Modal Interactions in Mushroom-Type Metamaterials with Thin Metal/Graphene Patches

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Abstract

In this paper, we study the modal characteristics of mushroom-type surfaces with thin metal/graphene patches (more generally, resistive film patches). The dispersion equation for natural modes is obtained from a nonlocal homogenization model with a generalized additional boundary condition at the connection of vias to thin metal patches. It is observed that by varying the metal thickness (or equivalently the surface conductivity) the modal spectrum is significantly perturbed and the modal interaction of complex modes (in the sense of modal transformation) occurs.

1. Introduction

In our recent paper [1] we have proposed a generalized additional boundary condition (GABC) for the analysis of reflection/transmission properties of mushroom-type and bed-of-nail-type wire media terminated with an arbitrary, electrically thin material acting as an imperfect ground plane and/or mushroom patches. It has been shown in [1] that the GABC is essential in order to obtain the correct homogenization results in these cases.

This paper focuses on the analysis of modal characteristics of mushroom-type surfaces with thin metal/graphene patches (or, in general, resistive film patches). The dispersion equation for TM^z natural modes of the structure is obtained from the nonlocal homogenization model with the GABC presented in [1]. Starting with the case of perfect electric conductor (PEC) patches analyzed in [2], it is observed that by gradually varying metal thickness of the patches (or equivalently the surface conductivity) the modal spectrum shown in Fig. 19 in [2] is significantly perturbed. With the presence of conduction losses (due to thin metal patches) the proper complex and improper complex leaky waves attenuate rapidly before radiating; i.e., they have high attenuation constant. However, the proper real (bound) modes become complex modes and experience peculiar modal interaction phenomena resulting in modal transformation with a small variation in the metal thickness (surface conductivity). In addition, it is observed that the reflection minima (absorption peaks) of the plane-wave incidence are directly related to properties of complex modes.

2. Nonlocal homogenization model

A nonlocal homogenization model, characterized by a nonlocal dielectric function of the effective permittivity along the wires, is obtained for the analysis of TM^z natural modes of the mushroom surface with thin metal/graphene patches and terminated with a PEC ground plane (with the geometry shown in Fig. 1). The dispersion equation is derived by matching the electric and magnetic fields at the air-to-patch interface at $x = L$ by using the two-sided impedance boundary conditions, at the PEC ground plane at $x = 0$ with the traditional boundary condition for the tangential electric field, and also applying the additional boundary condition for the microscopic current at the via-to-ground plane connection $dI(x)/dx|_{x=0^-} = 0$ [3] and the GABC at the via-to-patch connection

$\left[I(x)+\frac{\sigma_{2d}}{j\omega\varepsilon_0\varepsilon_r}\frac{dI(x)}{dx}\right]\Big|_{x=L^-}=0$ [1]. Here, σ_{2d} (S) is the complex surface conductivity of a 2-D material, and for a sufficiently thin 3-D material having complex conductivity σ_{3d} (S/m) we can write $\sigma_{2d}=\sigma_{3d}t$, where $t \ll \delta$ is material thickness and $\delta=\sqrt{2/\omega\mu_0\sigma_{3d}}$ is skin depth (see [4] for the surface conductivity of graphene).

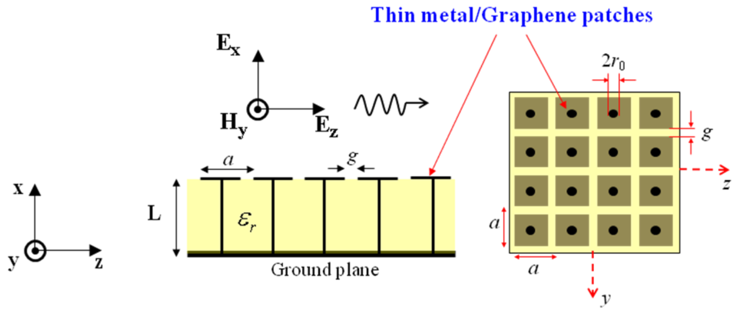


Fig. 1. TM^z natural modes on mushroom surface with thin metal/graphene patches.

The dispersion equation for the TM^z natural modes is obtained as follows:

$$K\coth(\gamma_{\rm TM}L)\cot(kL)+\left(\frac{1}{\gamma_0}-j\frac{\eta_0}{Z_g k_0}\right)=0 \quad (1)$$

