Bloch wave and particle resonances in stacked dogbone arrays

A.G. Schuchinsky[1], **A. Vallecchi**[2], **F. Capolino**[3], **A. Shitvov**[1]

[1] Institute of Electronics, Communications and Information Technology
Queen's University Belfast, Queen's Road, Queen's Island, Belfast, BT3 9DT, UK
email: a.schuchinsky@qub.ac.uk, a.shitvov@qub.ac.uk
[2] Dept of Information Engineering,
University of Siena, Via Roma 56, 53100 Siena, Italy
e-mail: andrea.vallecchi@unisi.it
[3] Dept. of Electrical Engineering and Computer Science
University of California, Irvine, CA 92697-2625, USA
e-mail: f.capolino@uci.edu

Abstract

This paper continues a series of works on metamaterials composed of the dogbone shaped conductor pairs. The analysis of the finite stacked arrays has shown that the transmission resonances can be cast into two major categories: (i) intrinsic resonances of individual pairs in each layer and (ii) Bloch wave resonances associated with the collective response of the whole stack. The effects of the layer spacing and the unit cell topology on the properties of stacked arrays will be discussed.

1. Introduction

Doubly periodic arrays of patterned conductors or inclusions stacked one over another represent one of the main classes of metamaterials. Such arrangements are compatible with the layer-by-layer fabrication processes providing reproducible shapes and controllable positions of the constituent elements. The resulting stratified media can be classified as generic anisotropic crystals whose properties are determined by both the lattice symmetry and internal content of individual unit cells.

Several types of the constituent elements such as double bars, split-ring resonators, double fishnet, and dogbone pairs have been proposed to obtain the magnetic response in the planar metamaterial structures. Arranged in the stacked doubly periodic arrays, they form artificial medium which is often described in terms of the *effective* parameters such as permittivity, permeability, refractive index, etc. of equivalent bulk materials. These parameters are usually retrieved from the scattering characteristics of a plane wave incident onto a single layer array or a few stacked arrays. Significant controversy in the literature surrounds these retrieval procedures and applicability of such obtained parameters to the description of multilayered structures, especially when they are extracted from data for a single layer array [1]. The shortcomings of these characterisation procedures become evident when the retrieved parameters violate the basic principles of causality and medium passivity. Moreover the existence of multiple resonances in the finite layered structures causes ambiguity in interpretation of the obtained results and hampers the physically meaningful description of the stratified metamaterial structures.

The studies of periodically stratified media have been performed in the past in the context of the dynamical theory of X-ray diffraction in perfect crystals [2] and more recently in connection with the design of thin film anti-reflecting coatings and photonic crystals [3]. To address these problems and assess the collective response of multilayer structures, Bloch waves in the infinite periodically spaced arrays have been analysed. The latter approach also provides insight into how the internal microscopic properties of the unit cell and interactions of the constituent particles affect the overall response of the stacked arrays. In this paper the resonant transmittance and reflectance by the finite stacks of layered arrays of dogbone shaped conductor pairs are considered. The effects of the layer spacing and the unit cell topology on the properties of stacked arrays are analysed.

ISBN 978-952-67611-0-7

2. Transmittance resonances in arrays of dogbone shaped conductor pairs

The doubly periodic arrays of dogbone pairs (Fig. 1) have been studied theoretically and experimentally in [4], [5]. It has been demonstrated that the lowest mode of the transmittance resonance in a single layer array is of magnetic type, and its fields are well confined inside the conductor pairs. To shed light on the properties of the multilayered arrays, eigenwaves in the infinite stack of periodically spaced layers have been analysed.

