

# Dimer and polymer metamaterials with both electric and magnetic coupling

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## Abstract

We tailor actively the dispersion curves of diatomic metamaterials with alternating electric and magnetic coupling and show that in 'dimer' metamaterials (coupling within the unit cell dominates) two resonance modes of the unit cell widen into two pass bands of slow waves, whereas in 'polymer' metamaterials (the coupling between the unit cells being as strong as the coupling within the cell) a single pass band with entirely new properties appears, including a zero-phase-velocity, non-zero-group-velocity state. Experimental and numerical verification is provided and possible applications are discussed.

## 1. Introduction

Magnetoinductive waves [1] (also referred to more recently as magnetization waves [2] and magnetic plasmons [3]) propagate on chains of magnetically coupled metamaterial elements. They were proved to exist on arrays of magnetic 'metaatoms' in a broad frequency range from MHz to the visible, with applications including guiding and processing of signals on a subwavelength scale, detection of nuclear magnetic resonance, and subwavelength imaging [4]. Recently, we introduced 'diatomic' metamaterials with pure magnetic coupling [5]. Such structures with two metamaterial 'atoms' per unit cell are analogous to diatomic solids capable of propagating both acoustic and optical phonons. In a subsequent work [6], we proposed a diatomic chain the interesting feature of which is that the coupling alternates between electric and magnetic. Typical for diatomic chains, the dispersion characteristic was shown to have two wide pass bands, above and below the resonant frequency of the split rings. In contrast to previously studied diatomic chains, however, the lower band was a backward wave and the upper band was a forward wave. In this paper we extend the analysis of diatomic metamaterials with alternating electric and magnetic coupling, to two distinct cases, with electric and magnetic coupling being either different or equal to each other in strength. We shall show that these two types of structures exhibit entirely different behaviour and confirm theoretical predictions by our experiments. Finally we shall discuss potential applications.

## 2. Dimer and polymer chains

In metamaterials comprising split ring resonators (SRRs), the type of coupling (electric or/and magnetic) depends crucially on the mutual orientation of the rings and on the distance between them [7]. In particular, for a dimer (a pair of split rings) in the 'near' configuration, with the gaps facing each

other, the dominant coupling is electrical and positive, whereas in the ‘far’ configuration, with the gaps being on opposite sides, the dominant coupling is magnetic and negative, so that in a diatomic chain formed from these dimers the coupling alternates between electric and magnetic, see Fig.1.

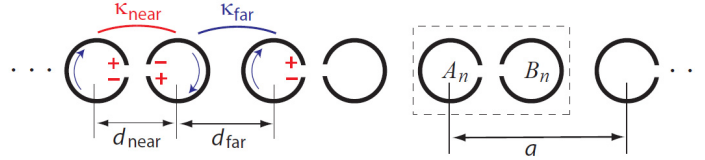


Fig. 1. Diatomic chain with alternating electric ( $\kappa_{\text{near}}$ ) and magnetic ( $\kappa_{\text{far}}$ ) coupling.

Assuming slow waves propagating by virtue of coupling along such a chain, with currents in both sublattices being  $A_n = A \exp[i(kan - \omega t)]$  and  $B_n = B \exp[i(kan - \omega t)]$  (see Fig.1) with complex amplitudes,  $A$  and  $B$ , and a wave number  $k = k' + ik''$  we obtain the dispersion equation in the form

$$4\zeta^2 - \kappa_{\text{far}}^2 - \kappa_{\text{near}}^2 v^4 + 2\kappa_{\text{far}}\kappa_{\text{near}} v^2 \cos ka = 0 \quad (1)$$

and the relationship between the amplitudes of currents within the same unit cell as

$$\frac{B}{A} = -\frac{2\zeta}{\kappa_{\text{far}} e^{-ika} + \kappa_{\text{near}} v^2}. \quad (2)$$

$\zeta = 1 - v^2 + i\nu/Q$  denotes the self-impedance of the SRR normalized to  $\omega L$ ,  $\omega$  is the frequency,  $L$  is the self-inductance,  $\nu = \omega_0/\omega$  is the reciprocal frequency normalized to the resonant frequency  $\omega_0$ , and  $Q$  is the quality factor.

