Treating the scattering problem at the interface between two metamaterials

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Abstract

We use the Bloch-mode orthogonality to derive simple closed-form expressions for the scattering coefficients at an interface between two periodic media, a computationally-challenging electromagnetic scattering problem that can be solved only with advanced numerical tools. The derivation relies on the assumptions that the interface is illuminated by the fundamental Bloch mode and that the two media have only slightly different geometrical parameters. Through comparison with fully-vectorial three-dimensional computations, the analytical expressions are shown to be highly predictive for various geometries, including dielectric waveguides and metallic metamaterials. They can thus be used with confidence for designing and engineering stacks of periodic structures.

1. Introduction

Periodic nanostructures, such as photonic crystals and metamaterials, have attracted a considerable deal of interest within the last decades. Although periodicity is indeed important, many devices are not fully periodic and include either local defects or tapers with a gradient of the geometrical parameters. Understanding wave scattering at an interface of two periodic media therefore has become a major issue.

Fully-vectorial, three-dimensional (3D) calculations of the scattering coefficients of such interfaces also represent a challenging electromagnetic problem, for which very few numerical tools are presently available [1-2]. In this work, we derive analytical expressions for the scattering coefficients between the fundamental Bloch modes (BMs) of two periodic media. Under the assumption that the media differ only weakly, we show that very simple, but accurate, closed-form expressions can be derived for the scattering coefficients of the fundamental BMs. Furthermore if we assume that the fundamental BM represents the main channel for the energy transport (single-BM approximation), the propagation in stacks of periodic media can then be handled analytically by multiplication of 2×2 transfer matrices involving those coefficients. The present derivation is motivated by the fact that simple intuitive theoretical formalisms have not been presented yet and that realistic expressions for coupling strengths at periodic interfaces may be useful for designing photonic devices, such as mode converters in periodic waveguides, tapered photonic-crystal (PhC) mirrors, or graded index beamers, for defining the impedance of PhCs, or for the theoretical study of the impact of fabrication imperfections in slow-light PhC waveguides, to quote a few of them.

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2. Theory

Let us consider the scattering problem shown in Fig. 1, where the incident fundamental mode $|\mathbf{P}_1\rangle$ in the left periodic medium impinges on an interface separating two periodic media. We adopt a Cartesian system hereafter with axes x, y, and z, the axis z being normal to the interface. The field on both sides of the interface can be expanded into a BM basis, and using the field continuity relations at $z = 0$, we have

$$|\mathbf{P}_1\rangle + r_1|\mathbf{P}_{-1}\rangle + \sum_{m>1} r_m|\mathbf{P}_{-m}\rangle = t_1\,|\mathbf{B}_1\rangle + \sum_{m>1} t_m|\mathbf{B}_m\rangle. \quad (1)$$

Equation (1), which is valid for the tangential field components, defines the reflection and transmission scattering coefficients r_m and t_m. Note that, on both sides of the equation, we have isolated the predominant outgoing fundamental Bloch modes, labeled $\mathbf{P}_{-1}\rangle$ and $|\mathbf{B}_1\rangle$. The positive and negative subscripts refer to BMs propagating towards the positive and negative z-directions, respectively. Our goal is to derive approximate expressions for the reflection and transmission coefficients, r_1 and t_1, by relying only on the knowledge of the fundamental BMs of the periodic media, the higher-order BMs being assumed to be unknown. Because a rigorous solution of Eq. (1) with fully-vectorial software is computationally expensive, such approximate expressions are anticipated to facilitate the preliminary stages of the design of complicated components involving stacks of periodic structures.

It is first tempting to neglect all the unknown quantities [2-4] i.e., the higher-order BMs, to end up with a simplified version of Eq. (1) that reads as

$$|\mathbf{P}_1\rangle + r_1|\mathbf{P}_{-1}\rangle = t_1\,|\mathbf{B}_1\rangle. \quad (2)$$

Assuming that $|\mathbf{P}_1\rangle$, $|\mathbf{P}_{-1}\rangle$ and $|\mathbf{B}_1\rangle$, are known at $z = 0$, the coefficients r_1 and t_1 can be obtained by projecting Eq. (2) on a complete set of functions $|\mathbf{F}_\mathrm{n}\rangle$ – for instance Fourier harmonics – and solving the resulting system of equations in the least-mean-squares (LMS) sense. As will be shown, however, the accuracy of the LMS solution is poor. This is because Eq. (2) authoritatively states an over-simplified expression for the fields on *both* sides of the interface and completely ignores the role of higher-order BMs involved in the coupling mechanism. Actually, it is possible to take them into account, at least approximately, without explicitly calculating them, and one may derive analytical expressions for r_1 and t_1 that are more accurate than those obtained with the LMS approach. This is at the heart of our contribution. The elimination of the unknown higher-order BMs in Eq. (1) can be achieved, under the sole assumption that the materials are reciprocal, by using BM orthogonality, which reads [1]

$$\langle\mathbf{B}_n|\mathbf{B}_m\rangle = \iint_S [\mathbf{E}_m \times \mathbf{H}_n - \mathbf{E}_n \times \mathbf{H}_m] \bullet \mathbf{z}\, \mathrm{d}S = 0 \text{ if } m \neq -n. \quad (3)$$

