Resonant three-wave interaction in the nonlinear anisotropic dielectric slabs

O.V. Shramkova, A. G. Schuchinsky

School of Electronics, Electrical Engineering & Computer Science, Queen's University Belfast
Queen's Road, Queen's Island, Belfast BT3 9DT, U.K.
Fax: +44-28-90971702; email: o.shramkova@qub.ac.uk

Abstract

The nonlinear interaction of waves in the anisotropic dielectric slabs illuminated by the plane waves of two tones is examined. A special case of Wolf-Bragg resonances at the combinatorial frequency is analysed analytically and numerically. The dependencies of the intensities of the reflected and transmitted waves of combinatorial frequencies on the layer thickness are studied.

1. Introduction

Nonlinear artificial materials and metamaterials have recently attracted increasing interest [1-3] owing to their unique functional capabilities at millimeter, terahertz (THz) and optical frequencies. Although the properties of an artificial material are determined by the microscopic features of the constituent particles and their arrangements, the macroscopic response of the medium can be described by the effective constitutive parameters, when the feature size of the unit cell is much smaller than the wavelength. Since majority of the currently available metamaterials are fabricated as stacked layers, it is expedient to investigate their nonlinear characteristics in the planar layered structures. Such an approach often provides insight in the fundamental mechanisms and phenomenology of the distributed nonlinear wave interactions. The recent works in this area have been primarily concerned with the second and third harmonic generation in dielectric layers [4]. However the problems of distributed frequency mixing and the related processes of nonlinear scattering by layers and films illuminated by two or more plane waves of different frequencies incident at different angles still remain scarcely studied and poorly understood. In this work we address these problems and examine Wolf-Bragg resonances at the combinatorial frequency in a weakly nonlinear anisotropic dielectric layer illuminated by the plane waves of two tones.

2. Nonlinear scattering by anisotropic dielectric layer

Let two plane waves of frequencies ω_1 and ω_2 be incident at angles Θ_{i1} and Θ_{i2} on a nonlinear dielectric layer of thickness L surrounded by a homogeneous linear medium (Fig.1).

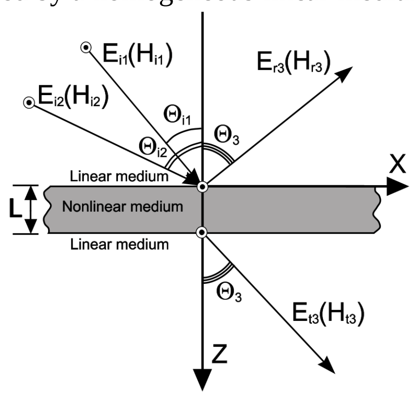

Fig.1: Geometry of the problem.

A layer with 6mm class of anisotropy is described by tensors of linear dielectric permittivity $\hat{\varepsilon} = (\varepsilon_{xx}, \varepsilon_{xx}, \varepsilon_{zz})$ and a second-order nonlinear susceptibility tensor $\hat{\chi}$. Since the layer is isotropic in the *xy* plane, the TE and TM waves can be analysed separately assuming the fields to be independent of the *y*-coordinate. We consider only the case of TM-polarization, whereas the TE case is somewhat simpler being unaffected by the anisotropy of $\hat{\chi}$. The fields in the nonlinear layer can be expressed as a superposition of 6 waves, which vary with *z* as $\exp(\pm i k_{zL3} z)$ and $\exp(\pm i k_{zL}^{\pm} z)$, where $k_{zL}^{\pm} = k_{zL1} \pm k_{zL2}$ and $k_{zL1,zL2,zL3} = \sqrt{\left(k_{1,2,3}^2 - \frac{k_{x1,x2,x3}^2}{\varepsilon_{zz}}\right)\varepsilon_{xx}}$ are the transverse components of the refracted waves in the layer, $k_{x1,x2,x3} = \frac{\omega_{1,2,3}}{c}\sqrt{\varepsilon_a}\sin\Theta_{i1,i2,i3}$, ε_a is the dielectric permittivity of the linear homogeneous medium. It should be noted that the wave vector components at the combinatorial frequency $\omega_3 = \omega_1 + \omega_2$ is determined by the requirement of the phase synchronism in the three-wave mixing process

