# Compact nonreciprocal optical dividers based on 2D photonic crystals 

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#### Abstract

We suggest and analyze a new nonreciprocal optical device based on 2D photonic crystals which fulfills simultaneously two functions: division of the input signal and isolation of the input port from two output ones.


## 1. Introduction

One of the ways to create highly integrated optical systems are photonic crystal chips. During the past decade, various components based on photonic crystals were suggested. Among them are three-port circulators based on microcavities [1], two-way dividers [2], and others. In this work, we investigate a possibility of further integration of photonic crystal chips, namely, we suggest a new nonreciprocal device which fulfills simultaneously two functions: division of optical signals and isolation of the input channel from two output ones. This will allow one to reduce the dimensions of optical subsystems.

## 2. General requirements to nonreciprocal divider and its idealized scattering matrix

We restrict ourselves by nvestigation of the nonreciprocal divider (ND) based on 2D photonic crystal structures. The minimum number of ports of the projected two-way divider is 4: input port 1 ; two output ports where the input signal is divided (numbered for example by 2 and 3 ); and an additional port 4 where the signals due to undesirable reflections in ports 2 and 3 are collected so that the input port 1 is isolated from output ports 2 and 3. The division of the input power between output ports can be in principle in any proportion, but for simplicity we shall consider a 3-dB divider. Firstly, we shall describe the divider using the scattering matrix in terms of power, i.e. we exclude from consideration phases of the scattering matrix elements. We assume also that our device is without losses. Thus, we can put the following restrictions on the elements of the 4 x 4 scattering matrix $[P]: P_{21}=P_{31}=1 / 2$. We shall also consider the ideally matched device with $P_{i i}=0, i=1,2,3,4$. We want also isolation of the output ports from input one: $P_{12}=P_{13}=0$. Using group-theoretical arguments, we will show below that for the device with antiplane of symmetry $P_{24}=P_{21}, P_{34}=P_{12}$ and $P_{43}=P_{21}$. Finally, from unitary property we obtain $P_{32}=1 / 2$ and $P_{42}=1 / 2$. Thus, the ideal matching, the division and isolation conditions, symmetry and unitary property allow us to write the desired scattering matrix as follows:

$$
[P]=\left(\begin{array}{cccc}
0 & 0 & 0 & 1  \tag{1}\\
1 / 2 & 0 & 1 / 2 & 0 \\
1 / 2 & 1 / 2 & 0 & 0 \\
0 & 1 / 2 & 1 / 2 & 0
\end{array}\right)
$$

The matrix (1) was obtained without any premise assumption of symmetry of the four-port. One can easily verified that this matrix has an antiplane of symmetry $T \sigma$. We can see a peculiarity of the above matrix: output ports 2 and 3 are not completely decoupled from each other. A half of the power goes from 2 to port 3 and vice versa.

## 3. Symmetry analysis

We choose the direction of uniform magnetization by a dc magnetic field $\mathbf{H}_{0}$ along the axis $z$ which is perpendicular to the plane xoy of a photonic crystal. It can be shown that for 2D structure of the fourport, the possible elements of symmetry are two-fold $\left(C_{2}\right)$ and four-fold $\left(C_{4}\right)$ rotations around the z -axis and planes of symmetry $\sigma$ combined with the Time reversal operator $T$, i.e. $T \sigma$. It can be also shown that the symmetries $C_{4}$ and $C_{2}$ do not serve for our purposes. Thus, the only symmetry which can give the desired scattering matrix is the antiplane of symmetry $T \sigma$. Two orientations of the ports with respect to the antiplane are shown in Fig. 1a and Fig. 1b. Using the commutation relations $\overline{\mathbf{R}} \cdot \overline{\mathbf{S}}=\overline{\mathbf{S}}{ }^{t} \cdot \overline{\mathbf{R}}$ for the antiuntary element $T \sigma$ we calculate the scattering matrices in terms of the voltages (see Table 1).


Fig. 1: Possible geometries of nonreciprocal four-ports. Continuous thin lines with arrows show desired division of incident wave, dotted lines with arrows demonstrate directions of reflected waves.

Table 1. Scattering matrices of NDs with different orientation of ports

| Figure | Fig. 1a | Fig. 1b |
| :---: | :---: | :---: |
| Matrix [S] | $\left(\begin{array}{llll}S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{13} \\ S_{31} & S_{32} & S_{22} & S_{12} \\ S_{41} & S_{31} & S_{21} & S_{11}\end{array}\right)$ | $\left(\begin{array}{cccc}S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{12} \\ S_{31} & S_{23} & S_{33} & S_{13} \\ S_{41} & S_{21} & S_{31} & S_{11}\end{array}\right)$ |

## 4. Simulation results

We shall consider transverse electric (TE) mode of the connected waveguides with the components $H_{z}, E_{x}, E_{y}$. Parameters of 2D crystal are as follows: hexagonal lattice of air holes of radius 0.3 a, $a=0.48 \mu \mathrm{~m}$ is the period of lattice. Magnetic material is bismuth iron garnet (BIG) material with refractive index $\mathrm{n}=2.5$. The magnetic media permittivity tensor of the BIG magnetized in $z$-direction has the diagonal element $\epsilon=6.25$ and the nondiagonal one $k=0.3$, the Voigt parameter is $k / \epsilon=0.048$. As a starting point for the ND simulation, one can choose for a given central frequency a resonator without magnetization which provides the resonance of lowest (dipole) rotating modes [3]. Then, connecting four waveguides to the magnetized resonator in the way discussed above one can try to realize the non-
reciprocal divider. Below, we show the calculated characteristics of the divider obtained by software COMSOL.


Fig. 2: Frequency responses of 3 dB divider excited at port 1 (a), port 2 (b) and port 3 (c).

## 5. Conclusion

Using the theory of magnetic groups, we have shown that in 2D geometry with homogeneous magnetization the only element admissible for the nonreciprocal divider is antiplane of symmetry. By numerical simulations we have shown realizability of the nonreciprocal divider based on 2D photonic crystals.

## References

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