Perfect tunneling in semiconductor heterostructures

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Abstract

We study the phenomenon of perfect tunneling (tunneling with unitary transmittance) in a 1D semiconductor heterostructure using a formal analogy of the electromagnetic wave equation and the Schrödinger equation. The Kane model of a semiconductor is used and it is shown that this phenomenon can indeed exist, resembling all the interesting features of the corresponding phenomenon in classical electromagnetism in which metamaterials are involved.

1. Introduction

The tunneling of electrons through a potential barrier is a phenomenon that has been known to physicists for a long time [1]. At first, the tunneling amplitudes were known to take appreciable values only on atomic scales. Later, however, use of the resonant tunneling in semiconductor heterostructures [2] opened the way to high tunneling amplitudes even in macroscopic devices, such as resonant tunneling diodes.

Phenomena equivalent to quantum tunneling are also known in other fields of physics, one example being classical electromagnetism, where for example the section of the waveguide below the cutoff frequency can serve as the potential barrier through which the photons can tunnel. It is also the field of electromagnetism, where so called perfect tunneling, i.e. tunneling with unitary transmission coefficient, has been proposed [3], theoretically studied [4] and experimentally proven [5] with the help of metamaterials.

The aim of this contribution is to show that perfect tunneling exists in the quantum domain. The proposal is based on the mathematical similarity of the Schrödinger equation and the electromagnetic wave equation.

2. Analysis and Results

Let us first show an example of a perfect tunneling setup in the case of electromagnetic waves. The structure is sketched in Fig. 1a, and follows the idea presented in [4]. The layers are assumed to be laterally infinite, and the plane waves are assumed to propagate perpendicular to the layers. It has been

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shown [4] that it is always possible to find such d_1, d_2 for which the tunneling (transmission) through this structure is equal to unity and can thus be called perfect, although the structure may contain potential barriers of theoretically any thickness.

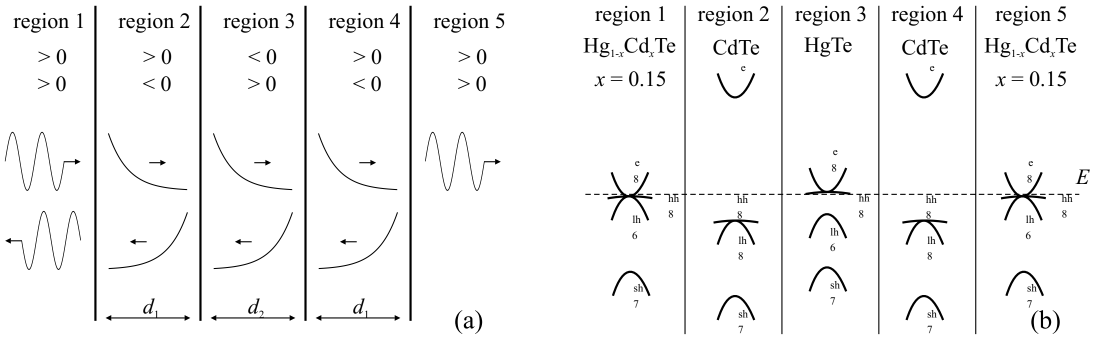

Fig. 1: Sketch of the electromagnetic perfect tunneling setup (a) and band diagram sketch of the realistic quantum tunneling structure (b).

The way to transform this structure into a quantum structure uses an analogy between the electromagnetic wave equation and the Schrödinger equation [6]. More specifically, if the longitudinal axis in Fig. 1a is denoted as the z-axis, the wave equation for a monochromatic plane wave propagating along this axis can be written as

$$\frac{\partial^2 E_x}{\partial^2 z} + \omega^2 \mu \varepsilon E_x = 0 \tag{1}$$

with boundary conditions

$$E_x^+ = E_x^- \quad ; \quad \frac{1}{\mu^+}\frac{\partial E_x^+}{\partial z} = \frac{1}{\mu^-}\frac{\partial E_x^-}{\partial z}. \tag{2}$$

On the other hand, if a semiconductor heterostructure is described by Kane model [7, 8], and the system is solved for electron envelope function f_{c}, the describing equation reads

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + E_{\Gamma_6}\right] f_{\mathrm{c}}(z) = E f_{\mathrm{c}}(z) \quad ; \quad \frac{1}{m} = \frac{2P^2}{3}\left[\frac{2}{E - E_{\Gamma_8}} + \frac{1}{E - E_{\Gamma_7}}\right]. \tag{3}$$

