Demonstration of enhancing second harmonic generation with doubly resonant metamaterial

T. Nakanishi[1], **T. Kanazawa, Y. Tamayama, and M. Kitano**

Department of Electronic Science and Engineering, Kyoto University, Kyoto 615-8510, Japan
[1]Fax: +81-753832324; email: t-naka@kuee.kyoto-u.ac.jp

Abstract

We investigate the enhancement of second harmonic (SH) generation in doubly resonant metamaterial. The unit cell consists of two coupled resonators, one of which resonates at the fundamental frequency, whereas the other resonates around the SH frequency. We observe that the SH generation in the doubly resonant metamaterial is 4.6 times as large as that in a singly resonant metamaterial in microwave frequency region.

1. Introduction

Resonant metamaterials, such as split ring resonators or cut-wire structures, effectively store electromagnetic energy in the constituents of the metamaterials. Pendry et al. predicted that if a nonlinear material is introduced into the places where the field is intensified, the nonlinear behavior of the material is enhanced [1]. This phenomenon has been applied to second harmonic (SH) and higher-order harmonic generations [2, 3, 4, 5, 6], nonlinear tunable metamaterials [4, 7, 8], and bistable media [7].

We propose a more efficient method of generating SH waves in a resonant metamaterial, as compared to previously proposed methods. We demonstrate that the SH generation can be enhanced by using composite metamaterial composed of two resonators: primary resonator designed for the fundamental frequency; secondary resonator designed for the SH frequency. The primary resonator effectively receives the incident electromagnetic waves and converts the frequency owing to the nonlinearity introduced in the structure. Furthermore, the second resonator contributes to the effective radiation of the SH waves stored in the metamaterial owing to the resonance effect.

2. Circuit model

A singly resonant metamaterial, such as split ring resonators, cut-wire structures, can be modeled as an inductor-capacitor-resistor series resonant circuit, as shown in Fig. 1(a). The voltage across the capacitor,

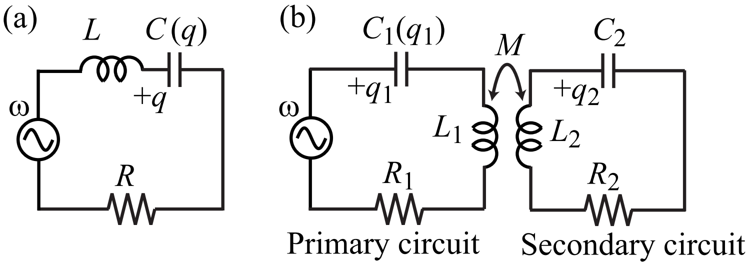

Fig. 1: (a) Singly resonant circuit and (b) proposed doubly resonant circuit for enhancing the SH current.

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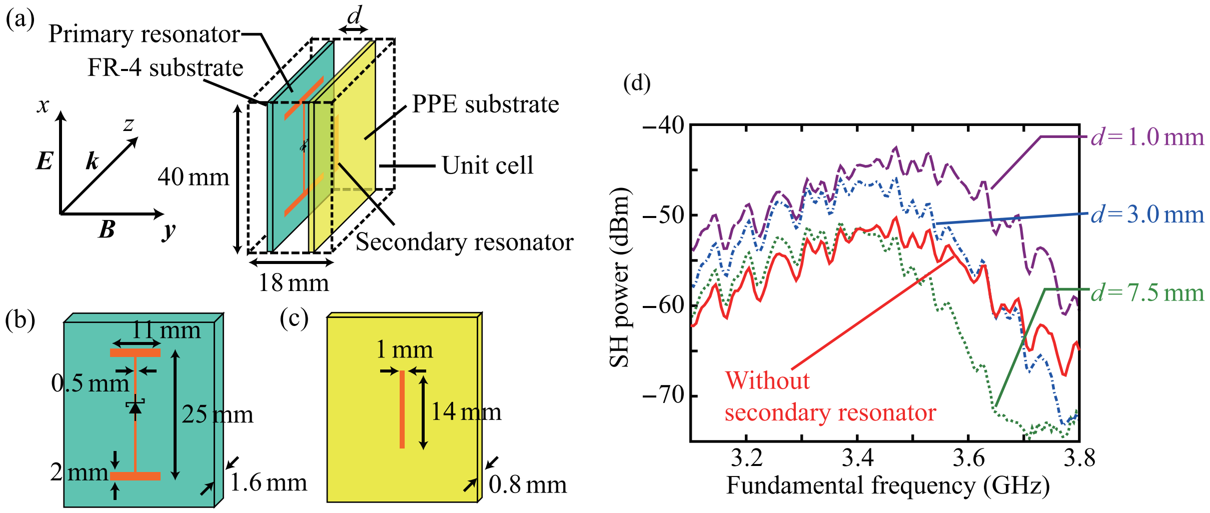

Fig. 2: (a) Unit cell of doubly resonant metamaterial. (b) Primary resonator. (c) Secondary resonator. (d) Measured SH power radiated from the doubly resonant metamaterial for different d.

