Highly Non-Linear RF Metamaterials

M.C.K.Wiltshire[1] **and D. R. Smith**[2]

[1]Condensed Matter Theory Group, Department of Physics, Imperial College London,
London SW7 2AZ, UK, *and* Robert Steiner MRI Unit, Imaging Sciences Department, Imperial College London, Hammersmith Hospital Campus, Du Cane Road, London W12 0NN, UK
Email: michael.wiltshire@imperial.ac.uk

[2]Duke University Center for Metamaterials and Integrated Plasmonics, 101 Science Drive,
2529 CIEMAS Building, Durham, North Carolina 27708, USA
Email: drsmith@ee.duke.edu

Abstract

We report on the development and characterisation of nonlinear, varactor-loaded Swiss Roll metamaterial elements. Two systems are considered: one with single varactors and the other with two, back-to-back varactors. Both types of element show strong nonlinear behaviour, as demonstrated by their harmonic generation. The single varactor system shows power-dependent response, second and third harmonic generation, and large hysteresis effects. The double varactor system does not saturate or show strongly power dependent effects, and has only weak second harmonic generation, because it is a symmetric structure. However, the third harmonic is very strong in this system. We expect bulk material based on these elements to exhibit very large χ^2 and χ^3 effects.

1. Introduction

Nonlinear phenomena in natural materials are weak, restricted by their small nonlinear susceptibilities. Accordingly they require large powers and long interaction lengths to achieve observable effects. Metamaterials, on the other hand, have no such restrictions, and can be constructed to have large nonlinear coefficients [1,2]. In recent years, effort has been concentrated on incorporating nonlinear elements, typically varactors, into metamaterials based on the split ring resonator (SRR), and both power-dependent resonance frequencies [3], harmonic generation [4] and hysteresis [5] have been observed. Methods for obtaining reliable estimates of the nonlinear coefficients, χ^2 and χ^3, using plane wave excitation of bulk metamaterials have been investigated [6] and used to interpret microwave experiments. In this work, we report on the development of RF elements based on the Swiss Roll structure, which exhibit very large nonlinear responses.

The Swiss Roll metamaterial element, originally proposed by Pendry et al [7], has proved to be very suitable for radiofrequency (RF) metamaterials. It has a strong magnetic response [8], showing a high, resonant permeability $\mu \sim 100$, and can be made massively chiral [9], with a chirality exceeding the permeability. Furthermore, it can be widely tuned by the incorporation of a varactor element [10], with a usable tuning range of ~40%. All these characteristics suggest it should be a suitable candidate for investigating non-linear effects in RF metamaterials.

2. Varactor loaded Swiss Rolls

Two different types of varactor loaded elements were constructed as shown in Fig. 1. The first set, shown schematically in Fig. 1(a), contained a single varactor that acted as a voltage controlled capacitor, C_v, along with blocking capacitors, C_1, and a decoupling resistor, R_1. In later versions, decoupling

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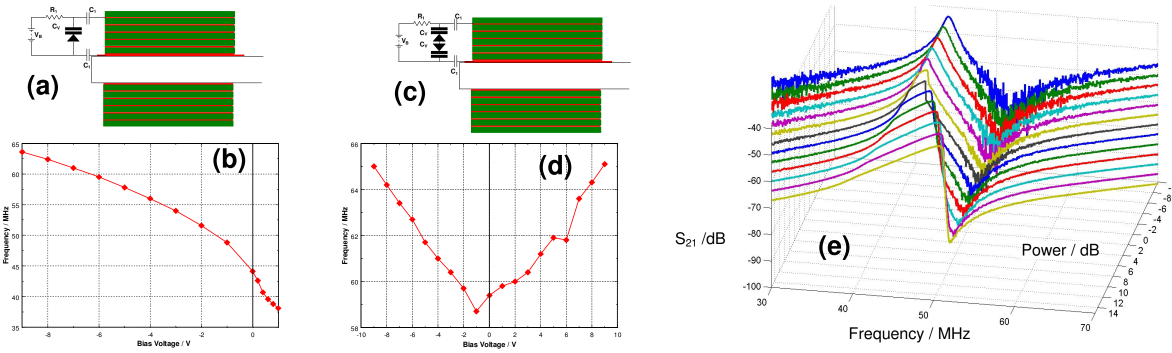

Fig. 1: (a) Varactor-loaded Swiss rolls element and (b) the tuning curve of resonant frequency vs bias voltage at low excitation power; (c) double varactor-loaded Swiss Roll element and (d) its tuning curve, showing approximately symmetric tuning for forward and reverse bias. (e) Transmission, S_{21}, vs frequency for a single varactor roll, with 0V bias applied, as a function of excitation power from -10dB to +14dB in 2dB steps. The saturation for excitation powers above about -2dB is evident.

resistors were placed on both leads, so that when they were assembled into bundles of elements, there was no effective return path for the RF signal through the ground lead. These elements are of the same form that were used in our tuning experiments [10], and their resonant frequency ranged from 40 MHz under 0.5 volts forward bias to 64 MHz for 9 volts reverse bias. Increasing the forward bias above 0.8V quickly led to conduction and loss of resonance, whereas the reverse bias could be increased to >30V when the resonant frequency saturated at 74 MHz. These elements are clearly very sensitive to applied voltage, see Fig. 1(b), and proved to be highly nonlinear. However, their saturation for even small amounts of forward bias leads to significant loss in the element, so a second diode, arranged back-to-back with the first diode [11], was used in a second set of elements, shown in Fig. 1(c). This arrangement can be driven into either forward or reverse bias, but has a smaller tuning range, because the two diodes act as capacitors in series, thus reducing their dynamic range. The tuning curve of the back-to-back diode samples is shown in Fig. 1(d), where the limited range of the (approximately) symmetric characteristic is clear.

