Free space polarizability measurement method

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Abstract

A simple and precise method for free space polarizability measurement of electrically small particles is presented. The method is based on a measurement of the scattering parameters of a waveguide loaded by a particle, and on knowledge of the dipolar polarizabilities of two calibration standards.

1. Introduction

The polarizability and the geometrical structure of a lattice are the two key properties of any material, by which the constitutive parameters, i.e. permittivity and permeability, can be uniquely determined. Methods for calculating or measuring polarizability are thus essential for material sciences. In the case of artificial media, including metamaterials, the polarizability of particles has in most cases been obtained theoretically by analytical models or by numerical methods, see for example [1, 2, 3]. However, few methods for measuring polarizability have been proposed so far. Pioneering work on polarizability measurement has been done by Cohn [4]. Unfortunately, a particle is placed in an electrolyte in his method, and this is not applicable at high frequencies and for particles of Split Ring Resonator type, which will not work inside an electrolyte. Recently, two other methods [5, 6] have been proposed. The first obtains the particle polarizability from a measurement of the scattering parameters of a waveguide loaded by the particle, while the second uses a measurement of the scattering parameters of a two dimensional square array of identical particles. The two methods are very similar, and even coincide in the case of a square TEM waveguide made from two electric and two magnetic walls.

The aim of this contribution is to present a generalization of the waveguide measurement method. Unlike the original method, the new method is free of any knowledge of the waveguide geometry, its modes and their coupling with the analyzed particle. This is effectively eliminated by proper calibration, using particles of known polarizability.

2. Analysis and Results

Let us assume a particle that is small in comparison with the used wavelength. The interaction of this particle with an electromagnetic field is well described by the induced dipole moments. In the following, we will assume that the particle is excited by the fields E_{loc}, H_{loc}, which are, in a good approximation, homogeneous in the volume occupied by the particle. In addition, we will assume that the scatterer has

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negligible cross-polarization and that it is non-bianisotropic. In this case, the induced dipole moments are given by

$$\begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} \alpha^{\mathrm{ee}} & 0 \\ 0 & \alpha^{\mathrm{mm}}\mu_0 \end{bmatrix} \begin{bmatrix} E_{\mathrm{loc}} \\ H_{\mathrm{loc}} \end{bmatrix}, \tag{1}$$

where α^{ee}, α^{mm} are the electric and magnetic polarizability respectively.

If this particle is inserted into a mono-mode waveguide (all higher order modes are evanescent), it will be excited and will scatter the impinging wave. A cell of this waveguide of infinitesimal length with the inclusion inside is sketched in Fig. 1a, where the particle is already described by its equivalent dipole moments. If the dominant mode of the waveguide is such that in the volume occupied by the particle the

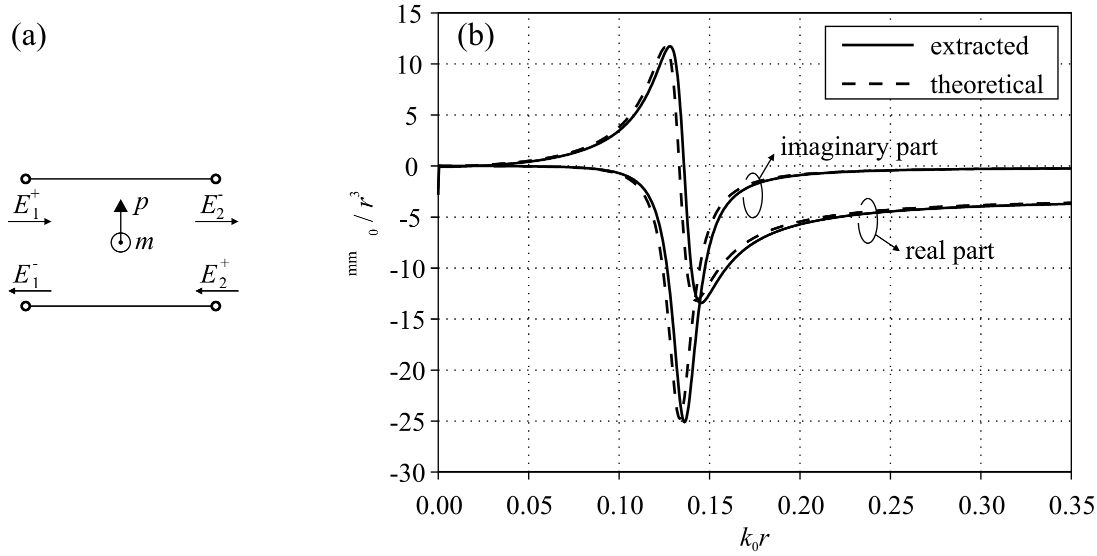

Fig. 1: (a) Sketch of incident and scattered fields at the dipolar discontinuity inside the mono-modal transmission line. (b) Comparison of extracted and theoretical polarizability for a planar perfectly conducting loop with $L/(\mu_0 r) = 3.19$, $k_{\mathrm{rez}} r = 0.13$, $R/Z_0 = 0.053$, see (7) for the theoretical polarizability.

exciting fields are mostly transversal (all TEM waveguides, center of TE10 metallic waveguide, etc.), then it is

$$\begin{bmatrix} E_1^- \\ E_2^- \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} E_1^+ \\ E_2^+ \end{bmatrix} + \begin{bmatrix} C^p & -C^m \\ C^p & C^m \end{bmatrix} \begin{bmatrix} p \\ m \end{bmatrix}. \tag{2}$$

In Eq. (2) the constants C^p, C^m characterize the conversion of the dipole moments into the field of the dominant mode, and they generally depend on the geometry of the waveguide, the used mode and the frequency.