v_C, reaches a maximum at the resonant frequency $\omega_0 = 1/\sqrt{LC}$. If the capacitor exhibits nonlinearity, $v_C = q/C(q) = q/C + \alpha q^2$, a large nonlinear effect is expected at ω_0. Using the second-order perturbation method under a weak nonlinearity condition, the SH current is obtained as

$$|I(2\omega)| = \frac{|\alpha|\, V^2}{\omega^2\, |Z(2\omega)Z(\omega)^2|}, \tag{1}$$

where $Z(\omega) = R - \mathrm{i}\left[\omega L - 1/\left(\omega C\right)\right]$ is the impedance of the circuit. When the circuit resonates, $\omega = \omega_0$, $|Z(\omega)|$ takes a minimum value and $|I(2\omega)|$ is maximized. As a result, the strong SH signal is generated from the singly resonant metamaterial. Here, we can expect that $|I(2\omega)|$ is further enhanced by also reducing $|Z(2\omega)|$ in some way.

We introduce a doubly resonant circuit shown in Fig. 1(b) for the enhancement of the SH current. This circuit is composed of two resonant circuits: the primary circuit with resonant frequency $\omega_1 = 1/\sqrt{L_1 C_1}$ and the secondary circuit with resonant frequency $\omega_2 = 1/\sqrt{L_2 C_2} \simeq 2\omega_1$. These resonators are coupled via a mutual inductance M. Applying the second-order perturbation method under a weak-coupling approximation, i.e., $|Z_1(\omega)Z_2(\omega)| \gg \omega^2 M^2$ and $|Z_1(2\omega)Z_2(2\omega)| \gg 4\omega^2 M^2$, the current flowing through each resonator can be approximated as follows:

$$|I_1(2\omega)| \approx \frac{|\alpha|\, V^2}{\omega^2\, |Z_1(2\omega)Z_1(\omega)^2|}, \quad |I_2(2\omega)| \approx \frac{|\alpha M|\, V^2}{\omega |Z_1(2\omega)Z_2(2\omega)Z_1(\omega)^2|}, \tag{2}$$

where $Z_i(\omega) = R_i - \mathrm{i}\left[\omega L_i - 1/\left(\omega C_i\right)\right]$ $(i = 1, 2)$. The SH current flowing in the primary circuit is independent of M and identical to that in the case of the singly resonant circuit. The SH current in the secondary circuit is proportional to M. We define the current ratio as $\beta \equiv |I_2(2\omega)/I_1(2\omega)| \approx |\omega M/Z_2(2\omega)|$. When $\beta \gg 1$, the SH radiation power can be enhanced approximately by a factor of β^2 compared with the case of the singly resonant metamaterial.

3. Experiments

Fig. 2(a) illustrates the unit cell of the doubly resonant metamaterial we used in the experiments. The primary resonator shown in Fig. 2(b) was fabricated on an FR-4 glass-epoxy printed circuit board of thickness 1.6 mm with a 35 μm-thick copper layer. A Schottky diode (Rohm RB886G) was loaded at the center of the I-shaped cut-wire structure. The periodic cascade of structures can be regarded as a series inductor-capacitor resonant circuit. The primary resonator resonates at 3.4 GHz. The secondary resonator shown in Fig. 2(c) was made of a 35 μm-thick copper film on a polyphenylene ether (PPE) substrate having a thickness of 0.8 mm. The secondary resonator was designed to resonate at 7.5 GHz considering the coupling M and the quality factors. The two resonators were separated from each other by distance d. For the experimental demonstration of the enhancement of the SH generation, we performed transmission measurements of the doubly resonant metamaterial. The mono-layer doubly resonant metamaterial was placed in a parallel plate waveguide composed of two copper plates parallel to the yz plane with a separation of 40 mm. The copper plates are equivalent to periodic boundaries because the electromagnetic fields are uniform in the x direction. A pair of ultra-wide band dipole antennas was used as a transmitter and a receiver. The transmitting antenna was excited by a signal generator and the receiving antenna was connected to a spectrum analyzer.

Fig. 2(d) shows the measured power of the SH wave generated from the doubly resonant metamaterial for three different distances d. The SH power generated from the metamaterial without the secondary resonator is also shown by the red line. In the absence of the secondary resonator, the radiated SH power reaches a peak around the fundamental frequency of $3.4\,\mathrm{GHz}$, which is the resonant frequency of the primary resonator. Focusing on the SH signal around the fundamental frequency $3.4\,\mathrm{GHz}$, the observed SH power increases with decreasing d. The radiated SH power for $d = 1.0\,\mathrm{mm}$ is 4.6 times as large as that in the case of the metamaterial without the secondary resonator.

4. Conclusion

We proposed the efficient method of generating SH waves by introducing additional component, which resonates at the SH frequency. We utilized composite metamaterial composed of two types of the cut-wire structures, and observed 4.6 times enhancement of the SH radiation at maximum. In this paper, we showed the results for the case where the structure of each resonator was fixed. By considering the shift in the resonant frequency due to strong coupling, or optimizing the shapes of the resonators for higher quality factor, we could expect further enhancement.

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