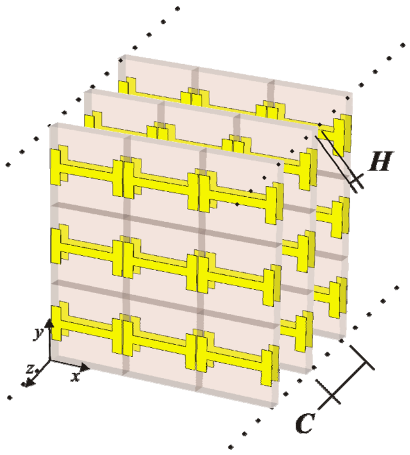
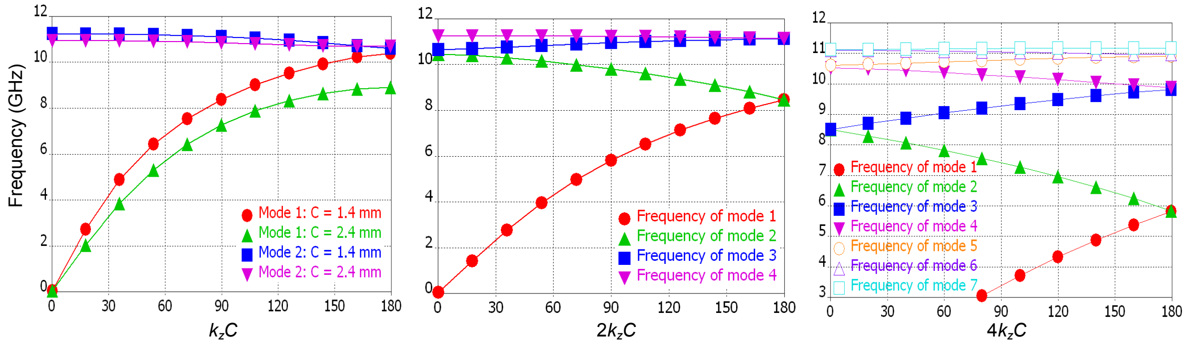


Fig. 1: Layout of the stacked doubly periodic arrays of dogbone shaped conductor pairs

The dispersion characteristics in Fig. 2a show that two modes of the fundamental Bloch waves exist in the considered frequency range. Mode 1 appears at lower frequencies and exhibits the normal dispersion whereas mode 2 represents a fundamental backward Bloch wave. The two modes are separated by a bandgap which varies with the structure periodicity C in the z-direction. When C decreases the bandgap reduces primarily because of mode 1 expansion towards higher frequencies. Conversely, the mode 2 dispersion is much less affected by the layer spacing that indicates tighter confinement of the mode 2 fields to an individual dogbone pair and weaker influence of the interlayer coupling. It is important to note that the transmittance resonance in a single layer array of dogbone pairs occurs only in the frequency band of mode 2 while no such resonances exist in the frequency band of mode 1 (Fig. 3). This transmittance resonance is of magnetic type and its frequency corresponds to $k_zC \approx \pi$, in the dispersion diagram of Fig. 2a where k_z is the propagation constant of fundamental Bloch wave in infinite periodic structure.

Stacking several arrays together qualitatively alters the transmittance characteristics of a single layer as illustrated in Fig. 3. Indeed, not only do the resonances existing in a single layer split but also additional transmittance resonances arise in the frequency band of mode 1. At first glance the latter resonances could be attributed to Bragg resonances similar to those in a stack of dielectric layers. However such an interpretation is hardly justifiable as overall thickness of the stack remains less than $\lambda/10$. Alternatively, inspection of the dispersion characteristics of the fundamental modes in the infinite periodic structures with the unit cells containing 1, 2 and 4 layers (Fig. 2) suggests that the transmittance resonances in the finite stacks can be attributed to Bragg resonances of the fundamental Bloch waves at $Nk_zC = m\pi$, $m = 1,2,\ldots$ where N is the number of layers. The examples of 2- and 4-layer stacked arrays exhibit very good correlation between the frequencies of the transmittance resonances in Fig. 3 and Bragg resonances of the fundamental Bloch waves in Fig. 2 and provide insight in the mechanisms of the resonance transmittance as further discussed in the next section.



(a) (b) (c)

Fig. 2: Dispersion of the normalised propagation constant k_zC (in degrees) along the z-direction in an infinite periodic stack of the dogbone pair arrays with (a) one layer, (b) two layers and (c) four layers per unit cell. The modes with negative slope correspond to backward propagation. The default unit cell size $C = 1.42$ mm and the unit cell size $C = 2.42$ mm has been additionally simulated in the one layer case; $H = 0.382$ mm, $\varepsilon_r = 2.2$.