We shall now make a distinction between two cases, the first one with the electric coupling dominating over the magnetic coupling,  $|\kappa_{\text{near}}| \gg |\kappa_{\text{far}}|$  and the second one with the coupling being constant in magnitude but alternating in sign from element to element,  $|\kappa_{\text{near}}| = |\kappa_{\text{far}}|$ . The corresponding dispersion curves following from Eq.(1) are shown in Figs.2a and b, with the insets showing the phasors of the magnetic field  $\{\Re(H), \Im(H)\}$  at the centre of the rings in subsequent elements. They are calculated analytically at a few typical points of the dispersion curve, at  $ka = 0, \pm\pi/2, \pm\pi$ . For clarity of presentation, losses are disregarded in both examples.

It is possible to interpret the case of Fig. 2a, with the dominant electric coupling, as a chain of electric dimers, each consisting of two SRRs coupled electrically and having two eigenfrequencies, the lower one (with currents in the two rings being in phase) and the upper one (in antiphase). At each of the two frequencies there is a pass band of the magnetoinductive wave due to the negative magnetic coupling between the dimers so the lower branch is a backward wave, whereas the upper branch is a forward wave, which is in agreement with results of Ref. [8]. The phasors of the magnetic field on the insets in Fig. 2a confirm fully this qualitative picture.

In the case of Fig.2b with  $|\kappa_{\text{near}}| = |\kappa_{\text{far}}|$ , the dispersion curve changes dramatically. The stop band at resonance closes up, and at the resonant frequency a mode can propagate with zero phase velocity but non-zero group velocity. The phasors of the magnetic field along the chain can be seen to differ dramatically from those for the dimer line of Fig.2a. The phase between two elements of the dimer change gradually from zero to  $\pi$  as the frequency increases from the bottom to the top of the pass band. Therefore, this configuration could be seen rather as a ‘polymer’ that has limited memory of the properties of the dimer.

In the presentation we shall show in more detail the predictions of our theoretical model and demonstrate good agreement between theory, experiments and numerical results. Applications, that include leaky-wave antennas and metamaterial based parametric amplifiers, will also be discussed in a broad frequency range from microwaves to visible frequencies.

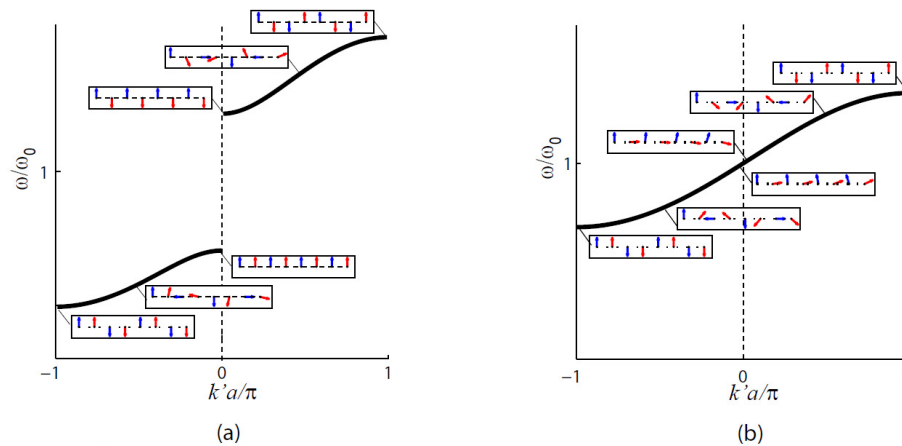


Fig. 2. Dispersion for (a) a dimer chain ( $|\kappa_{\text{near}}| \gg |\kappa_{\text{far}}|$ ) and (b) a polymer chain ( $|\kappa_{\text{near}}| = |\kappa_{\text{far}}|$ ).

### 3. Conclusions

Properties of dimer and polymer metamaterials with alternating electric and magnetic coupling are described analytically, and verified experimentally and numerically. Possible applications are discussed.

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