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ABSTRACTS

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Development of high vorticity structures in incompressible 3D Euler equations

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We study numerically the development of singularities of vorticity field for 3D Euler equation on grids equivalent to up to 2048^3 nodes. We examine nearly arbitrary smooth symmetric and non-symmetric initial data. We track the evolution for not only the global maximum of the vorticity field, but also for all of the local maximums. For several realizations of the initial data, both symmetric and non-symmetric, and as long as our simulations are reliable, we observe exponential development of singularities with time. To the end of our simulations we also observe the initial stage of formation of Kolmogorov energy spectrum $k^{(}-5/3)$. Our results suggest that exponentially developing singularities may lead to formation of Kolmogorov energy spectrum even before viscosity comes in play.

Characteristic initial value problems for integrable reductions of Einstein's field equations and gravitational interaction of short electromagnetic pulses on an expanding cosmological background

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Discovery of integrability of symmetry reduced vacuum Einstein equations and formulation of the well known now Belinski-Zakharov inverse scattering approach to solution of these equations (including the soliton generating transformations and reformulation of the problem for "non-soliton" part of the solutions in terms of some matrix Riemann-Hilbert problem) more than thirty years ago opened the ways for construction in General Relativity of many physically interesting solutions as well as for development of similar (or based on the same basic ideas and appropriately modified) approaches to solution of other (non-vacuum) integrable symmetry reductions of Einsteins field equations (the Einstein-Maxwell and Einstein-Maxwell-Weyl equations in General Relativity, the bosonic field equations in some string gravity and supergravity models in four and higher dimensions and some others). In this talk, we recall a general construction of some form of the linear integral equations found later by the present author and available for solution of any known today integrable reductions of Einstein's field equations. This form of (quasi-Fredholm) linear integral equations is most appropriate for solution of the characteristic initial value problems for these equations. An application of this integral equation method to construction of solution for nonlinear gravitational interaction of short electromagnetic pulses colliding on the expanding cosmological background is considered.

Modelling of squall with the generalised kinetic equation

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We study the long-term evolution of random wind waves using the new generalised kinetic equation (GKE). The GKE derivation does not assume quasi-stationarity of a random wave field [1]. In contrast with the Hasselmann kinetic equation, the GKE can describe fast spectral changes occurring when a wave field is driven out of a quasi-equilibrium state by a fast increase or decrease of wind, or by other factors. In these cases, a random wave field evolves on the dynamic timescale typical of coherent wave processes, rather than on the kinetic timescale predicted by the conventional statistical theory. Besides that, the generalised theory allows to trace the evolution of higher statistical moments of the field, notably the kurtosis, which is important for assessing the risk of freak waves and other applications.

A new efficient and highly parallelised algorithm for the numerical simulation of the generalised kinetic equation is presented and discussed. Unlike in the case of the Hasselmann equation, the algorithm takes into account all (resonant and nonresonant) nonlinear wave interactions, but only approximately resonant interactions contribute to the spectral evolution. However, counterintuitively, all interactions contribute to the kurtosis. Without forcing or dissipation, the algorithm is shown to conserve the relevant integrals. We show that under steady wind forcing the wave field evolution predicted by the GKE is close to the predictions of the conventional statistical theory, which is applicable in this case. In particular, we demonstrate the known long-term asymptotics for the evolution of the spectrum. When the wind forcing is not steady (in the simplest case, an instant increase or decrease of wind occurs), the generalised theory is the only way to study the spectral evolution, apart from the direct numerical simulation. The focus of the work is a detailed analysis of the fast evolution after an instant change of forcing, and of the subsequent transition to the new quasi-stationary state of a wave field. It is shown that both increase and decrease of wind lead to a significant transient increase of the dynamic kurtosis, although these changes remain small compared to the changes of the other component of the kurtosis, which is due to bound harmonics.

A special consideration is given to the case of the squall, i.e. an instant and large (by a factor of 2–4) increase of wind, which lasts for $O(10^2)$ characteristic wave periods. We show that fast adjustment processes lead to the formation of a transient spectrum, which has a considerably narrower peak than the spectra developed under a steady forcing. These transient spectra differ qualitatively from those predicted by the Hasselmann kinetic equation under the squall with the same parameters.

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Self-similarity of wind-driven seas and universality of wind wave growth

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For wind-driven growing waves we found a simple universal relationship that relates the instant wave steepness with the number of waves appeared since the beginning of the process of wave development. The relationship holds both in fetch and duration domains and does not contain wind-sea interaction parameters explicitly. It relies upon recent advances of the theory of weak turbulence of water waves where wave nonlinearity is assumed to be a leading physical mechanism. Self-similar solutions as reference cases of the theory are generalized for arbitrary rates of wave growth by developing a simple adiabatic approach where fetch and duration become key physical scales replacing habitual wind speed scaling. With the new scaling the dependencies of non-dimensional wave height on non-dimensional wave period are universal power-law functions with exponents 5/2 (fetch-limited case) and 9/4 (duration-limited).
The validity of the proposed theory is illustrated by results of numerical simulations, in situ measurements of growing wind seas and wind-wave tank experiments. The impact of the new vision of sea wave physics is discussed in the context of conventional approaches to wave modeling and forecasting.

Raman backscattering in plasma as method for the phase correction of intense ultrashort laser pulses

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One of the most promising methods for obtaining ultraintense ultrashort laser pulses is currently based on the use of Raman backscattering in a plasma. It can provide an output power higher by a factor of 104 105 than that with the usual technique for amplifying frequency modulated pulses in the plasma [1]. The compression regime based on Raman backscattering in the plasma was experimentally implemented [2, 3]. In particular, the implementation of a nonlinear regime with depletion of the pump pulse was demon strated. However, the nonlinear regime achieved in experiments was not transferred to the stage of a significant amplification of an output pulse. The reason was various parasitic effects responsible either for the enhancement of noise (thermal fluctuations of the plasma and the prepulse of the amplified pulse) or for the violation of the conditions of three wave matching for Raman backscattering because of the inhomogeneity of the plasma density. Here, another method of the use of Raman backscattering in the plasma is proposed to obtain intense ultrashort laser pulses with a given phase front. In this case, requirements on the parameters of the plasma and on the duration of its existence in this regime are softer than those in the usual scheme of Raman compression. The method is based on the fact that phase perturbations of the pump pulse that are smooth in the longitudinal direction are concentrated in the weakest wave (plasma wave). As a result, the amplified pulse will have a weakly perturbed initial phase front and, correspondingly, good focusability.

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Projective differential geometry of multidimensional dispersionless integrable hierarchies

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We introduce a general setting for multidimensional dispersionless integrable hierarchy in terms of differential m-form Ω_m with the coefficients satisfying the Plücker relations, which is gauge-invariantly closed and its gauge-invariant coordinates (ratios of coefficients) are (locally) holomorphic with respect to one of the variables (the spectral variable). We demonstrate that this form defines a hierarchy of dispersionless integrable equations in terms of commuting vector fields locally holomorphic in the spectral variable. The equations of the hierarchy are given by the gauge-invariant closedness equations.

Bottleneck phenomena in capillary turbulence on the surface of cryogenic liquids

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We report the experimental observation of two bottleneck phenomena near the high frequency boundary of the inertial range on the spectrum of the turbulence in the system of capillary waves on the surface of liquid hydrogen and superfluid helium driven by a harmonic force. Both effects are manifested as a local maximum on the spectrum of pair correlation function of the surface elevation. The first phenomenon has been observed on the steady state spectra of capillary turbulence on the surface of superfluid helium. The other one has been observed in experiments with liquid hydrogen; it can be seen only during reorganization of the turbulent cascade due to a generation of waves below the driving frequency. Despite the similarities between two effects we argue that they have different underlying nature. While the bottleneck on the surface of superfluid helium is caused by the detuning between the high-frequency harmonics and the resonant frequencies of the surface oscillation in a finite container [1], the bottleneck on the surface of liquid hydrogen can be associated with a distortion of the Kolmogorov-Zakharov spectrum caused by the viscous damping [2].