$$k_{x3} = k_{x1} + k_{x2}. \tag{1}$$

Intensities $|F_{r,t}|^2$ of the waves of frequency ω_3 outside the layer emitted in backward (z<0) and forward (z>0) directions depend on the layer thickness L. The case of Wolf-Bragg resonance when L equals an integer number of half-waves ($k_{zL3}L = \pi q,\ q = 0, \pm 1, \pm 2...$) is of particular interest. In this case the expressions for $F_{r,t}$ can be simplified and take the form

$$\begin{aligned} F_r &= \frac{1}{2}\sum_{n=\pm}\left[(-1)^q\left(1-\frac{\varepsilon_a k_{zL}^n}{\varepsilon_{xx}k_{za3}}\right)N_1^n - e^{-ik_{zL}^n L}\left(1+\frac{\varepsilon_a k_{zL}^n}{\varepsilon_{xx}k_{za3}}\right)N_2^n\right]\left(1-(-1)^q e^{ik_{zL}^n L}\right) \\ F_t &= -\frac{1}{2}\sum_{n=\pm}\left[(-1)^q\left(1+\frac{\varepsilon_a k_{zL}^n}{\varepsilon_{xx}k_{za3}}\right)N_1^n - e^{-ik_{zL}^n L}\left(1-\frac{\varepsilon_a k_{zL}^n}{\varepsilon_{xx}k_{za3}}\right)N_2^n\right]\left(1-(-1)^q e^{ik_{zL}^n L}\right) \end{aligned}, \tag{2}$$

where $k_{za3} = \sqrt{k_3^2\varepsilon_a - k_{x3}^2}$ is the longitudinal wave number of the wave at frequency ω_3 in the surrounding homogeneous media, ε_a is the dielectric permittivity of the linear homogeneous medium.

The analysis of (2) shows that $|F_{r,t}|$ possess nulls and absolute maxima when the following relations are satisfied simultaneously

$$q\pi - k_{zL}^{\pm}L = m^{\pm}\pi, \qquad m^{\pm} = 1, 2, ... \tag{3}$$

where m^+ and m^- have the same parity (nulls occur at even $m^\pm$ and maxima - at odd $m^\pm$). Since the nonlinear layer acts as a resonant source of the waves emitted into surrounding homogeneous medium at frequency ω_3, both F_r and F_t reach their extremes concurrently. The frequencies $\omega_{1,2}$, corresponding to the nulls and absolute maxima of $|F_{r,t}|$, have been expressed in terms of the layer parameters in closed form.

3. Simulation results

Based upon the analytical estimates of the $|F_{r,t}|^2$ nulls and maxima at frequency ω_3, the $|F_{r,t}|^2$ have been simulated in a broad range of the layer thicknesses L. Fig. 2 demonstrates that at $\omega_1/\omega_2 = 1.944$, $|F_{r,t}|^2$ attain the global maxima at the incidence angles $\Theta_{i1} = 15°$, $\Theta_{i2} = 40°$. $|F_{r,t}|^2$ reaches peak values of at $L = 2$ mm ($q = 308$, $m^+ = 1$, $m^- = 205$), $L = 6$ mm ($q = 924$, $m^+ = 3$, $m^- = 615$), and has the nulls at $L = 4$ mm ($q = 616$, $m^+ = 2$, $m^- = 410$) and $L = 8$ mm ($q = 1232$, $m^+ = 4$, $m^- = 820$). The fine details of $|F_{r,t}(L)|^2$ behaviour in the proximities of the global maximum and minimum are shown in Fig. 3. It can be observed that these dependencies are asymmetric. This fact can be attributed to the amplitude and phase disbalance of the pump waves inside the layer due to the difference of their reflection coefficients $R(\omega_{1,2})$. The $|F_{r,t}|^2$ maxima correspond to the layer thicknesses for which $|R(\omega_1)|^2 = |R(\omega_2)|^2 = 0$.

