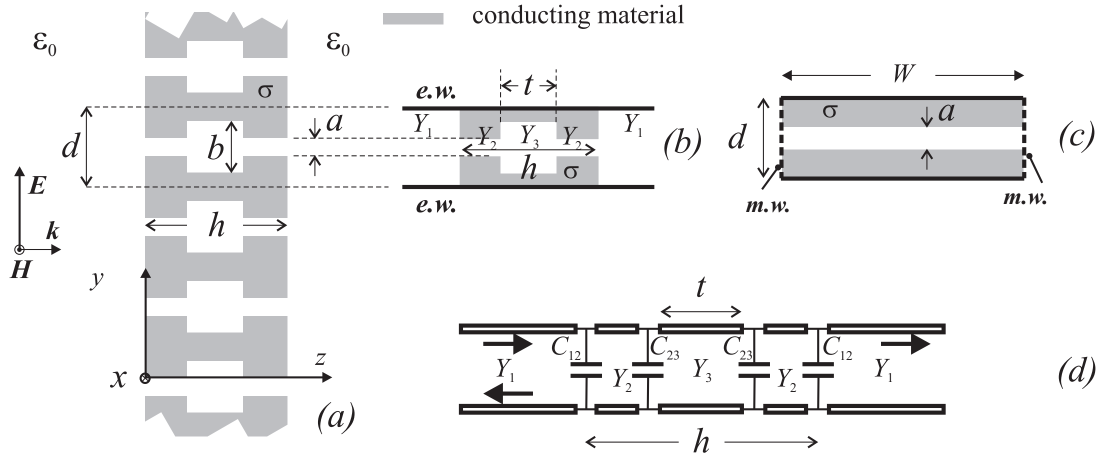

Fig. 1: (a) Cross section of a 1-D electrically thick diffraction grating with stepped sub-wavelength slits. (b) Equivalent unit cell for normal incidence. (c) Front view of the unit cell. (d) Proposed equivalent circuit.

3. Results

The full-wave analysis of the structure under consideration has been carried out by means of a conventional mode matching scheme. Finite elements method (HFSS) has also been used to test our numerical code. The numerical results have been compared with analytical results coming from the solution of the elemental circuit in Fig. 1(d). A couple of comparisons are depicted in Fig. 2. These figures show perfect agreement between numerical and analytical data as well as the ability of controlling the transmission spectrum by tuning the internal groove in the slit. It is interesting to note that frequency-independent capacitances can be used provided that the operation frequency is not close to the Rayleigh-Wood (RW) anomaly frequency. In the frequency range close to the RW anomaly, closed-form frequency-dependent expressions have been used.

4. Conclusion

One dimensional diffraction gratings using stepped slits have been analyzed using an analytical circuit model. All the parameters of such circuit have been obtained in closed form. This method simplifies the design of devices based on these geometries avoiding lengthy numerical calculations. Optimization codes can be applied if analytical models are used.

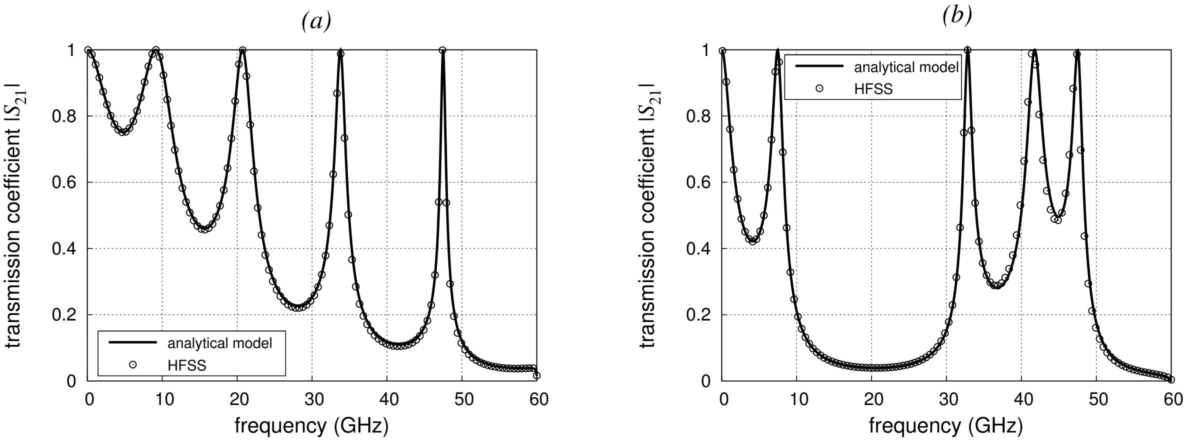

Fig. 2: Transmission spectra of two gratings with the following dimensions (unit cell parameters): (a) $d = 5\,\text{mm}$, $t = 10\,\text{mm}$, $h = 10.2\,\text{mm}$, $a = 0.5\,\text{mm}$, $b = 2.5\,\text{mm}$. (b) $d = 5\,\text{mm}$, $t = 4\,\text{mm}$, $h = 10.0\,\text{mm}$, $a = 0.5\,\text{mm}$, $b = 2.5\,\text{mm}$.

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