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Robust energy transfer mechanism via precession resonance in nonlinear turbulent wave systems

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A robust energy transfer mechanism is found in nonlinear wave systems, which favours transfers towards modes interacting via triads with nonzero frequency mismatch, applicable in meteorology, nonlinear optics and plasma wave turbulence. We introduce the concepts of truly dynamical degrees of freedom and triad precession. Transfer efficiency is maximal when the triads' precession frequencies resonate with the system's nonlinear frequencies, leading to a collective state of synchronised triads with strong turbulent cascades at intermediate nonlinearity. Numerical simulations confirm analytical predictions.

New reductions of Gauss-Codazzi equations in three-dimensional Euclidean space to the sixth Painlevé equation

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The Gauss-Codazzi equations govern the geometry of surfaces in R^n. In 1897, Hazzidakis found a reduction to a codimension three P6 equation in the case $n = 3$. Our motivation is to find a reduction to the full (codimension zero) P6. Since the Gauss-Codazzi equations are underdetermined (three equations in four unknowns), we first restrict them to a determined system and compute its Lie point symmetries. This allows us to find three more reductions to P6, with respective codimensions three, two, two.

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Local analysis of wave fields from hindcasted sea states for rogue wave risk evaluations

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Global-scale wave climate models, such as Wavewatch III, are widely used in oceanography to hindcast the sea state that occurred in a particular geographic area at a particular time. These models are applied in rogue-wave science for characterizing the sea states associated with observations of rogue waves (e.g., the well know "Draupner" [1] or "Andrea" [2] waves) or ship accidents attributed to their presence (e.g., the Suwa-Maru [3] or the Louis Majesty accidents [4]).
While wave climate models are generally successful in providing realistic representations of the sea state and are able to handle a large number of physical factors, they are also based on a very coarse-grained representation of the wave field and therefore unsuitable for a detailed resolution of the wave field and refined wave-height statistics. On the other hand, local wave models based on first principle fluid dynamics equations are able to represent wave fields in detail, but in general they are hard to interface with the full complexity of real-world sea conditions.
In this talk we will discuss our current efforts in the direction of filling this gap by a combined global- and local-scale approach, and the challenges we face in doing so. We shall discuss some feasible ways of including physical effects such as wave breaking and wind forcing in 3D random sea states simulated by the high order spectral (HOS) method [5], and will show some preliminary results on 3D simulations from hindcasted wave spectra.

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Freak-waves

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We applied canonical transformation to water wave equation not only to remove cubic nonlinear terms but to simplify drastically fourth order terms in Hamiltonian for water waves equation. After the transformation well-known but cumbersome Zakharov equation is drastically simplifyed and can be written in X-space in compact way. This new equation is very suitable for analytic study as well as for numerical simulation. At the same time one of the important issues concerning this system is the question of its integrability. The rest part of the work is devoted to numerical and analytical study of the integrability of the equation obtained. In the second part we present generalization of the improved Zakharov equation for the "almost" 2-D water waves at the surface of deep water. When considering waves slightly inhomogeneous in transverse direction, one can think in the spirit of Kadomtsev-Petviashvili equation for Korteveg-de-Vries equation taking into account weak transverse diffraction. Equation can be written in terms of canonical normal variable b(x; y; t). This equation is very suitable for robust numerical simulation. Due to specific structure of nonlinearity in the Hamiltonian the equation can be effectively solved on the computer. It was applied for simulation of sea surface waving including freak waves appearing

Nonlinear harmonic surface waves on a deep water. Experimental results in “Marintek” ocean basin (Trondheim, Norway).

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Nonlinear harmonic surface waves on a deep water. Experimental results in “Marintek” ocean basin (Trondheim, Norway).
We studied propagation of one-dimensional harmonic waves with different amplitudes in Marintek Ocean Basin. Waves were investigated within the frequency range 0.41 Hz, for amplitudes 0.0680.2 m. The deep of the basin was 3 m. The main results were got for waves with $f = 1$ Hz (wavelength = 1.56 m), so we worked mainly with waves on a deep water. We observed formation of direct and inverse energy cascades at pumping of the system by single harmonic frequency high amplitude. In the report we will discuss the Fourier spectrum of the initial harmonic waves and its evolution with time and distance, the formation of energy cascades, the interactions of two and three co-linear harmonic waves, and the influence of the noise on these interactions in the quasi-one-dimensional case.

Features of resonant tunneling of electromagnetic waves through gradient barriers plasma. Exactly solvable models.

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Exactly solvable model of resonant tunnelling of electromagnetic waves through gradient barriers in an inhomogeneous plasma are considered. Interrelations of permitivity profile of an inhomogeneous plasma and the amplitude of the wave vector have been studied for the gradient wave barriers depending on the incoming parameters of the model. It is shown that full enlightenment of the plasma layer for electromagnetic waves is possible for a even very wide wave barriers. It was found that in a nonuniform layer may contains the narrow sublayers where the wave field may have strong splashes. It is shown that in the considered exactly solvable models the wave vector profiles can automatically satisfy the reflectionless conditions on the boundaries of plasma-vacuum and contain an arbitrary function, which describes the very variable plasma inhomogeneity in the layer.

A study of two color optical breakdown of gas by investigation of irradiated terahertz pulses properties

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1. Background

During last decade laser-plasma methods of terahertz radiation generation were investigated intensively [1, 2]. These methods are characterized by using of femtosecond laser pulses ionizing neutral media and exciting electron currents which radiates broadband terahertz pulses. A number of experimental techniques were proposed in this area [2]. Most of theoretical interpretations focus on low frequency current excitation process, while spectral and spatial properties of terahertz radiation depends on nonlinear dynamics of femtosecond pulse propagating in media. From experiments [3] it is known that spectrum maxima could exceed inverse envelope time. In work [4] it is stated that terahertz radiation pattern produced by two color laser pulses (coherent admixture of laser pulse and its second harmonic usually generated in BBO crystal) has a hollow in the center. These features can not be interpreted without thoroughly investigation of nonlinear laser pulse dynamic peculiarities. On the other hand terahertz radiation features can bring some information about laser pulse propagation. In this work we investigated low frequency current (residual current excited in plasma by laser pulse) properties calculated in numerical simulations of nonlinear two color laser pulse propagation in gas. Using calculated currents we concluded some properties of terahertz radiation and compared them to experimental observations.

2. Model

We used scalar wave equation for electric field of laser pulse in paraxial approximation (similar approach was used in relativistic plasma wake field studies [5]). According to our estimations ionization supposed to be of tunneling type. Ionization rate formula was adopted from [6]. We used hydrodynamic model for current: $\partial j/\partial\tau = nE$, where j is electric current (both optical and low frequency), n is plasma density, E is laser pulse electric field, $\tau = t - z/c$ is a time in co-moving laser pulse coordinates.

3. Results

In our simulations we used real experimental parameters. We found that a region exciting low frequency currents shrinks in time due to refraction of laser pulse on breakdown plasma. This effect can significantly shift the upper bound of terahertz spectra. Despite of our expectations we saw no oscillations of low frequency current excitation efficiency due to phase detuning of first and second optical harmonics on plasma. Instead of changing the sign the source of low frequency current decays before the end of nonlinear interaction of laser pulse and gas. The source of low frequency current exists only in areas where breakdown is taking place. The most efficient low frequency current generation occurs at very first sub-front of breakdown plasma, where phase between first and second harmonic is yet preserved. According to our estimations the hollow structure of terahertz radiation pattern can be interpreted by superluminal velocity of breakdown front.

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Localization-delocalization transition of inertial particles in inhomogeneous turbulence.

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A new effect in the old field of thermophoresis (and turbophoresis) going back to Maxwell. A new theory describes motion of inertial particles in inhomogeneous turbulence and predicts particle segregation according to their sizes and densities around turbulence minima.