The quantities $E_{\Gamma_6}, E_{\Gamma_8}, E_{\Gamma_7}$ are the band edge energies of $\Gamma_6, \Gamma_8, \Gamma_7$ bands which are position dependent in step like manner along the heterostructure and the quantity P is the element of the Kane matrix. The boundary conditions in this case read

$$f_{\mathrm{c}}^+ = f_{\mathrm{c}}^- \quad ; \quad \frac{1}{m^+}\frac{\partial f_{\mathrm{c}}^+}{\partial z} = \frac{1}{m^-}\frac{\partial f_{\mathrm{c}}^-}{\partial z}. \tag{4}$$

Comparing (1,2) with (3,4), we can see that the two physical problems are mathematically identical assuming

$$E_x \to f_{\mathrm{c}} \quad ; \quad \mu \to m \quad ; \quad \varepsilon \to 2\left(E - E_{\Gamma_6}\right) \quad ; \quad \omega \to 1/\hbar. \tag{5}$$

Using this analogy, we can arrive at the possible quantum implementation of the perfect tunneling setup using the $Hg_{1-x}Cd_xTe$ ternary alloy as is depicted in Fig. 1b.

The parameters needed for calculating the envelope function can be obtained from reliable measurements or from first principle calculations, which suggest $2m_0P^2 \approx 18.5$ eV, $E_{\Gamma_6} \approx (1.47x + 0.08)$ eV, $E_{\Gamma_8} \approx (-0.36x + 0.36)$ eV and $E_{\Gamma_7} \approx (-0.36x - 0.59)$ eV, where x represents the $Hg_{1-x}Cd_xTe$ mole fraction. The transmission coefficient and the amplitude of the envelope function at the transmission maximum were calculated for $d_1 = 1.26$ nm, $d_2 = 10$ nm, and are depicted in Fig. 2, which clearly presents the perfect tunneling that we are looking for, including the interface maxima at the boundaries.

At this point it is important to stress that the unitary transmission is achieved at energies for which the regions 2,3,4 support only evanescent waves. Such phenomenon is thus quite different from the usual resonant tunneling for which the region 3 is propagative with either $m > 0, (E - E_{\Gamma_6}) > 0$ (resonant tunneling diode) or $m < 0, (E - E_{\Gamma_6}) < 0$ (interband resonant tunneling diode).

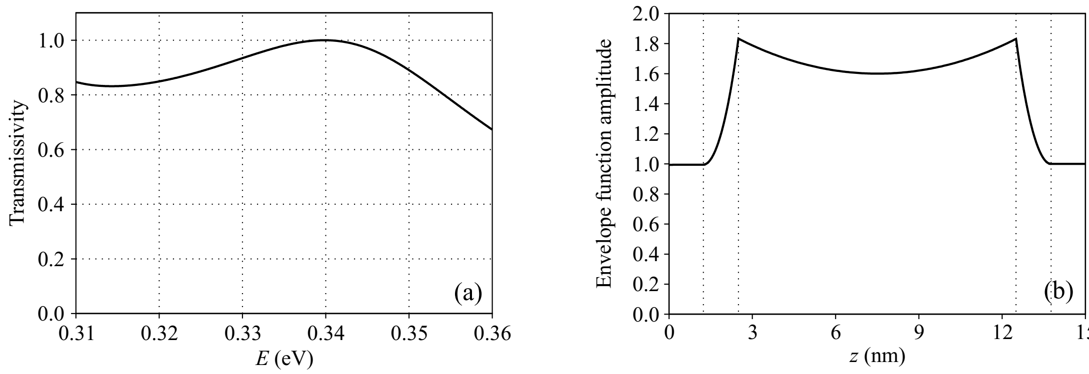

Fig. 2: Amplitude of the transmission coefficient (a) and amplitude of the envelope function (b) for the energy of maximum transmittance. The parameters are $d_1 = 1.26$ nm, $d_2 = 10$ nm.

3. Conclusion

We have exploited the formal analogy of the electromagnetic wave equation and the Schrödinger equation to transfer the idea of perfect tunneling into the semiconductor domain. We have particularly shown that perfect tunneling can be found in 1D semiconductor heterostructures composed of HgCdTe ternary alloys exhibiting all the features of the electromagnetic phenomenon. We think that the results reported here can excite greater interest in extending the physical concept of metamaterials into the semiconductor domain.

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