Sub-diffraction-limited imaging in the far-field

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Abstract

Perfect imaging in the far-field becomes possible in refractive-index-profile devices such as e.g. Maxwell's fish-eye that can be described in transformation optics by a spherical geometry. We present experimental results in the microwave regime, which show sub-wavelength imaging with a positive refractive-index profile at several wavelengths away from the objects.

1. Introduction

In 1882 Ernst Abbe found that any image created by a conventional optical devices is limited by diffraction [1]. Consequently, the only way to increase the resolution significantly is by decreasing the wavelength, a principle that is used e.g. in UV lithography and electron microscopy. In 2000 Pendry revisited the idea of Veselago that a planar slab of negative index material acts as a lens [2, 3]. He found that such a device will create a perfect, not diffraction-limited image as it will focus propagating waves and, in addition, reconstruct evanescent waves in the same image plane [3]. It was believed, that evanescent waves are needed to create a perfect image and, therefore, sub-diffraction imaging can only be achieved close to material interfaces.

With the increasing accuracy of lithography it became possible to taylor the optical properties of matter by metamaterial structures. Consequently, a new theoretical description on light-matter interaction was developed describing the optical properties of matter in a virtual space for light by transformation optics [4, 5, 6]. This method allows for designing complex optical devices such as e.g. optical cloaks. In this paper, we show that certain geometries of the virtual space show extraordinary optical behavior such as e.g. imaging that is not limited by diffraction. Intriguingly this sub-diffraction imaging is possible even at several wavelengths away from objects and interface. Hence, it seems to be possible to create sub-diffraction-limited resolution without the direct utilization of evanescent fields. In this paper, we discuss theoretical and experimental results for the example of Maxwell's fish-eye.

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2. Maxwell's fish-eye

In 1854, Maxwell had the idea of using a index-profile lens to bend all light arising from an object towards an image point [7]. He found that the required refractive index profile has the form

$$n = \sqrt{\varepsilon_r} = \frac{n_0}{1 + (\frac{r}{R})^2} \tag{1}$$

with n being the local refractive index, ε_r the local permittivity, n_0 the refractive index at the center of the lens, r the distance from the center and R the radius of the device. On such a structure, light arising from a source follows circles in space which intersect in the image point (Fig. 1b). The corresponding virtual space of Maxwell's fish-eye is the surface of a sphere, which allows us to illustrate the unique properties of such a device [8]: the light on the surface of a sphere follows its geodesics and all the light arising from a source is focus in its antipodal point (see Fig. 1a). Consequently, each point in the device has a unique focus point to which the source is perfectly transformed and which is not limited by diffraction (Fig. 1c). Please note that for Maxwell's fish-eye, source and image are located within the actual device in contrast to conventional lenses which are located between source and image.

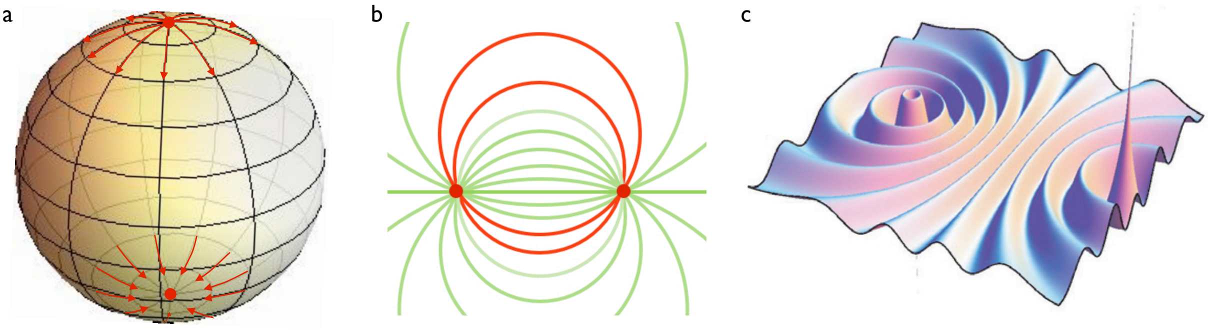

Fig. 1: (a) Geometry of Maxwell's fish-eye in virtual space and the corresponding ray-paths (b) and field-distribution (c) in real space [8].