Dispersionless integrable systems in 3D and Einstein-Weyl geometry

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For several classes of second order dispersionless PDEs, we show that the symbols of their formal linearizations define conformal structures which must be Einstein-Weyl in 3D (or self-dual in 4D) if and only if the PDE is integrable by the method of hydrodynamic reductions. This demonstrates that the integrability of dispersionless PDEs can be seen from the geometry of their formal linearizations. The talk is based on joint work with Boris Kruglikov.

Random switching in two-level atomic system

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In two level atoms with doubly degenerated lower levels ($lambda$ - configuration), transitions from the upper level to the lower sublevels correspond to left or right polarization of the light. When relative population of lower sublevels is different, propagation of optical pulses in such medium is followed by polarization switching. This polarization switching occurs randomly in the presence of spatial inhomogeneity of the relative population of the lower sublevels. Maxwell-Bloch equation describing interaction of ultra-short electromagnetic pulses with such atomic system is known to be integrable. Integrality of Maxwell-Bloch equation allows to obtain analytic description for statistical properties of polarization switching. Analytic results are verified by direct numerical simulations

Nonlinear stage of modulation instability in the scalar and vector nonlinear Schrodinger equations.

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Superregular solitonic solutions recently discovered by Vladimir E. Zakharov and Andrey A. Gelash [1] is the important scenario of modulation instability in the frame of the one-dimensional focusing nonlinear Schrodinger equation (NLSE). This 2N-solitonic solutions describe the evolution of a broad class of an initial plane wave (a condensate) localized perturbations. In this scenario a perturbation develops into 2N quasi-Akhmediev breathers. Reverse is also true at a certain moment of time 2N quasi-Akhmediev breathers are almost annihilate into small localized solution. Recently we have generalized the theory to the degenerate case [2]. Superregular solitonic solutions form the full solitonic part of localized perturbation spectrum with rational solutions (Peregrine soliton [3] and its multisolitonic analogs [4-5]) and Kuznetsov solitons with spectral parameter close to branch point [6]. In this work we discuss the theory of superregular solitonic solutions and the possibility of their experimental observation. We present the most appropriate solution parameters for hydrodynamics and optics. The second part of this work is devoted to vector generalizations of superregular solutions. By using the dressing method we construct the N-solitonic solution on the condensate background for two-component coupled NLSE (Manakov system) and three-component coupled NLSE. In the scalar NLSE the mechanism of almost full quasi-Akhmediev breathers annihilation mathematically explained by the nulling of two-solitonic Akhmediev solution with the same spectral parameter [1,2]. We find the similar conditions in the vector case. Then we construct and discuss the vector superregular solitonic solutions.

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Riemann-Hilbert Problems and Soliton equations. The Reduction problem and Hamiltonian properties.

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Our main tool is the Riemann-Hilbert problem (RHP) with canonical normalization

$$\xi^+(\vec{x},t,\lambda) = \xi^-(\vec{x},t,\lambda)G(\vec{x},t,\lambda), \qquad \lambda^k \in \mathbb{R}, \qquad \lim_{\lambda\to\infty} \xi^+(\vec{x},t,\lambda) = \mathbb{1}, \tag{1}$$

where the functions $\xi^\pm(\vec{x},t,\lambda)$ are taking values in a simple Lie group $\mathfrak{G}$ and allow analytic extension for $\lambda^k \in \mathbb{R}$ respectively, see [1, 2, 3]. We also assume that the sewing function $G(\vec{x},t,\lambda)$ depends on the auxiliary variables $\vec{x},t$ as follows:

$$i\frac{\partial G}{\partial x_s} - \lambda^k [J_s, G(\vec{x},t,\lambda)] = 0, \qquad i\frac{\partial G}{\partial t} - \lambda^k [K, G(\vec{x},t,\lambda)] = 0. \tag{2}$$

where J_s belong to the Cartan subalgebra of the relevant simple Lie algebra $\mathfrak{h} \subset \mathfrak{g}$.
The canonical normalization of the RHP allows us to use the asymptotic expansions:

$$\xi^\pm(\vec{x},t,\lambda) = \exp Q(\vec{x},t,\lambda), \qquad Q(\vec{x},t,\lambda) = \sum_{k=1}^{\infty} Q_k(\vec{x},t)\lambda^{-k}. \tag{3}$$

where all $Q_k(\vec{x},t) \in \mathfrak{g}$. Next we use Zakharov-Shabat theorem [4] to construct a family of commuting operators L_s, M provided the coefficients $Q_j(\vec{x},t)$, $j = 1,\ldots,k$ in (3) satisfy a certain set of soliton equations. New examples of such equations [1, 2] are obtained by applying Mikhailov's $\mathbb{Z}_h$-reduction [5] on $C\xi^\pm(\vec{x},t,\lambda)C^{-1} = \xi^\pm(\vec{x},t,\lambda\omega)$ where $C^h = \mathbb{1}$ and $\omega^h = 1$. One can also derive their soliton solutions using the dressing Zakharov-Shabat method [4].
It is natural to expect that the Hamiltonian properties of the new types of soliton equations can be obtained from the the generic Hamiltonian structures of polynomial bundles [6] by considering the reduction conditions as Dirac constraints.

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Earthquakes as turbulence in the Earth's core

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Geothermal heat flux causes inhomogeneous convection in the mantle provoking earthquakes. As a consequence the brittle earth core is brocken on plates and most quakes are observed at the plate boundaries. The frequency size distribution of quakes, the Gutenberg-Richter law can be obtained at a single assumption that the correlation time of the stochastic forces causing quakes is much less than reaction time of the core in the form of quakes. The size distribution of the plate areas was found to be . The physical form of this distribution is explained by dimensional analysis and the numerical prefactor was found to be O(1).

The Cauchy problem for the Pavlov equation

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Commutation of multidimensional vector fields leads to integrable nonlinear dispersionless PDEs arising in various problems of mathematical physics and intensively studied in the recent literature. This report is aiming to solve the scattering and inverse scattering problem, recently introduced just at a formal level, for integrable dispersionless PDEs, concentrating on the prototypical example of the Pavlov equation, and to justify an existence theorem for global bounded solutions of the associated Cauchy problem with small data.

Stability of autoresonance under persistent perturbation

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Deterministic dynamical system which has an asymptotical stable equilibrium is considered under persistent perturbation by white noise. If the perturbation does not vanish in the equilibrium position than there is not Lyapunov's stability. New concept of stability on a large time interval is discussed. The measure of stability is expectation of the square distance from the trajectory till the equilibrium position. The method of parabolic equation is applied to both estimate the expectation and prove such stability. The results are applied to analyse the autoresonance phenomenon.

Nonlinear waves in a two-component Bose-Einstein condensate

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In a two-component Bose-Einstein condensate there exist two types of motions with in-phase and counter-phase oscillations of the components. Correspondingly, they can be called waves of density and waves of polarization. It is shown that in the case of small difference between values of the nonlinear interaction constants the polarization waves are described by the Gardner equation [1]. This leads to essentially new dynamics compared with the case of the density waves described by the Korteweg-de Vries equation. In particular, it is found [2] that the flow of two-component condensate past a polarized obstacle leads to generation of oblique breathers, on the contrary to the case of a non-polarized obstacle yielding generation of oblique solitons. In the limit of equal nonlinear interaction constants the condensate dynamics is described by the Manakov system whose periodic solution is found [3] which explains the results of recent experiments [4].

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Coupled Ostrovsky equations for internal waves on a shear flow

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Coupled Ostrovsky equations for internal waves on a shear flow
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We study the behaviour of weakly nonlinear oceanic internal waves in the presence of background rotation and shear flow, when two distinct linear long wave modes have nearly coincident phase speeds. The waves are described by a system of coupled Ostrovsky equations, derived from the full set of Euler equations for incompressible density stratified fluid with a free surface and rigid bottom boundary conditions. We show that in the generic case, in the absence of the shear flow, initial solitary-like waves are destroyed and replaced by two coupled nonlinear wave packets, being the counterpart of the same phenomenon in the single Ostrovsky equation. I will also discuss results obtained when there is a three layer shear flow, when the dynamics is much more diverse.

