The impedance of metamaterials and nanoplasmonic structures

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Abstract

We discuss our efforts to assign an effective impedance to metamaterials while considering the field profile of the Bloch modes supported by the bulk structure. We show that an unambiguous introduction is only possible for carefully designed metamaterials for which at their interface only a single Bloch mode is excited. This renders the metamaterial homogenous and all effective properties are linked to this Bloch mode. We extend this concept to other nanooptical systems, e.g. plasmonic nanowaveguides, and provide new insights into the broader definition of an impedance.

1. Introduction

The homogenization of metamaterials (MM) and the assignment of effective properties is a problem of paramount importance. Their availability is decisive to consider MMs in the design of future applications. It requires that the effective properties do not just predict the optical response for exactly the scenario at which they have been retrieved, but they should preserve their predictive power beyond.

This is exactly the problem with most retrieval procedures where an effective permittivity and permeability is assigned to MMs, e.g. upon illuminating a slab at normal incidence and inverting the complex reflection and transmission. For deviating incidence angles other parameters were retrieved, contradicting the assumption usually imposed, i.e. the medium is biaxial anisotropic. Moreover, the appearances of anti-resonances or a thickness dependence of the effective properties are indications that such description oversimplifies the complexity of MMs. It has been concisely shown that most of such discrepancies can be linked to strong spatial dispersion [1,2]. This asks to consider more complicated constitutive relation to properly account for the response of an actual MM and also for additional boundary conditions. This tends to be tedious and new approaches are required to homogenize MMs.

For the description of light propagation in the bulk this approach was found by relying on the dispersion relation of Bloch modes. Such treatment requires the MM to be composed of periodically arranged unit cell, an assumption which is valid for most top-down MMs. Then, light propagation can be entirely understood while considering the dispersive properties of a single Bloch mode if the imaginary part of its propagation constant is much smaller than the imaginary part of all the other Bloch modes. The Bloch mode is called the fundamental Bloch mode and in addition to an effective index, quantities as a refraction or diffraction coefficient can be derived from the dispersion relation [3].

A problem not yet properly accounted for in that framework is that of treating coupling issues at an interface, i.e. considering the necessary finiteness of all MMs in applications and the impact of the emerging interface. Understanding these coupling issues is urgently required because this fundamental mode not just needs to be existent but it also has to be excited dominantly at the interface. Otherwise the MM cannot be understood as homogenous. In our contribution we will detail the mathematical de-

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tails leading to expressions for an effective impedance while considering only the fields of the Bloch mode. If that impedance can be used to predict the details of the coupling peculiarities at an interface between a homogenous material and a MM, or even between two MMs, we argue that the MM is homogenous. We finally outline extensions of this approach to other nanooptical systems, rendering it a broad and versatile mean to treat the interface among nanooptical elements.

2. Mathematical derivation of the Bloch mode impedance

The mathematical derivation starts with the actual boundary problem at the interface between vacuum and a periodic MM. We will assume that we have access to all Bloch modes $\left|R_n\right\rangle(\mathbf{k}_\perp,\omega)$ of the MM as well as the plane waves $\left|L_n\right\rangle(\mathbf{k}_\perp,\omega)$ of vacuum. They parametrically depend on the tangential wave vector $\mathbf{k}_\perp$ and the frequency ω. For simplicity we assume that the MM is symmetric with respect to the lateral dimensions and that the surface of the MM offers mirror symmetry with respect to the z-direction. At the boundary the amplitudes of all modes have to be adjusted such that

$$\sum_n i^n \left|L_{n+}\right\rangle + \sum_n r^n \left|L_{n-}\right\rangle = \sum_n t^n \left|R_{n+}\right\rangle . \tag{1}$$

We assume illumination from the left of the interface as a superposition of forward propagating modes $\left|L_{n+}\right\rangle$ with amplitudes i^n. At the interface there are backward propagating modes excited in reflection $\left|L_{n-}\right\rangle$ and forward propagating modes excited in transmission $\left|R_{n+}\right\rangle$. By assuming illumination with the *fundamental Bloch mode* indexed with 0 and assuming that the overlaps between dissimilar modes of both media approximately vanish, provides scattering coefficients according to [4]

$$r_0 = -\frac{\left\langle R_{0-}^\dagger \middle| L_{0+}\right\rangle}{\left\langle R_{0-}^\dagger \middle| L_{0-}\right\rangle} \quad \text{and} \quad t_0 = \frac{\left\langle L_{0+}^\dagger \middle| L_{0+}\right\rangle}{\left\langle L_{0+}^\dagger \middle| R_{0+}\right\rangle} . \tag{2}$$