3. Qualitative analysis and discussion

The scattering problems for finite periodically stratified media were treated in the past using Abelés matrix formalism [2], [6] and direct multiplication of the transmittance matrices [3], [7]. For one-dimensional structures, the transfer matrices for the whole stack have been obtained in closed form [3], [7] implicitly assuming that only a single wave can propagate. In all these cases Fresnel coefficients of individual layers employed in the transfer matrices have been readily available. A generalisation of these techniques to the case of periodic lateral perturbations in the plane of interface has been proposed in [8].

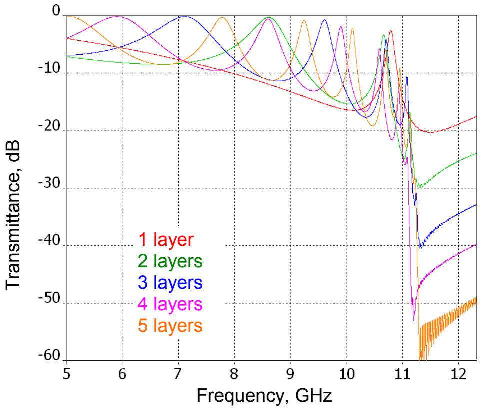


Fig. 3: Transmittance of the stacked arrays of dogbone pairs at the variable number of layers, C = 1.42 mm.

Unfortunately none of these approaches is directly applicable to the analysis of layered metamaterials because the array constituent elements (dogbone pairs) distort the fields of incident wave and interact through evanescent fields as well. Nevertheless, in the sparsely spaced stacks where the layer interaction is dominated by the fundamental propagating waves, the analytical form of the transfer matrices [3] using Fresnel coefficients of isolated layers provides a good qualitative estimate of the characteristics of the finite stacks. The latter approximation proved to be particularly useful for the analysis of the magnetic resonances whose fields are confined to the interior of the dogbone pairs.

When spacing between the layers decreases and coupling between the arrays becomes stronger the basic transfer matrices lose their accuracy due to increased contribution of the evanescent fields. This effect of the interlayer coupling can be somewhat taken into account by using the actual phase of the fundamental Bloch waves inside the stack instead of the phase inferred from the transmission coefficient of an isolated layer [3]. Such a modification of the transfer matrix for the finite stack improves the accuracy of the transmittance resonances and the estimation of the band edge position. However this approximation still needs further refinement by including the higher order Bloch waves, which are the propagating modes inside the stack. At small spacing C between the layers, these higher order modes may make significant contribution to the field distributions at the stack interfaces with the surrounding media. In the meantime, the qualitative analysis based upon the closed form transfer matrices provides insight into the properties and mechanisms of the resonance transmittance in the stacked arrays of dogbone shaped conductor pairs and similar multilayered metamaterial structures.

References

[1] C. R. Simovski, On electromagnetic characterization and homogenization of nanostructured metamaterials, *Journal of Optics*, vol. 13, p.013001 (1-22), 2010.

[2] D.W. Berreman, Dynamical theory of x-ray diffraction in flat, focusing, and distorted crystals by Abelés's matrix method, *Physical Review B,* vol. 14, p. 4313-4316, 1976.

[3] J. M. Bendickson, J. P. Dowling and M. Scalora, "Analytic expression for electromagnetic mode density in finite, one-dimensional, photonic band-gap structures", *Physical Review E*, vol. 53, p. 4107-4121, 1996

[4] G. Donzelli, A. Vallecchi, F. Capolino, and A. Schuchinsky, "Metamaterial made of paired planar conductors: Particle resonances, phenomena and properties", *Metamaterials*, vol. 3, pp. 10–27, 2009.

[5] A.P. Shitvov, A. Vallecchi, A.G. Schuchinsky, F. Capolino, "Characterization of Metamaterials Made of Stacked Layers of Dogbone Conductor Pairs", 2010 *IEEE AP-S Int. Symp./URSI Meeting*, July 11-17, 2010, Toronto, Canada, 115.5

[6] F. Abelés, La théorie générale des couches minces, *Le Journal de Physique et le Radium*, vol. 11, pp. 307–310, 1950.

[7] D.W.L. Sprung, H. Wu, J. Martorell, Scattering by a finite periodic potential, *American Journal of Physics*, vol. 61, pp. 1118-1124, 1993

[8] D.W. Berreman, A.T. Macrander, Asymmetric x-ray diffraction by strained crystal wavers: 8×8 dynamical theory, *Physical Review B,* vol. 37, pp. 6030-6040, 1988.