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Superradiant Lasing: Quasi-regular and Quasi-chaotic Regimes

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Analytic theory, numerical simulation, and qualitative analysis of the threshold conditions, nonlinear dynamics, and spectral features of the superradiant emission and cooperative radiative behavior of a dense many-particle system in a low-Q Fabry-Perot cavity with distributed feedback of counter-propagating electromagnetic waves are given. Novel lasing regimes, from regular to chaotic ones, are found to be originated from collective spontaneous emission of active centers and may contain well-correlated or weakly connected dynamical spectral components. Various systems with extreme spatial-spectral density of radiating particles as active media of superradiant lasers are discussed, including those with almost homogeneous broadening as well as strongly inhomogeneous broadening of a spectral line. In the case of experimental verification, the phenomenon of CW superradiant lasing will be promising in the information optoelectronics and condensed matter physics, in particular, for managing novel oscillators with complicated dynamical spectra and for creating unprecedented diagnostics of quantum coherent many-particle effects.

Strongly Nonlinear Surface Dynamics of Non-Conducting Fluid under the Action of Horizontal Electric Field

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In the present work we consider the problem of free surface motion of the fluid with high dielectric constant in the strong horizontal electric field. It is shown that nonlinear surface waves can propagate nondispersively in the direction of electric field. Moreover, the counter-propagating solitary weakly nonlinear waves preserve the initial form after collision. The question arises as to whether the same tendency is observed for strongly nonlinear waves. We have shown that collisions of strongly nonlinear waves are elastic, i.e., the energy of solitary wave does not change. However, results of our numerical simulations indicate that, in general case, the interaction of strongly nonlinear waves leads to the shape distortion and formation of regions with high surface steepness.
This study was supported by the Russian Foundation for Basic Research (Project No. 14-08-31194), and by the Dynasty Foundation.

Whitham type equations revisited: critical points and Lauricella functions.

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Semi-Hamiltonian integrable hydrodynamic type systems which describe critical points of functions obeying the linear Darboux systems are discussed. It is shown that a wide class of such systems can be constructed using the Lauricella type solutions of the Euler-Poisson-Darboux equations as sead functions. Classical multi-phase Whitham equations for the Korteweg- de Vries and nonlinear Schrodinger equations are the particular examples of such systems.

TO THE ORIGIN OF SEXAGESIMAL SYSTEM OF NUMERATION

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The Sexagesimal Numeration System, all-known from the traditional clock face, came to us from the deep Ancient Times. The system is apparently survived due to the fact that 60 has many divisors. But how could it arise? The statement of this report is that the calculus arose on the basis of finger accounts and completely identical in principle the numeration systems on the base '4' , '20' and '40'. The last is survived in Siberia to the early 20th century and can serve as a proof of our version of arising the sexagesimal system.

Waves' instabilities on a discrete grid.

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We investigated possibility of realization of different resonant interaction on a discrete grid, characteristic for wave tanks and numerical simulations for both gravity and capillary waves. New type of instability of a standing wave proposes a simple mechanism for generation of an isotropic spectrum, convenient for wave turbulence experiments, through pumping of a one standing wave. This instability is realizable for any type of waves with dispersion relation depending only on the magnitude of a wave vector in systems with present four-waves interaction, for example both surface capillary and gravity waves. All analytical results were confirmed by numerical simulation, extending our previous work on the matter (A.I. Dyachenko, A.O. Korotkevich, V.E. Zakharov, Decay of the monochromatic capillary wave, JETP Lett., 77, 9, 477-481 (2003)).

Confined low-frequency radiation from femtosecond filaments in gases

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An extended femtosecond filament is a nonlinear light structure, which can emit a continuum of frequencies as well as quasi-isolated pulses in certain spectral ranges [1-4]. If there is a seed created by means of optical parametric amplifier or other techniques, the conversion efficiency to the desired spectral range can be increased significantly [5]. The specific case is the dual-frequency (800 nm and 400 nm) interaction in the gaseous filament leading to an efficient production of terahertz pulses [6-8]. However, the major part of terahertz radiation from filaments diverges forming a ring in the far-field [9]. The degree of divergence of the newly formed frequencies is crucially important for applications including a distant object inspection.

In this paper we use a unidirectional pulse propagation equation model [10] to study numerically the intensity growth, spectral content and spatial divergence of quasi isolated pulses generated by extended femtosecond filaments in gases. All the newly-created frequencies are generated self-consistently in the course of propagation with the resolution of 0.25 THz. We demonstrate that the major reason for the on-axis propagation of the quasi-isolated pulse centered in the vicinity of 900-1000 nm is the third-order nonlinearity of neutral molecules. The THz pulse at 1-5 THz frequency originates from the free electron photocurrent and propagates in the cone surrounding the filament. We numerically obtained the spectrum from terahertz to the 3d harmonic of 800 nm initial pump pulse due to the dual-frequency interaction in argon filament. The initial 800 nm pump pulse has 3.2 mJ energy and 50 fs duration; the initial second harmonic energy is 10 mJ.

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Generalized Ronkin function for quasiperiodic dimer configurations

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A generalization of Kenyon-Okuounkov results on periodic dimers configurations will be presented. An integrable quasi-periodic structures are constructed. The corresponding surface tension is identified with function that is Legandre dual to generalized Ronkin function.

Variational principle and stationary mirror structures in a plasma with pressure anysotropy

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Based on Grad Shafranov like equations, a gyrotropic plasma where the pressures in the static regime are only functions of the amplitude of the local magnetic field is shown to be amenable to a variational principle with a free energy density given by the parallel tension. This approach is used to demonstrate that small amplitude static holes constructed slightly below the mirror instability threshold identify with lump solitons of KPII (Kadomtsev Petviashvili) equation and turn out to be unstable. It is also shown that regularizing effects such as finite Larmor radius corrections cannot be ignored in the description of large amplitude mir ror structures.

Universal profile of the vortex condensate in two-dimensional turbulence

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An inverse turbulent cascade in a restricted two-dimensional periodic domain leads to the creation of condensate – a pair of coherent system-size vortices. We perform extensive numerical simulations of this system and carry on detailed theoretical analysis based upon momentum and energy exchanges between the turbulence fluctuations and the mean coherent condensate (vortices). The theory predicts the vortex profile and amplitude which perfectly agree with the numerical data.

Observations of low-frequency surface waves in a vertically oscillating elastic container

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An initial goal of our experiment was to study mechanisms of the inverse cascade in surface wave turbulence using a well-controlled laboratory set-up with energy pumping scale in gravitational range. For this we used an open rectangular cell with the horizontal sizes 42 x 42 cm and the height 30 cm. The cell was filled with water to a depth 15-20 cm and the surface waves were generated parametrically by vertical vibrations of the whole cell. We checked all possible range of vibration frequencies corresponding to the parametric wave wavelengths from 2 to 10 cm and used different types of excitation with single or multi frequencies, but we did not observe any waves with the frequency below than the lowest parametrically forced mode has. Nevertheless, when we move to the higher frequencies corresponding to the parametric waves in capillary range, we found a narrow excitation frequency interval around 82 Hz, where the low frequency modes of large amplitudes appeared as a result of the single frequency excitation. The frequencies of these stationary waves are 2.3 and 2.7 Hz and their k-vectors are oriented along the diagonals and the side walls of the cell correspondingly. Our further studies showed that these low frequency waves are coupled with bending oscillations of the cell side walls which resonate at the frequency about 82 Hz. The amplitude of bending side walls vibrations in the open cell is minimum (zero) at the cell bottom and corners were the walls are fixed and is maximum at the middle and the top of the side wall. The forcing is provided by the oscillating hydrostatic pressure. Above some critical vibration amplitude, in addition to Faraday waves, the oscillating walls produce the cross waves at a half excitation frequency with the wave vector parallel to the wall. Further increase of the vertical vibration amplitude leads to a significant growth of the cross wave amplitude. And finally, after the amplitude of cell vibrations exceeds the next threshold the slow modes arise. We characterized the observed wave configurations using a profilometry technique allowing us to measure time evolutions of the 2D surface wave elevations $\eta(x, y, t)$. We show that the slow mode appears as the result of an instability and present k- and ω-spectra for the stationary wave field regimes. We draw a parallel between our observations and the 'dragon wash' phenomenon and discuss possible mechanisms responsible for the low frequency mode instability.