The local field can now be quite generally written as

$$\begin{bmatrix} E_{\mathrm{loc}} \\ H_{\mathrm{loc}} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ Y_0 & -Y_0 \end{bmatrix} \begin{bmatrix} E_1^+ \\ E_2^+ \end{bmatrix} + \begin{bmatrix} C_{\mathrm{INT}}^{\mathrm{EE}} & C_{\mathrm{INT}}^{\mathrm{EM}} \\ C_{\mathrm{INT}}^{\mathrm{ME}} & C_{\mathrm{INT}}^{\mathrm{MM}} \end{bmatrix} \begin{bmatrix} p \\ m \end{bmatrix}, \tag{3}$$

where Y_0 is the modal admittance of the dominant mode, and $C_{\mathrm{INT}}^{\mathrm{EE}}, C_{\mathrm{INT}}^{\mathrm{EM}}, C_{\mathrm{INT}}^{\mathrm{ME}}, C_{\mathrm{INT}}^{\mathrm{MM}}$ are some constants that reflect the environment around the particle, similarly as C^p, C^m in (2).

Putting everything together, we can write

$$\begin{bmatrix} 1 & 1 \\ Y_0 & -Y_0 \end{bmatrix} \left([\mathrm{S}] - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} C^p & -C^m \\ C^p & C^m \end{bmatrix} + \begin{bmatrix} C_{\mathrm{INT}}^{\mathrm{EE}} & C_{\mathrm{INT}}^{\mathrm{EM}} \\ C_{\mathrm{INT}}^{\mathrm{ME}} & C_{\mathrm{INT}}^{\mathrm{MM}} \end{bmatrix} = \begin{bmatrix} \alpha^{\mathrm{ee}} & 0 \\ 0 & \alpha^{\mathrm{mm}}\mu_0 \end{bmatrix}^{-1}, \tag{4}$$

where [S] is the measured scattering matrix. At this point it is important to note that matrix [S] describes only the dipolar discontinuity. In any real system, one will first have to make a standard vector calibration with reference planes at the plane of the particle.

It can be seen in (4) that any two scatterers with known electric and magnetic polarizabilities fully characterize all unknown constants. In the following we will use the known polarizabilities of an electrically small perfectly conducting sphere [1]

$$\frac{1}{\mu_0 \alpha^{\mathrm{mm}}} = -\frac{1}{2\pi r^3} + j\frac{k_0^3}{6\pi}\ ;\ \frac{\varepsilon_0}{\alpha^{\mathrm{ee}}} = \frac{1}{4\pi r^3} + j\frac{k_0^3}{6\pi} \tag{5}$$

and of an electrically small infinitelismaly thin perfectly conducting disc (oriented so that the magnetic field is perpendicular to its surface) [1]

$$\frac{1}{\mu_0 \alpha^{\mathrm{mm}}} = -\frac{3}{8r^3} + j\frac{k_0^3}{6\pi}\ ;\ \frac{\varepsilon_0}{\alpha^{\mathrm{ee}}} = \frac{3}{16r^3} + j\frac{k_0^3}{6\pi}, \tag{6}$$

where r is the radius of the sphere or of the disc.

To show the actual performance of the method, the magnetic polarizability of a planar perfectly conducting loop of mean radius r, inductance L, area $A = \pi r^2$, loaded by lumped capacitance C and resistance R will be extracted from a full wave simulation of this particle placed in a rectangular TEM waveguide of width $a = 10r/3$. For the loop oriented with the normal along the magnetic field, the free space magnetic polarizability can be calculated theoretically and has the following form

$$\frac{1}{\mu_0 \alpha^{\mathrm{mm}}} = \frac{L}{\mu_0 A^2}\left(\frac{\omega_0^2}{\omega^2} - 1\right) + j\left(\frac{k_0^3}{6\pi} + \frac{R}{\omega \mu_0 A^2}\right), \tag{7}$$

where ω_0 is the resonant frequency. A comparison of the polarizability extracted by the method presented above and its theoretical value (7) is shown in Fig. 1b. The measured scattering matrices were substituted by matrices calculated by the CST Microwave Studio.

3. Conclusion

In summary, we have developed a simple method that can potentially serve for measuring free space dipolar polarizabilities. The method is based on knowledge of the polarizabilities of two objects (standards). By means of full wave simulation it has been shown that the method is capable of giving precise results, but further investigation focused on a realistic measurement system and on tolerance analysis will be necessary.

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