Theoretical round. Problems to solve

General note. Maybe not all problems have correct questions. Some questions (maybe the main question of the problem, maybe one of the subquestions) may make no real sense. In this case you have to write in your answer (in English or Russian): «**impossible situation – ситуация невозможна**». Of course, this answer has to be explained numerically or logically.

Data from the tables (Planetary data, stars, constants, etc.) may be used for solving every problem.

The answers «**Да-Yes**» or «**Нет-No**» have to be written in English or Russian.

1. **Dreams of the Polar Bear.** Terrestrial Polar Bear-observer, watching the lunar disk in the phase of the full moon, in his dreams became thousand times closer to it. What did the magnitude of the Moon become? The solution has to include an artistic picture with an image of the Bear-observer on the North Pole.

2. **Great oppositions.** Now great oppositions of Mars happen every 15 or 17 years. To simplify colonization of Mars, to improve environmental conditions on it, and to increase the effect of the great oppositions, our civilization has conceived an ambitious project: to reduce by 6.0% the semi-major axis of this planet's orbit, while keeping the eccentricity of the orbit unchanged. How often will the great oppositions of Mars occur in the case of the new orbit?

3. **Proxima Centauri.** Can the star Proxima Centauri be visible from the vicinity of Alpha Centauri by naked eye? (**«да-yes»** or **«нет-no»**). Justify your solution by calculations of the corresponding apparent magnitude of Proxima Centauri.

4. **Hydroplanet.** Hydroplanet consists of a rocky "core" of radius ***R*** and a thick layer of water surrounding it from all sides. Local humans live at the bottom of this world's ocean (i.e., on the surface of the "core"), and hydrosphere is an analogue of our atmosphere for them. Local scientists observe astronomical objects from the bottom of the ocean. Like on our Earth, the duration of day-night period on Hydroplanet is T = 24 hours.
   4.1. Find the minimum depth ***H*** of the ocean, for which celestial bodies will be visible at the horizon.
   4.2. What will be the duration of the day for inhabitants of the planet's equator? The disk of the central star can be considered as a point source of light.
   4.3. Calculate the value of "atmoshperical" refraction at the horizon on such a planet.
   Points of your solution have to include drawings with all necessary sizes or angular sizes. The outer surface of the ocean is smooth, no waves or ripples.

5. **Climate.** The XXXII (32^{nd}) century AD. Due to unknown reasons, the Sun became cooler, not changing its size, and the inclination of ecliptic to the celestial equator plane decreased to zero. Scientists proved that the problem of preserving the Earth's climate and the annual change of seasons can be solved by a small star, negligible mass, but similar to a main-sequence star, moving in a circular orbit around the Sun. The star will revolve around the Sun as an inner planet, resulting in establishing a temperature regime on the Earth similar to the actual one at Issyk-Kul: the average winter and summer temperatures are -3 and +17 degrees respectively.
   5.1. Estimate the temperature of the cool Sun.
   5.2. Estimate to what spectral class the star which will be placed in the desired orbit will correspond.

6. **Sombrero Galaxy.** The Sombrero Galaxy, M104, is known as a holder of one of supermassive black holes. Images of the galaxy and its center (left side) have angular sizes 10'×7' and 10.8"×7.2" respectively. To the right, three spectra from the Hubble space telescope (in the region containing the emission lines from ionized oxygen and nitrogen) are shown for regions around the center of the galaxy. The bottom spectrum is from a region around the very center, while other two spectra are from flanking region. The values on top of the dotted lines indicate the rest (laboratory) wavelength in Ångströms.
   6.1. Is M104 approaching us or receding from us? (Answer this question by sketch drawing.)
   6.2. Estimate the velocity of M104 towards us or away from us.
   6.3. Roughly estimate the approximate mass of the supermassive black hole in the center of M104 (answer in solar masses $\mathbf{m}_\odot$).