Branch Cut Singularity of Stokes Wave

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Stokes wave is the fully nonlinear gravity wave propagating with the constant velocity [1,2]. We consider Stokes wave in the conformal variables which maps the domain occupied by fluid into the lower complex half-plane. Then Stokes wave can be described through the position and the type of complex singularities in the upper complex half-plane. Similar idea was exploited for other hydrodynamic systems in different approximations, see e.g. [3-6]. We studied fully nonlinear problem and identified that this singularity is the square-root branch point. That branch cut defines the second sheet of the Riemann surface if we cross the branch cut. Second singularity is located in that second (nonphysical) sheet of the Riemann surface in the lower half-plane. As the nonlinearity increases, both singularities approach the real line forming the classical Stokes solution (limiting Stokes wave) with the branch point of power 2/3. We reformulated Stokes wave equation through the integral over jump at the branch cut which provides the efficient way for finding of the explicit form of Stokes wave [7].

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Turbulence in superfluids: ideas, experiments, mumerics and theory

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Turbulence in superfluid helium is unusual and presents a challenge to fluid dynamicists because it consists of two coupled, inter penetrating turbulent fluids: the first is inviscid with quantized vorticity, the second is viscous with continuous vorticity. Despite this double nature, the observed spectra of the superfluid turbulent velocity at sufficiently large length scales are similar to those of ordinary turbulence. I present experimental, numerical and theoretical results which explain these similarities, and illustrate the limits of our present understanding of superfluid turbulence.

Haantjes manifolds and Witham equations

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Haantjes manifolds are a mild generalization of the concept of bihamiltonian manifold. In the talk I will present an example of Haantjes manifold related to the theory of Whitham equations

Blowup as a driving mechanism of turbulence in shell models

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Since Kolmogorov proposed his phenomenological theory of hydrodynamic turbulence in 1941, the description of the mechanism leading to the energy cascade and anomalous scaling remains an open problem in fluid mechanics. Soon after, in 1949, Onsager noticed that the scaling properties in the inertial range imply nondifferentiability of the velocity field in the limit of vanishing viscosity. This observation suggests that the turbulence mechanism may be related to a finite-time singularity (blowup) of incompressible Euler equations. However, the existence of such blowup is still an open problem too. In this work, we give a numerical evidence that the blowup indeed represents the driving mechanism of the inertial range for a simplified (shell) model of turbulence. Here, blowups generate coherent structures (instantons), which travel through the inertial range in finite time and are described by universal self-similar statistics. The anomaly (deviation of scaling exponents of velocity moments from the Kolmogorov theory) is related analytically to the process of instanton creation using the large deviation principle.

On the averaged multi-dimensional Poisson brackets

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We consider the averaging of local field-theoretic Poisson brackets in the multi-dimensional case. As a result, we construct a local Poisson bracket for the regular Whitham system in the multidimensional situation. As can be shown, the special features of the space of modulated parameters in higher dimensions lead to a different natural class of the averaged brackets in comparison with the one-dimensional situation. We suggest a direct procedure of construction of the bracket for the Whitham system for $d > 1$ and discuss the conditions of applicability of the corresponding scheme. At the end, we discuss canonical forms of the averaged Poisson bracket in the multidimensional case.

Nonlinear dynamics of cosmological scalar fields with singular potentials

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We investigate the dynamics of the inflaton scalar field $\phi(t, \mathbf{r})$ governed by the nonlinear Klein-Gordon equation in the Friedmann-Robertson-Walker Universe,

$$\phi_{tt} + 3H\phi_t - a^{-2}\Delta\phi + U'(\phi) = 0,$$

where $a(t)$ is the scale factor, $H = a_t/a$ is the Hubble parameter. We consider the potentials having singularity at their minimum, $|U''(\phi)| \to \infty$ $(\phi \to 0)$.
First, the rapid oscillations of the homogeneous background $\phi(t)$ near the minimum are studied. These damped oscillations determine growth of the scale factor $a(t)$ through the Friedmann equations with the effective pressure p and energy density ρ [1]. To describe the oscillations we make the transformation $(\phi, \phi_t) \to (\rho, \theta)$, representing the field as $\phi(t) = \varphi(\rho, \theta)$, where the function $\varphi(\rho, \theta)$ is given by a quadrature, ρ and θ are slow and fast variables. Equations governing the evolution of these variables are obtained in the Van der Pol approximation.
As examples, we consider two potentials with logarithmic and fractional power singularities. For these potentials we calculate the equation of state parameter $w = p/\rho$ and show that in some range of ρ it lies in the interval $-1 < w < -1/3$, providing the accelerated expansion of the Universe.
Then we examine the resonant amplification of the field fluctuations $\delta\phi(t, \mathbf{r})$ on the oscillating background $\phi(t)$. The Fourier k-modes of $\delta\phi$ satisfy the singular Hill equation with slowly varying parameters ρ and k/a. Using the stability-instability chart, calculated by the generalized Lindemann - Stieltjes method [2,3], we argue that the fluctuations can be significantly amplified when the representative trajectories of the parameters cross the resonance zones. At the nonlinear stage these fluctuations can transform into well localized oscillating lumps, the pulsons.

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Experiments on the Visualisation and Properties of Quantum Turbulence

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Recent experiments on quantum turbulence [1] in He II are reported and discussed. They introduce and exploit the use of metastable He_2^* molecules as a means of visualising and tracing the turbulence, as well as using the scattering of very small vortex rings.

Experiments in the $T \to 0$ regime address what is, in principle, a very simple regime. In the almost complete absence of normal fluid component, quantum turbulence (QT) exists in the superfluid alone. It involves the chaotic motion of tangles of linear topological defects in the superfluid order parameter field – quantized vortices. Each vortex line contributes a flow field defined by the Biot-Savart relation, and in ^{4}He the velocity circulation around each vortex is equal to the ratio of Planck's constant to the ^{4}He atomic mass: $\kappa = h/m = 1.00 \times 10^{-3}\,\mathrm{cm}^2/\mathrm{s}$. QT consists of a dynamically evolving tangle of such quantized vortices and is essentially a macroscopic quantum phenomenon. QT decays even at the lowest temperatures, and the mechanism for such decay in superfluid ^{4}He is thought to involve a Kolmogorov-like energy cascade [2] on the Kelvin waves leading eventually to the radiation of phonons [3].

Characterization of QT can in principle be effected by tagging the vortex lines, and monitoring the evolution of the tangle. Micron-sized particles of solid hydrogen have been used [4] for this purpose, but are unsuitable for use at very low T because their introduction heats the liquid too much and because of their relatively large size. An alternative, which we discuss here, is the use of metastable He_2^* molecules. They can be created by a laser pulse or in the course of field emission from a sharp metal tip immersed in the liquid. They are bound to the vortex cores. Their arrival can be detected either by the ionization they produce in a metal electrode [5] or by their fluorescence [6] when illuminated by a laser.

A long-term aim is an experiment on pure QT in the $T \to 0$ limit where the evolution of a decaying tangle of quantized vortex lines is monitored and measured quantitatively by He_2^* fluorescence. All the elements for this are now in place: methods of creating QT frictionlessly, to minimise heating [7]; confirmation that He_2^* molecules can be created at high densities by laser excitation and that they fluoresce within the liquid [8] above 1 K; and a demonstration that the molecules are indeed trapped on vortex cores in the low T limit [9].