3. Nonlinear behaviour

The behaviour of these materials was measured for both single elements and groups of elements, by placing the samples as a linking core between two coaxial loops and using these as the signal transmitter and receiver attached to a network analyser; here we report the single element behaviour. The first series of experiments recorded the transmission between the loops, S_{21}, as a function of bias voltage for low applied power; the peak positions provided the tuning curves in Fig. 1(b) and (d). In a second series of experiments, we recorded S_{21} for fixed bias as the excitation power from the network analyser was varied; this is shown in Fig. 1(e) for the case when $V_B = 0$, and the effect of the saturation of the varactor when the system is dynamically driven into forward bias, is clear for powers exceeding -2dB. If the reverse bias DC voltage is increased, this saturation point moves to higher incident power, as expected, and above $V_B = 3$V reverse bias, no saturation is observed. The double varactor system shows no saturation in these experiments, as there is always one element operating in reverse bias. Therefore, although the varactor in forward bias does add to the loss, the other diode provides capacitance and hence prevents breakdown.

The experiments were then moved to an Agilent PNA-X 5245 vector network analyser, so that harmonic generation could be investigated. In this instrument, the receiver channel can be set to measure signals at a fixed factor of (and, if required, off-set from) the transmitter channel frequency. It is therefore straightforward to make harmonic generation measurements: we scan the transmitter over some frequency range, say 30 – 70 MHz as in Fig. 1, and monitor the signal in the range 60 – 140 MHz for the second harmonic, or 90 – 210 MHz for the third harmonic. Some results are shown in

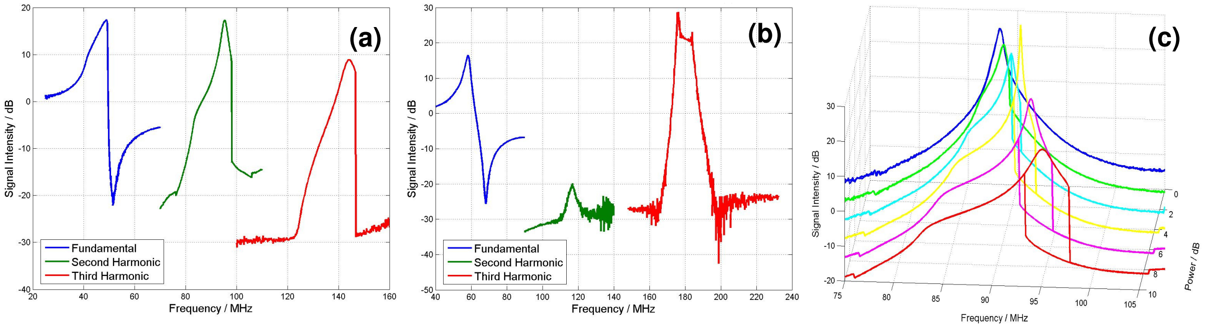

Fig. 2: Fundamental response and second and third harmonic signals for (a) a single varactor roll that shows very strong harmonic generation, and (b) a double varactor roll, which shows little second harmonic signal. The bias voltage was set to 0V and the excitation power was +10dB in both cases. (c) shows the hysteresis in the second harmonic for the single varactor roll as a function of excitation power for 0V bias. Hysteresis appears for excitation power > 4dB.

Fig. 2(a) and (b): here, both samples were excited using +10dB power, and the bias voltage was set to $V_B = 0V$.

There are three features in these data that should be pointed out. First, the harmonic signals are extremely strong. Although we have not attempted to obtain absolute values of the harmonic coefficients, it is clear from the low noise floor in both plots that the signals are large (note the logarithmic scale). Second, we note that the second harmonic signal for the double varactor system, shown in Fig. 2(b), is very weak. This is to be expected because the system has been symmetrised by the addition of the second varactor, so no even harmonic signals should appear. The odd harmonics are unaffected, and the third harmonic is correspondingly strong. Third, we can see clearly the saturation effects in Fig. 2(a), whereas the double varactor roll in Fig. 2(b) does not show these effects. The saturation affects the fundamental and both the second and third harmonics, and hysteresis measurements have been made on the second harmonic as a function of both power and bias, and these are shown in Fig. 2(c) for $V_B = 0V$. Here, hysteresis is apparent for excitation powers exceeding 4dB, opening out to about 10% for +10dB excitation power. The data for $V_B = 1V$ shows much reduced hysteresis, as expected.

4. Conclusion

This paper presents preliminary measurements of the non-linear behaviour of varactor-loaded Swiss roll metamaterial elements. Both single and back-to-back varactor configurations have been considered. The single varactor elements are readily saturated, and show large second and third harmonic generation, along with significant hysteresis. The double varactor elements do not saturate, and do not display second harmonic generation, but have a large third harmonic response. Assembling these elements into bulk material should provide a metamaterial with extremely large values of χ^2 and χ^3.

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