Modes defined with † are adjoint fields which are solutions to the same eigenvalue problem but for the negative tangential wave vector component $-\mathbf{k}_\perp$. The equations can be simplified and an impedance can be introduced by assuming a plane wave as illumination from a homogenous isotropic medium with a tangential impedance defined as $Z_0 = \sqrt{\mu_0/\varepsilon_0}\,/\cos\alpha$ with α being the angle of incidence. Then, the scattering coefficients are given by

$$r_0 = \frac{Z_R - Z_0}{Z_R + Z_0} \quad \text{and } t_0 = \frac{2Z_R}{Z_R + Z_0}\frac{\left\|E_y^P\right\|}{\left\|E_y^B\right\|} , \tag{4}$$

where the tangential impedance of the MMs is defined as

$$Z_R = \frac{\left\|E_y^B\right\|}{\left\|H_x^B\right\|} . \tag{5}$$

and $\left\|\cdot\cdot\right\|$ denotes the cross-section average over the interface. With that at hand we arrived at the final contribution: an expression of the optical effects occurring at the interface while considering the fundamental Bloch mode only. The MM is then entirely characterized by a single wave vector and an impedance. Both parameters depend on the frequency and the tangential wave vector component. Moreover, a unique definition asks to fix the interface of the MM since the cross-sectional average of the Bloch modes depends on the plane of averaging. In contrast to previous work where both quantities were linked to a permittivity and a permeability, we stress that this is not possible for most MMs since their angular dispersion is well beyond and much more complicated than that of ordinary biaxial anisotropic media. This is exactly what makes MMs unique. They are unfortunately not just materials characterized by ε and μ; but they are equally not that complicated as to require each time a rigorous treatment. They are a truly mesoscopic media which has to be considered.

3. Application to a swiss-cross metamaterial

To answer the question whether the retrieved impedance has predictive power we show in Fig. 1 the reflection and transmission coefficient at an interface between air and a swiss-cross MM. It was calculated rigorously and by using Eqs. 4, respectively. The illuminating frequency was fixed at 170 THz, being the frequency were the propagation constant takes most negative values at normal incidence. All quantities are shown as a function of the separation of the functional layers, i.e. an effective dilution of the MM. Next to the optical coefficients, the impedance and the effective index retrieved from a single layer by inverting reflection and transmission coefficient of the MM are shown for comparison.

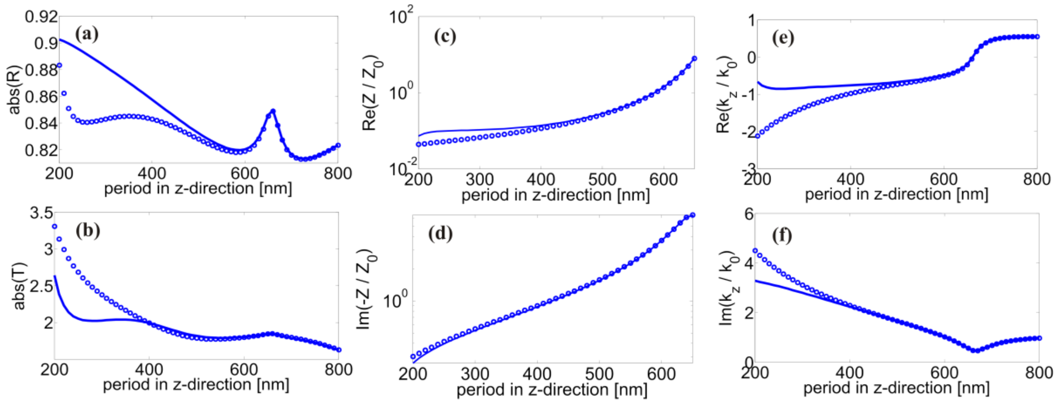

Fig. 1: (a),(b) Scattering coefficients at the single interface. Solid lines represent rigorous results and symbols are calculated using Eqs. (4). In figures (c)-(f) the solid lines are achieved using the properties of the fundamental Bloch mode of the swiss-cross MM. The symbols are those results achieved by a parameter retrieval procedure of the associated single layer slab MM.

It can be seen that various approaches to predict the effective properties converge quantitatively above a certain separation. Then, the properties as retrieved for the bulk and as retrieved from a finite slab all provide the same results. The MM can be homogenized and its effective properties are entirely linked to the fundamental Bloch mode. This Bloch mode, and this is decisive for the success of the description, provides a field distribution that resembles a plane front at the chosen interface. This assures an effective excitation of only that mode. This requires basically to define an interface that avoids proximity to nanometric inclusion in the unit cell. This asks for a sufficient strong dilution which eventually is detrimental for the strength of the desired dispersive effects. Nevertheless, the material is not diluted in excess and it will be shown that for a careful design all fascinating dispersive effects are witnessed, e.g. negative refraction or an anomalous diffraction; properties not found in nature.

4. Conclusion

We provide an effective description of MMs and suggest to use an effective index and an effective impedance. Both quantities can be derived from the fundamental Bloch mode; being in full analogy to natural materials where index and impedance can be derived from solving Maxwell's equations with the suitable Ansatz of a plane wave.

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