Recent measurements [10] have enabled a comparison to be made between the QT created by a moving grid, by ions, and by spin-down when the Manchester rotating cryostat is stopped, and will be discussed.

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Vortical laser bullets in femtosecond pulse propagation

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We present numerical modelling of nonlinear propagation of ultra-short laser pulses with vorticity (vortical bullets) in dielectrics. Such bullets are used in femtosecond laser modification of dielectric materials by producing an ionized medium in the focal domain. We show that vorticity significantly modifies dynamics energy absorption which is essential for this method of laser modification of material.

Generation of subharmonics in the turbulent spectra of liquid hydrogen

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The results of experimental investigations of the waves generated on the surface of liquid hydrogen in the rectangular cell are presented. It is observed that in case of the monochromatic pumping in addition to direct Kolmogorov-Zakharov turbulent spectrum of the capillary waves under certain conditions one can see generation of the low frequency harmonics placed in the range of transition from the capillary to the gravitational waves.

Darboux transformations with tetrahedral reduction group and nonlocal symmetries

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We study Lax-Darboux scheme which is invariant with respect to the tetrahedral reduction group. We have found a generic and four degenerate elementary Darboux maps and corresponding differential-difference (DΔEs) and partial-deference (PΔEs) integrable systems. Some of these PΔEs have a reduction to a scalar 6-point equation, which can be regarded as a difference analogue of Kuprschmidt's KdV6 equation. Differential-difference equations are non-local symmetries of the corresponding partial-difference systems. We are making steps towards the extension of the Symmetry approach to the case of non-local symmetries.

On the symmetries of the Manakov-PMD equation

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Singular KdV solutions and isospectral deformations

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TBA

Hamiltonian Formalism of the Benney system in Three Dimensions.

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We construct Benney hydrodynamic lattice associated with three dimensional Benney system describing a long waves of a fluid of finite depth on a plain. Simultaneously we show that this hydrodynamic lattice is connected with three dimensional Vlasov kinetic equation. We present simple multi-component reductions for these models.

Soliton interaction and turbulence in KdV-like models

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An approximate theoretical description of solitonic gases in the integrable models like the Korteweg-de Vries equation was proposed by V. Zakharov [1]. He has shown that the pair interaction of solitons plays important role in the dynamics of the ensemble of solitary waves. Later this research direction has been successfully pursued by G. El and his co-authors using the inverse scattering technique [2]. In our study we analyse theoretically the pair soliton interaction and numerically the dynamics of random soliton ensembles in integrable (Korteweg-de Vries, modified KdV, Gardner equations) systems and non-integrable (Benjamin-Bona-Maxoni equation) systems. Firstly, the known two-soliton solutions of the integrable KdV, mKdV and Gardner equations are used to calculate the moments of wave field. We have shown that the two soliton interaction leads to significant variations of the third and fourth moments in the dominant interaction region while the first two moments are integrals of these equations and remain unchanged due to the conservation of the mass and momentum. In particular, higher moments are decreased if solitons are of the same polarity and increased if solitons have opposite polarities. The magnitudes of the relative variations of the third and fourth moments turn out to be non-monotonic functions of the soliton amplitude ration having a maximum in the transition region between the exchange and overtaking scenarios of two soliton interactions. The qualitative implication of this dynamical effect for the soliton turbulence theory will be a change of the skewness and kurtosis of the turbulent wave field in the regions of higher density of solitons. For the KdV equation these results are published in [3]. Several numerical experiments on the solitonic gas turbulence in the framework of an integrable KdV and a nonintegrable regularized KdV BBM equation are presented. First of all, we showed that the probability distribution for the solitonic gas remains quasi-invariant during the system evolution for both KdV and KdV BBM cases. The special attention was paid to the statistical characteristics such as kurtosis and skewness which measure the 'heaviness' of tails and the asymmetry of the free surface elevation distribution. In particular, using the asymptotic methods and Monte Carlo simulations we showed that both skewness and kurtosis increase with the Stokes Ursell number S and decrease when the BBM term coefficient. When both parameters are increased gradually and simultaneously, these effects are in competition: first we observe the increase of these statistical characteristics, but then, this tendency is inversed and they decrease after reaching their respective maximal values. We would like to underline that the proposed Monte Carlo methodology is much less computationally expensive than direct numerical simulations. Despite the small number of Monte Carlo runs (100) the estimated statistical error is sufficiently small for the purposes of this study. On the other hand, this approach is restricted, strictly speaking, to the situations where the solitons are well separated in space. These results are published in [4]. The present study opens a number of perspectives for future investigations. More general nonlinearities could be included into the model along with some weak dissipative and forcing effects. This could allow us to observe Kolmogorov spectra of a solitonic gas. The nonintegrable effects need some time to be accumulated. Consequently, even longer simulation times are needed. The interaction of a solitonic gas with a random radiation field has to be studied as well.

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KPII: Cauchy–Jost function, Darboux transformations and totally nonnegative matrices

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Properties of the Cauchy–Jost (know also as Cauchy–Baker–Akhiezer) function in the case of pure solitonic solution are discussed in detail. In terms of this function action of the Baäcklund transformation is derived. Transformation of an auxiliary totally nonnegative matrix parameterizing generic soliton solution under such transformations is given. Relation of the Baäcklund transformation with property of total nonnegativity property of matrices is discussed.

Self-similarity of wind input terms and magic numbers

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We study numerically solutions of Hasselmann equation for different wind input terms, including those used in operational wave prediction models, for limited fetch growth statement. Some of the wind input terms are well physically justified and some are not, but simulation results with first ones exhibit remarkable universal properties – self-similar behavior of total energy and frequency as a functions of the fetch distance, described by power laws obtained from self-similarity analysis of Hasselmann equation. It is more remarkable that even for not so well physically justified wave input terms, we do observe absence of good self-similar behavior of the total energy and spectral peak frequency, but still can construct specific "magic" combinations of specific powers of total energy and peak frequency, obtained from self-similar analysis of Hasselmann equation for limited fetch waves growth, which are preserved with good accuracy along the fetch. We explain this "strange" coincidence as locally realized self-similar regimes in individual locations of the fetch. In summary, we observe the universality of the wind wave growth for wide spectrum of the wind input terms historically developed for Hasselmann equation, exhibiting itself in either direct manifestation in the form of self-similar laws of the total energy and peak frequency as function of fetch, or preservation of the specific combinations of the powers of theoretical self-similar solutions. In our opinion, the observed properties emphasize the key role of nonlinearity in the evolution of wave ocean surface.

Self-organization and generation of large scale flows in quasi 2D turbulence

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The self-consistent generation of large scale flows - zonal flows - by the rectification of small scale turbulent fluctuations is a generic feature of quasi-2D turbulent flows. It is of great importance both in geophysical flows and in magnetically confined plasmas. These flows, which are generally sheared, will regulate the turbulence, suppressing the small scale structures and set up effective transport barriers.
The purpose of this presentation is to discuss the morphology of zonal flows, basic mechanisms for their generation and their influence on turbulence and the associated transport in magnetized plasmas and rotating fluids.
Zonal flow generation will be illustrated in a simple fluid experiment performed in a rotating tank with radial symmetric bottom topography [1]. The results are discussed by the concept of the potential vorticity, PV, which is a Lagrangian conserved quantity. An effective mixing that homogenises PV will lead to replacing the high PV near the centre of the tank with low PV from the outside, and this will appear as an anti-cyclonic vortex over the centre, hence a large scale flow.
In magnetically confined hot plasma turbulent transport is the dominating channel for radial density, energy and momentum transport across the confining magnetic field. However, spontaneously generated sheared poloidal flows strongly reduce the radial turbulent transport and are instrumental in the rapid transition to an enhanced confinement state (the H-mode), with suppressed turbulent transport. The transition from low (L-mode) to the high (H-mode) confinement is still not understood from first principles. Recent investigations of zonal flow generation in plasma turbulence and the role of these in the L-H transition will be discussed, and illustrated by low-dimensional models of the predator-prey type [2].