3. Experimental realization

The experimental realization of Maxwell's fish-eye is done by a microwave experiment using copper structures and dielectric filler to create the required refractive index profile of eq. (1) (Fig. 2a, [9]). The structures are implemented in a waveguide consisting of parallel plates, which is single mode at the wavelength of interest. The source consists of active antennas whereas passive antennas are used for drains (see section 4) and probes, with the latter being scannable over the device area. Typical distances between source and image are around twice the free-space wavelength of 3 cm.

Maxwell's fish-eye is not limited to the microwave range or to metamaterial structures. It can e.g. be realized in integrated silicon photonics for the near-IR range by using greyscale lithography [10]. In contrast to the metamaterial approach, this method allows one to create smooth refractive index profiles rather than those with discrete steps due to the copper structures.

4. Results

In first experiments, we study the field distribution within our microwave device with a single antenna as point source. When using a scannable probe only, we found that the field distribution shows a maximum at the image point, which is still limited by diffraction. This effect is due to interference of incoming and outgoing waves at the image point, showing a standing wave pattern with typical periodicity in the order

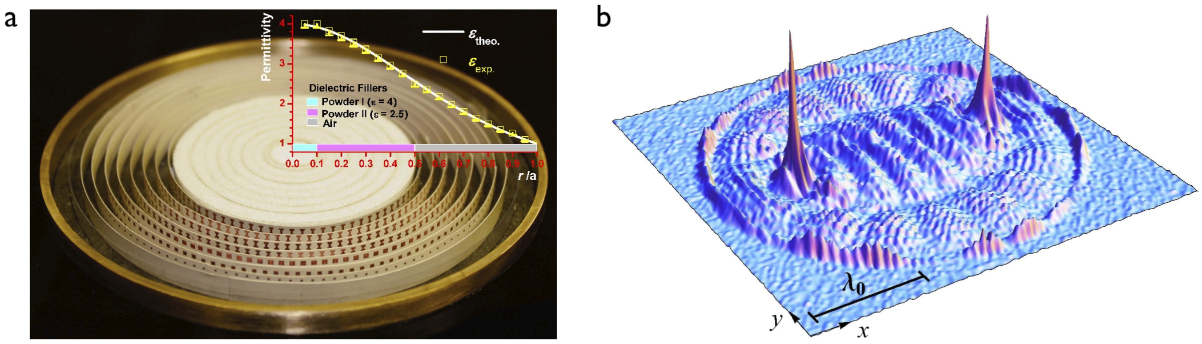

Fig. 2: (a) Microwave realization of Maxwell's fisheye consisting of copper structures in a single-mode waveguide. (b) Experimentally obatined field amplitudes with source (left) and image point (right) [9].

of the wavelength. It is only when placing a drain at the position of the image point that we can observe a subwavelength-sized image point (Fig. 2b).

When implementing a drain at the position of the image one needs to carefully consider whether the observed field distribution is truely an image or just an artifact of the drain, that might create localized fields at the image position. In order to verify the true field distribution, we position an array of drains in the image plane with distances of $\lambda/20$. Here, we use two point sources with a separation of $\lambda/5$ as objects. When scanning the probe over the array of drains, the observed signal is not correlated to the expected near-field distribution of the drains, but clearly reconstructs the object distribution of the two point sources with a resolution of at least $\lambda/5$ [9].

5. Conclusion

Transformation optics allows us to construct and describe complex optical devices such as e.g. Maxwell's fish-eye which can be described by the surface of a sphere in virtual space. With such a device, it is possible to overcome the diffraction limit in the far-field without using evanescent waves at interfaces. Obviously, complex geometries with non-eucledian coordinates open the possibility to overcome classical limitations and create a pathway to new optical devices.

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