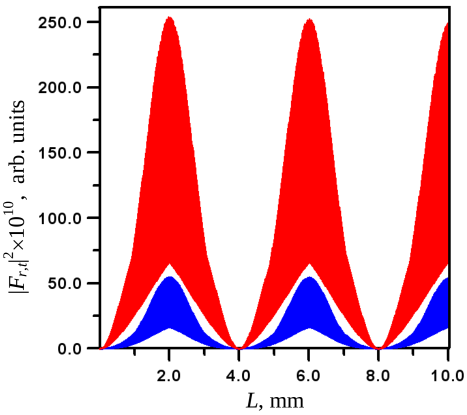

Fig. 2: Intensity of the field at frequency ω_3 radiated from the layer of thickness L in the reverse ($|F_r|^2$ - blue line) and forward ($|F_t|^2$ - red line) directions of the z-axis at Θ_{i1} =15°, Θ_{i2} = 40° and $\omega_1 = 4.195 \cdot 10^{13}\ s^{-1}$, $\omega_2 = 2.158 \cdot 10^{13}\ s^{-1}$, $\varepsilon_{xx} = 5.382$, $\varepsilon_{zz} = 5.457$, $\chi_{xxz} = 2.1 \cdot 10^{-7}$, $\chi_{zxx} = 1.92 \cdot 10^{-7}$, $\chi_{zzz} = 3.78 \cdot 10^{-7}$, $\varepsilon_a = 1$

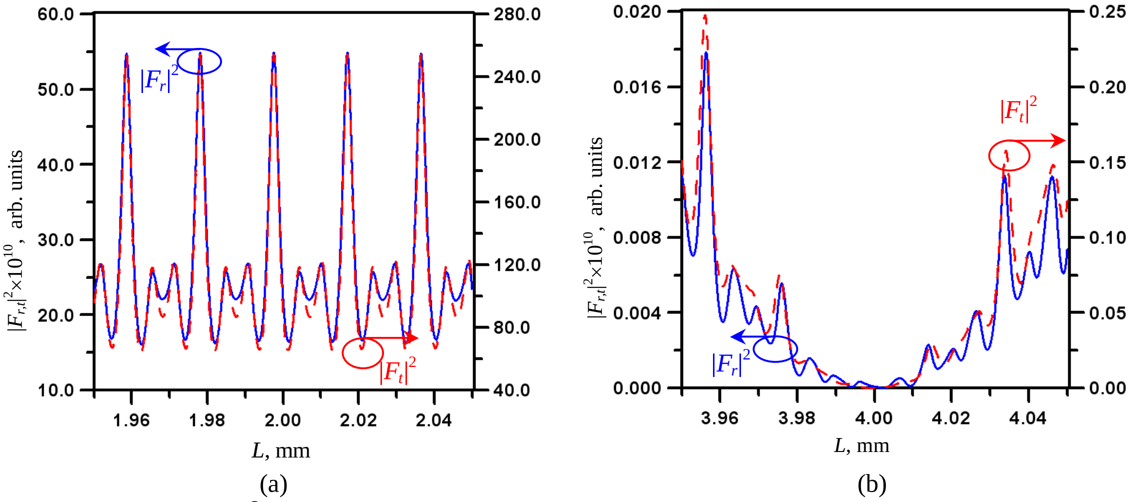

Fig. 3: Details of $|F_{r,t}(L)|^2$ at frequency ω_3 in the vicinities of maximum at L = 2 mm (a) and minimum at L = 4 mm (b). $|F_r|^2$ - solid lines and $|F_t|^2$ - dashed lines.

4. Conclusion

The nonlinear scattering of two plane waves of frequencies ω_1 and ω_2 incident onto anisotropic dielectric slabs with nonlinear permittivity has been analysed. It has been shown that Wolf–Bragg resonances of very high orders may lead to the global maxima and nulls of the scattered field at the combinatorial frequency $\omega_3 = \omega_1 + \omega_2$. It has been demonstrated that the intensity of the emitted field at the global maxima can exceed that in the low-order Wolf-Bragg resonances for several orders of magnitude thus providing the conditions for giant enhancement of the nonlinear scattering effects.

References

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