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The Fermi – Ulam Problem and Dynamical Billiard with Solitons

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The Fermi Ulam problem deals with the complex dynamics of a classical point particle bouncing between harmonically oscillating walls (a dynamical billiard) [1,2]. Though the problem can be applied to various fields of physics, similar problems with respect to solitons were not considered yet, as far as we know. In the talk, we analyze features of "longitudinal" and "transverse" solitons in a dynamical billiard. For definiteness, we consider the atomic Bose Einstein condensate (BEC) in a dynamical trap with oscillating walls [3-5].

The initial governing equation is the Gross Pitaevskii equation for the wave function of weakly non-ideal atomic gas at zero temperature. For BEC confined by the trap in the transverse directions and ideal barriers with square potential moving along the longitudinal direction, the BEC wavefunction obeys the nonlinear Schrödinger equation with zero conditions at oscillating boundaries. In unbounded scheme (without barriers) the equation is solvable by the inverse scattering method and support sech-type solitons [6]. Such "longitudinal" soliton interaction with an ideal motionless barrier can be described as interaction of this soliton with its antiphased mirror image. For harmonically oscillating barriers, we demonstrate various scenarios of the interaction including soliton quasi-periodic and chaotic motion.

In regimes when the BEC wave packet diffuses over the whole trap length, the governing equation for the case of harmonically oscillating barriers has highly non-equidistant quasienergy spectrum. If the oscillation frequency is close to the frequency of transition between a pair of the levels, it is possible to realize their resonance interaction in the framework of two-level scheme. For these levels' amplitudes the transverse dynamics is governing by a Manakov-type system of equations with coherent linear and incoherent nonlinear coupling. We present families of "transverse" two-component (vector) solitons corresponding to these equations and various scenarios of solitons' interaction including breather formation.

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Ideal hydrodynamics inside as well as outside non-rotating black hole: Hamiltonian description in the Painlevé-Gullstrand coordinates

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It is demonstrated that with using Painlevé-Gullstrand coordinates in their quasi-Cartesian variant, the Hamiltonian functional for relativistic perfect fluid hydrodynamics near a non-rotating black hole differs from the corresponding flat-spacetime Hamiltonian just by a simple term. Moreover, the internal region of the black hole is then described uniformly together with the external region, because in Painlevé-Gullstrand coordinates there is no singularity at the event horizon. An exact solution is presented which describes stationary accretion of an ultra-hard matter ($\varepsilon \propto n^2$) onto a moving black hole until reaching the central singularity. Equation of motion for a thin vortex filament on such accretion background is derived in the local induction approximation. The Hamiltonian for a fluid having ultra-relativistic equation of state $\varepsilon \propto n^{4/3}$ is calculated in explicit form, and the problem of centrally-symmetric stationary flow of such matter is solved analytically.

Spontaneous breaking of the spatial homogeneity symmetry in wave turbulence leading to the formation of radiating pulses

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I discuss recent results that show that spatial homogeneity, the symmetry property that all statistical moments are functions only of the relative geometry of any configuration of points, can be spontaneously broken by an instability of the finite flux Kolmogorov-Zakharov spectrum in certain (usually one dimensional) systems. As a result, wave turbulence is not a statistical attractor in this case. As I show for a particular system (the one-dimensional Majda-McLaughlin-Tabak equation), the new nonequilibrium state is dominated by radiating solitary pulses. The evolution of these pulses becomes the main mechanism of transfer of energy from long spatial scales to short scales. The spectrum of this system (the Majda-McLaughlin-Tabak spectrum) can be derived from the evolution of these pulses.

Resonant control of solitons

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Control of envelope solitons and, in particular, amplification of solitons, is a fundamental problem in nonlinear optics, spin waves in magnetics and another applications in condensed matter physics. An effective method to excite and control envelope solitons was proposed in Refs.[1,2]. The method was based on the autoresonant phenomenon when the soliton was captured by the resonant driving with the frequency close to the internal frequency of the soliton. However, the method appears very difficult for applications because the capture of the soliton of a finite amplitude occurs only in a narrow range of the driving frequencies and with strong restrictions on the phase of the driving. In this paper we propose that, in contrast to the autoresonance, the promising method of control and amplification of solitons can be based on the "scattering on resonance" [3,4] without capture of the soliton. The control occurs by the frequency modulated driving with multiple crossing of the resonant frequency of the soliton [5]. The proposed method overcomes strong restrictions on the driving which are required to control solitons in the autoresonant approach.

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Processes of concentration of energy during the formation of rogue waves

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Consider rogue waves. We are studying these waves using computational experiments. We are seeing the emergence of rogue waves in detail. In this case we consider the local concentration of energy in a wave. It is shown that rogue wave has an order of magnitude greater energy than the average energy of the waves.

The results obtained allow reformulating the definition of the killer waves. Also, these results contribute to a better understanding of the mechanisms of these waves.

Waves on jet currents: a new paradigm and novel mechanisms of freak wave formation

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We develop a new paradigm of how to describe linear and weakly nonlinear dynamics of waves on jet currents with a particular emphasis on new mechanisms of freak wave formation.
From numerous seamen accounts and insurer records it has been known for long time that rogue waves events are quite frequent on certain currents, e.g. on notorious in this respect Agulhas current. The theoretical explanation of this fact is still lacking, which is due to our overall poor understanding of wave nonlinear evolution on currents. We address this challenge by developing a new systematic asymptotic theory of waves dynamics on jet currents which does not rely on the WKB approximation. The solutions for jet currents with arbitrary lateral profiles are found by first solving a linear 2D boundary-value problem in terms of an asymptotic series in natural small parameters. In essence, we employ approximate separation of variables, which is always justified for the oceanic conditions.
There are three types of waves on the current: passing through, reflected and trapped. The key role in the new approach is played by the trapped modes. The trapped modes themselves (rather than comprising them harmonic components) participate in the nonlinear interactions. A general weakly nonlinear theory of trapped mode evolution is being developed. In particular, the corresponding interaction coefficients have been derived. The properties of resonant interactions are qualitatively different from those between the waves in the absence of a current. In particular, three-wave interactions are always allowed in deep water and may play an important role in wave field evolution. There are three main advantages of the developed approach: (i) it is systematic and can identify and address the situations where the commonly adopted paradigm is not applicable; (ii) the current could be almost arbitrary, i.e. weak/strong, or smooth/with sharp edges; (iii) the initially 3-D problem is reduced to solving 1D evolution equations with the lateral and vertical dependence being prescribed by the corresponding modal structure. The modes can participate in both three and four- interactions. The prevalence of one type over another depends on parameters of waves and currents. The rich variety of possible regimes is being explored.
From the perspective of rogue wave occurrence we have identified several new mechanisms which have no analogues in the absence of currents.

Solitons of trapped waves on jet currents

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The asymptotic modal approach developed in Shrira & Slunyaev (2014) for waves trapped by an opposing jet current is extended by examining the weakly nonlinear dynamics of trapped waves due to four-wave resonances. Evolution equations governing dynamics of an arbitrary number of wave packets have been derived. In particular, for a single mode the asymptotic procedure yields the integrable one-dimensional nonlinear Schrodinger equation (NLS). The NLS describes the evolution of modes along the current, while the modal structure is specified by the corresponding boundary value problem (BVP). When the current is weak in comparison with the wave celerity, the BVP reduces to the classic stationary Schrodinger equation with conditions of decay outside the jet, which allows exact solutions for a number of model current profiles. This enables us to find analytically the interaction coefficients in the dynamic equations. Thus, to the leading order a variety of analytic solutions to the evolution equation and the BVP specifying the trapped modes is readily available. A few such asymptotic solutions are tested in numerical simulations of the Euler equations. The equations are solved by means of the adapted High Order Spectral Method (West et al, 1987). Single trapped mode solutions are simulated: the uniform waves train, modulated wave train, and solitary wave packets. The weakly nonlinear theory is shown to be a reasonable first approximation to the solution even in the case of rather steep waves. Solitary patterns of trapped waves were found to be robust, though an insignificant radiation is observed in the course of their propagation, which suggests that the solitary wave patterns represent important elements of nonlinear dynamics of gravity waves on jet currents. Their presence in the stochastic wave field may result in significant deviation from the Gaussianity, and increase the extreme wave probability.

