# Magnonic crystals and their metamaterials properties theoretical considerations

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#### Abstract

Magnonic crystals (MCs), or composite materials with periodically arranged magnetic inclusions, can have properties not found in bulk samples. In the long-wave limit the MC can be characterized by its effective continuous properties. We calculate the effective permeability of two-dimensional MCs to estimate their potential usage as negative index metamaterials.

#### **1** Introduction

The properties of spin waves (SWs) in magnonic crystals (MCs) are determined by interactions of different type: the short-range exchange interactions and the long-range dipolar ones. The effect of these two types of interactions on the spin-wave spectrum depends on the constituent materials and the scale of their modulation, i.e., on the lattice constant of the MC and the thickness of the MC film. For large lattice constant values the dispersion of low-frequency bands is determined by the magnetostatic interactions. With decreasing lattice constant the boundary of the first Brillouin zone (1BZ) will stretch, the band frequencies will shift up, and the exchange interactions will gradually gain in importance to finally prevail over the dipolar interactions.

Also the potential applications of the MC depend on the lattice constant. With decreasing modulation scale the spin-wave frequencies will grow as high as to the sub-terahertz range. This opens the door to applications of the dynamic properties of such MCs in devices operating in this range of electromagnetic radiation frequency, e.g., by modeling the effective response of the system.

## 2 Model and structure of magnonic crystal

We consider thin slabs of 2D magnonic crystals in the form of an antidot lattice (ADL), shown in Fig. 1(a), or a bi-component magnonic crystal, presented in Fig. 1(b), with ferromagnetic cylindrical dots embedded in a magnetic matrix. Using the plane wave method (PWM) [1], we calculate the frequencies of the magnetic excitations and the dynamic components of the magnetization vector, with the demagnetizing field taken into account. The calculations are performed in the linear approximation with damping neglected. The obtained dynamic components of the magnetization vector allow us to plot their profiles and calculate the relative intensities of the resonance absorption. We investigate the magnonic band structure, the magnonic modes and the relative absorption versus the structural parameters, including spontaneous magnetization, lattice constant and filling fraction.



Fig. 1: Thin film of MC in the form of (a) an ADL with a hexagonal lattice of holes in a magnetic matrix material; (b) a bi-component 2D magnonic crystal with the same symmetry as in (a). The structure is characterized by the lattice constant p, the hole/dot radius R, and the film thickness t. A static bias magnetic field  $H_0$  is oriented along one of the crystallographic axes, z-axis.

## **3** Results

Figure 2 shows sample magnonic band structures calculated for a permalloy ADL with holes arranged in a hexagonal lattice. The calculations were performed for two lattice constant values, p = 100 nm and 300 nm (plots (a), (b) and (c), (d), respectively) and for different hole radii R: (a) 30 nm, (b) 40 nm, (c) 60 nm, (d) and 100 nm. A very small MC thickness is assumed – 10 nm in (a), (b) and 24 nm in (c), (d) – to avoid the formation of standing waves across the film thickness. The external bias magnetic field  $H_0$  is applied along the z-axis, and strong enough to saturate the sample. The depicted magnonic band structures differ significantly. The largest magnonic band gap is seen in Fig. 2(b), where due to the small lattice constant (100 nm) of the MC and the large holes (R = 40 nm) the SW amplitude is confined to a small volume of the ferromagnetic material. A wide magnonic band gap extends from 21 GHz to 29 GHz and is delimited by flat magnonic bands. The frequency of the modes is seen to decrease and the width of the bands to increase with decreasing hole radius.

The increase in the lattice constant from 100 nm to 300 nm results in reduced magnonic band frequencies. For p = 300 nm the spectrum starts at 2 GHz and 6 GHz for 2R = 200 nm and 2R = 120 nm, respectively. Two narrow magnonic band gaps, between the 2<sup>nd</sup> and 3<sup>rd</sup> bands and between the 8<sup>th</sup> and 9<sup>th</sup> bands, are observed in the case of larger holes. In contrast, no gap is seen to occur in Fig. 2(c).

This suggests a magnetostatic nature of the low-frequency magnonic bands. The dispersion relation of the lowest mode in the direction perpendicular to  $H_0$ , i.e., along the  $\Gamma$ -M direction in the first Brillouin zone (shown in Fig. 2, top-right corner), resembles that of the Damon-Eshbach (DE) magnetostatic mode known from the spectrum of excitations in uniform thin films.

Comparing the evolution of the magnonic band structure with increasing hole size, we observe a completely different behavior of the spectra obtained for two different lattice constants. The bands shift up the frequency scale for p = 100 nm, and down for p = 300 nm; the group velocity of the first band increases for p = 100 nm and decreases for p = 300 nm. The observed dependence of the magnonic dispersion on the scale of material modulation in the MC is related to the type of predominating interactions. When the lattice constant is small (e.g., p = 100 nm) and the holes are relatively large the exchange interactions prevail, while for large values of p the magnetostatic interactions play the main role in the formation of magnonic bands.

Having established the interaction type predominating in the system, we calculate the effective parameters, namely the magnetization and the exchange constant, characterizing the 2D slab of MC as if it were a uniform film. The effective parameters are obtained by fitting the PWM results to analytic dispersion curves: the exchange SWs and the magnetostatic DE modes, for p = 100 nm and p = 300 nm, respectively. With the same method we can study the magnonic spectra of the bi-component MC obtained by filling the holes with a ferromagnetic material other than Py (see Fig. 1). The calculated effective parameters of the MC and the obtained SW mode profiles (Fig. 2(e)) can be used for determining the



Fig. 2: Magnonic band structure of the 2D ADL presented in Fig. 1(a), for two different values of lattice constant p: (a), (b) 100 nm and (c), (d) 300 nm. The antidot radius R is (a) 30 nm, (b) 40 nm, (c) 60 nm and (d) 100 nm. (e) The three highest-intensity spin-wave modes in the ADL corresponding to (d).

effective permeability of thin slabs of MC in the form of either an ADL or a bi-component structure with the method developed in [2]. This can allow to estimate the possibility of using the MCs as materials with a negative permeability.

# 4 Conclusions

We present a theoretical study of the magnonic band structure of 2D MCs in the form of a thin slab of Py with antidots, or a bi-component structure in Py. In this systematic theoretical research we find the structures and material compositions optimum for the opening of magnonic gaps in the spin-wave spectrum, and elucidate the role played in its formation by the exchange and magnetostatic interactions. Using the effective magnetic parameters found by a fitting procedure, we estimate the permeability of the MCs. We show that these properties may help obtain metamaterials with a negative permeability in a relatively high frequency range by properly designing the structure of the 2D MC.

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## References

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