In Eq. (3), $|\mathbf{B}_n\rangle$ and $|\mathbf{B}_m\rangle$ are two BMs of the same periodic medium, S represents an (x, y) cross-section of the medium, and $\mathbf{E}_n$ and $\mathbf{H}_n$ are the electric and magnetic transverse components of $|\mathbf{B}_n>$, $\mathbf{z}$ being the unit vector normal to the interface. Equation (3) defines an antisymmetric bilinear form: $\langle\mathbf{B}_m|\mathbf{B}_n\rangle = -\langle\mathbf{B}_n|\mathbf{B}_m\rangle$. Hereafter, the BMs are normalized so that $\langle\mathbf{B}_n|\mathbf{B}_{-n}\rangle = 4$, implying that non-evanescent BMs propagating in lossless periodic media have unit power flow along the z axis [1].

Coming back to Eq. (1), we start by deriving an approximate closed-form expression for r_1. Under the assumption that the two periodic media are only slightly different, it is expected that the scattering process predominantly consists in exciting the fundamental BMs, $|\mathbf{P}_{-1}\rangle$ and $|\mathbf{B}_1\rangle$, the excitation of the higher-order BMs being a weaker process induced by the transverse mode-profile mismatch between the fundamental BMs $|\mathbf{P}_1\rangle$ and $|\mathbf{B}_1\rangle$. Therefore, an accurate expression for the reflection coefficient r_1 can be derived by neglecting the high-order transmitted BMs, and we obtain $|\mathbf{P}_1\rangle + r_1|\mathbf{P}_{-1}\rangle + \sum_{m>1} r_m|\mathbf{P}_{-m}\rangle \approx t_1 |\mathbf{B}_1\rangle$. Then, without any further approximation, we use the BM orthogonality and project the previous equation onto $\langle\mathbf{P}_1|$ and $\langle\mathbf{P}_{-1}|$, to get $r_1\langle\mathbf{P}_1|\mathbf{P}_{-1}\rangle = t_1\langle\mathbf{P}_1|\mathbf{B}_1\rangle$ and $\langle\mathbf{P}_{-1}|\mathbf{P}_1\rangle = t_1\langle\mathbf{P}_{-1}|\mathbf{B}_1\rangle$, from which we eliminate t_1 to obtain

$$r_1 = -\langle\mathbf{P}_1|\mathbf{B}_1\rangle/\langle\mathbf{P}_{-1}|\mathbf{B}_1\rangle. \tag{4}$$

Consistently, to calculate t_1, we first neglect the high-order reflected BMs and, after projecting onto $\langle\mathbf{B}_1|$ and $\langle\mathbf{B}_{-1}|$, we obtain $r_1\langle\mathbf{B}_{-1}|\mathbf{P}_{-1}\rangle + \langle\mathbf{B}_{-1}|\mathbf{P}_1\rangle = t_1\langle\mathbf{B}_{-1}|\mathbf{B}_1\rangle$ and $\langle\mathbf{B}_1|\mathbf{P}_1\rangle + r_1\langle\mathbf{B}_1|\mathbf{P}_{-1}\rangle = 0$. The elimination of r_1 leads to

$$4t_1 = \langle\mathbf{B}_{-1}|\mathbf{P}_1\rangle - \langle\mathbf{B}_{-1}|\mathbf{P}_{-1}\rangle\langle\mathbf{B}_1|\mathbf{P}_1\rangle \,/\, \langle\mathbf{B}_1|\mathbf{P}_{-1}\rangle. \tag{5}$$

The closed-form expressions of Eqs. (4) and (5) constitute the main result of the present work [5]. At the conference, we will show that they are much more accurate than Eqs. (2), for various geometries, such as metallic metamaterials and dielectric periodic waveguides.

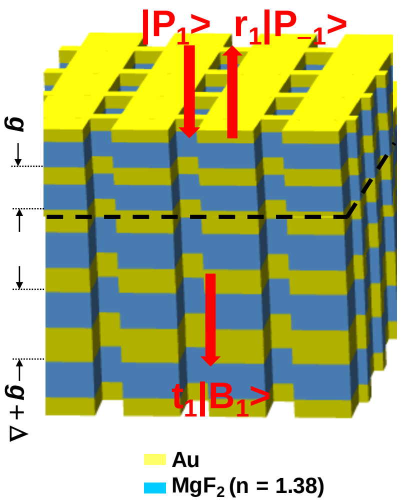

Fig.1 Scattering at the interface of two *z*-periodic media. In the transverse *x*- and *y*-directions the media can be periodic, like a fishnet, or aperiodic, like in the *z*-periodic waveguide. Only the fundamental BMs, $|\mathbf{P}_1\rangle$ (incident), $|\mathbf{P}_{-1}\rangle$ (reflected), and $|\mathbf{B}_1\rangle$ (transmitted), are shown. The goal is to derive accurate expression of r_1 and t_1, assuming that only the fundamental BMs $|P_1>$ and $B_1>$ are known.

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