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Dynamics of two-dimensional dark solitons in the inhomogeneous flow of the Bose-Einstein condensate

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Dynamics of vortex structures and their interaction with each other in many respects determine key aspects of evolution for the ultracold Bose gas cloud with repulsing interaction between atoms. Quantum vortices (topological defects or phase singularities) are rightly associated with the breaking of the superfluidity mode and the transition of the Bose-Einstein condensate (BEC) to the turbulent state. Thus, it is important to make a maximal progress in solving the problem of describing different kinds of motion for interacting vortex formations. As it was shown in our works [1-3], in the case when the distance between a vortex and an antivortex is substantially smaller than the characteristic scale of the medium inhomogenity the distributions of both the density and the velocities field in vortex pairs are similar to those that take place for the homogeneous flow-free condensate in correspoding solitary solutions of Gross Pitayevsky (GP) equation (equivalent to nonlinear Shrodinger equation (NSE) in terms of dimensionless variables). In this case one may introduce a concept of the "two-dimensional dark quasisolitons". Such quasisolitons represent the holes of BEC concentration, propagating in the inhomogeneous Bose gas at subsonic velocities with acceleration and, generally speaking, along the curved paths. We have developed the asymptotic theory describing behaviour of two-dimensional quasisolitons structures in a smoothly inhomogeneous resting condensate and taking into account many peculiarities of the Bose gas. In the present work this theory was essentially modified and generalized to the case of the two-dimensional dark quasisolitons motion in the smoothly inhomogeneous flows of BEC with repulsing interaction between atoms. Using this theory we have succeeded in detailed analizing and explaining peculiarities of scattering for both vortex and vortex-free quasisolitons formations on a single vortex and on condensate flows arising in the process of laminar flowing around barriers which move in the Bose gas at the constant subsonic velocity. The results of the direct numerical simulation performed within the framework of the GP equation are demonstrated a good agreement with the developed theory.

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Polynomial forms for Calogero-type Hamiltonians

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A transformation of the quantum elliptic Calogero-Moser Hamiltonian to a differential operator with polynomial coefficients is found. A classification of special integrable quantum Hamiltonians with polynomial coefficients is considered.

Nonlinear vector waves in the atomic chain model

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Flexural transverse waves in the anharmonic chain of atoms is considered and the nonlinear vector equation for the phonon modes is derived in the long-wave approximation, when the dispersion effects are relatively weak. Depending on the relative strength of the coupling constants characterising the chain different model equations can be obtained from the basic equation. In one particular case the equation derived can be reduced to the non-integrable vector mKdV equation earlier obtained in the paper by Gorbacheva and Ostrovsky (Physica D, 1983). In another limiting case two new model equations can be obtained. One of them is the vector two-wave equation dubbed here the 'second-order cubic Benjamin Ono (socBO) equation', and another one is its one-wave version. The last two equations describe weakly nonlinear transverse phonon modes in a chain with the quadratic dispersion law. Another interesting particular case takes place for the specific intermediate value of the coupling constant, when the nonlinear term in the equation derived is balanced by the sixth-order dispersion. Stationary solutions to the derived equation are studied. Conditions of existence of physically reasonable periodic and solitary type solutions are found. It is shown that among solitary solutions there are plane vector solitons and helical solitons. Interactions of plane solitary waves with different polarisations as well as helical solitons are studied by means of numerical simulation. It is shown that two plane solitons with the same polarisation interact elastically, whereas the interaction of solitons lying initially in the different planes is inelastic. Interesting features of interaction of helical solitons are presented.

Nonlinear phenomena with whispering gallery modes

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Whispering gallery modes (WGMs), which are strongly localized near the surface of a resonator owing to the total internal reflection, are known since the time of Lord Rayleigh. During the last decade, a lot of research interest has been attracted to optical WGM micro-resonators made of different transparent solid-state materials – glasses and crystals (both χ^2 and χ^3). The sizes range from tens of μm to several mm, the shape is typically axisymmetric (spherical or not), and different fabrication techniques can be used. The problem of coupling light in and out of the resonator is reliably solved: Different couplers (prism, fiber, etc.) can transfer up to 100% of pump light to individual WGMs.

The main figures of merits for WGMs are the quality factor Q and the modal cross-section σ. The values of Q reach nowadays 10^{11} bringing the line width to a sub-MHz range, and the values of σ can be as small as $\sim 10^2\,\mu\text{m}^2$. Correspondingly, the light intensity in WGM, $I \propto PQ/\sigma$ can be huge even for a very small pump power P. Thus, very weak coherent continuous-wave light sources can initiate strong nonlinear effects. The latter can be very specific. Furthermore, a weak material nonlinearity can be easily compensated by high quality factors of the interacting waves. The window of transparency becomes a crucial issue.

The back side of the ultra-strong intensity enhancement is discreteness of the frequency spectrum. It causes problems for the phase matching, so that additional tuning means should be used.

Among the nonlinear effects in WGM resonators are generation of frequency combs and transition to chaos, excitation of mechanical vibrations and acoustical WGMs, and also modified optical parametric oscillation and second-harmonic generation. My goal is to overview these and some other nonlinear phenomena.

Inverse Scattering Transform in Coherent Optical Communications

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I will overview recent progress in application of the inverse scattering transform in coherent optical fibre communications.

Inverse and direct cascades in 2d Gross-Pitaevskii equation

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We study inverse and direct cascades in 2D Gross-Pitaevsky (nonlinear Schrodinger) equation. This is the case where no local cascades can exist and yet we derive analytically the exact flux law (analog of Kolmogorov's 4/5-law) and confirm it in direct numerical simulations. We compare turbulence spectra to analytical predictions of Dyachenko, Newell, Pushkarev, Zakharov (1992) and Malkin (1996). A unified picture of optical turbulence emerges.

Analytical theory of wind-driven sea

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TBA

Particular solutions to a mutltidimensional version of n-wave type equation

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We represent a version of the dressing method allowing one to construct a new class of n-wave type nonlinear partial differential equations (PDEs) whose solution space may be parametrized by arbitrary functions of several variables. The algorithm is based on the integral equation of special type. Its reduction to the classical D-bar problem is discussed. There is no restrictions on the dimensionality of nonlinear PDEs. The associated solution manifold is parametrized by the arbitrary functions of several variables, but this freedom is not enough to provide the full integrability of nonlinear PDE. Some integrable reductions of the derived nonlinear system are studied. We also represent matrix multi-lump 5-dimensional particular solutions to the reduced system.

Formation of singularities at the interface of two fluids as a result of the Kelvin-Helmholtz instability

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The dynamics of singularity formation on the interface between two ideal fluids is studied for the Kelvin-Helmholtz instability development within the Hamiltonian formalism. It is shown that the equations of motion derived in the small interface angle approximation (gravity and capillary forces are neglected) admit exact solutions in the implicit form. The analysis of these solutions shows that, in the general case, weak root singularities are formed on the interface in a finite time for which the curvature becomes infinite, while the slope angles remain small. For Atwood numbers close to unity in absolute values, the surface curvature has a definite sign correlated with the boundary deformation directed towards the light fluid. For fluids with comparable densities, the curvature changes its sign in a singular point. In the particular case of fluids with equal densities, the obtained results are consistent with those obtained by Moore based on the Birkhoff-Rott equation analysis.

Index