where $K=N/D$,

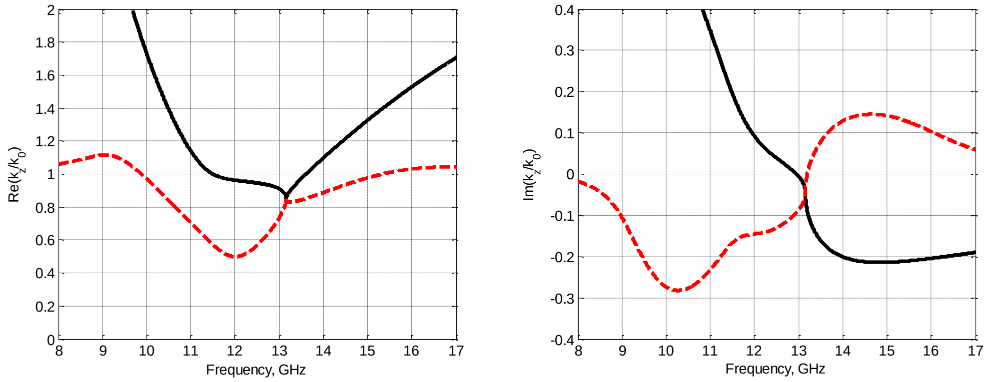
$$N=\left(\frac{1}{\varepsilon_{xx}^{\rm TM}}-1\right)\left(\frac{\sigma_{2d}\gamma_{\rm TM}}{j\omega\varepsilon_0\varepsilon_r}\tanh(\gamma_{\rm TM}L)+1\right)+\left(1-\frac{\sigma_{2d}k}{j\omega\varepsilon_0\varepsilon_r}\tan(kL)\right) \quad (2)$$

$$D=-\frac{k}{\varepsilon_r}\left(\frac{1}{\varepsilon_{xx}^{\rm TM}}-1\right)\left(\frac{\sigma_{2d}\gamma_{\rm TM}}{j\omega\varepsilon_0\varepsilon_r}+\coth(\gamma_{\rm TM}L)\right)+\frac{\gamma_{\rm TM}}{\varepsilon_r}\left(\cot(kL)-\frac{\sigma_{2d}k}{j\omega\varepsilon_0\varepsilon_r}\right). \quad (3)$$

In (1)-(3), $k=k_0\sqrt{\varepsilon_r}$, $\gamma_0^2=k_z^2-k_0^2$, $\gamma_{\rm TM}^2=k_p^2+k_z^2-k_0^2$, $\varepsilon_{xx}^{\rm TM}=1-k_p^2/(k_z^2+k_p^2)$; k_p is the plasma wavenumber defined in [1] and references therein, and Z_g is the continuous surface impedance of the patch array [1], [5]. The dispersion equation (1) can be solved numerically for the propagation constant k_z. Also, it should be noted that the above dispersion equation (1) can be obtained from the plane-wave incidence problem by setting the denominator in the expression of the reflection coefficient to zero; see (9) in [1].

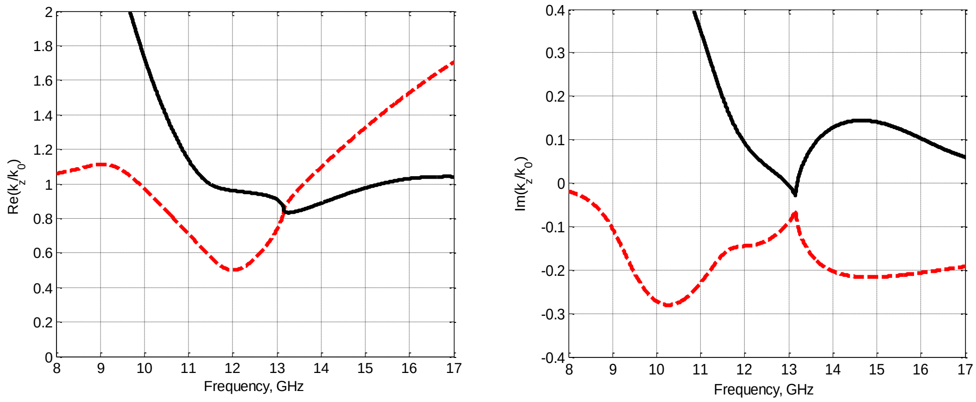
3. Results and discussions

The numerical results for the normalized propagation constant k_z/k_0 are obtained for the TM^z natural modes of the mushroom surface with thin metal/graphene patches (Fig. 1) for the following geometrical and material parameters: L = 1 mm, a = 2 mm, g = 0.1 mm, r_0 = 0.05 mm, and ε_r = 10.2. Starting with the case of PEC patches analyzed in [2] and gradually decreasing the metal thickness (or equivalently the surface conductivity), it is observed that proper real (bound) modes, which become complex modes due to the presence of conduction losses in thin metal, are significantly perturbed. An interesting observation is that for some values of metal thickness these complex modes interact, and a small change in the value of metal thickness may result in modal interchange (modal transformation) as it is shown in Figs. 2 and 3. Here we assume the bulk conductivity of the metal $\sigma_{3d} = 2.9 \times 10^6$ (S/m) and vary metal thickness.



(a) (b)

Fig. 2. Real (a) and imaginary (b) parts of the normalized propagation constant versus frequency for two complex modes with the metal thickness $t = 21.6$ nm.



(a) (b)

Fig. 3. Same as in Fig. 2 with $t = 21.4$ nm.

4. Conclusion

The modal spectrum in mushroom structures with thin metal/graphene patches is studied based on the nonlocal homogenization model with the GABC. It is observed that the modal transformations occur for complex modes with small variation in the metal thickness (surface conductivity) leading to interesting wave phenomena, and, in general, to nonuniqueness in the classification of modal spectrum in such lossy structures.

References

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