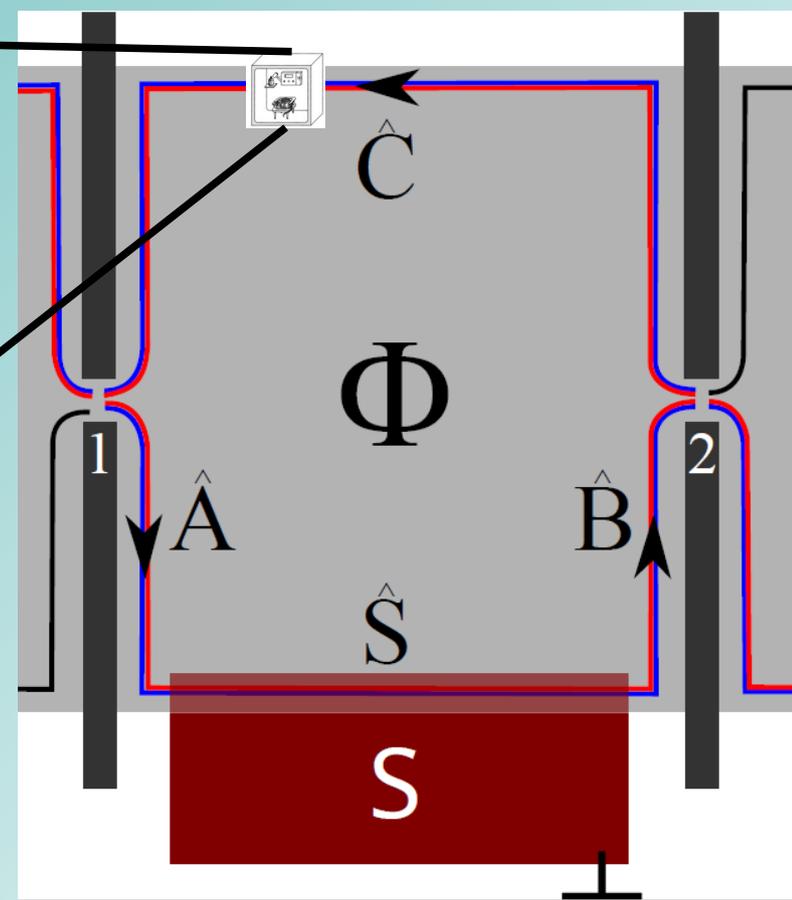
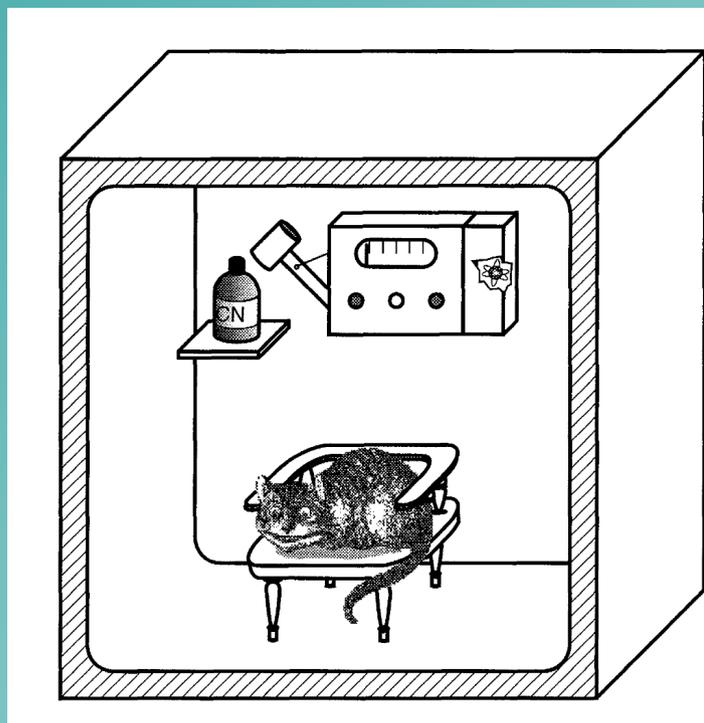


Шрёдингеровский кот в интерферометре Боголюбова

Вадим Храпай

ИФТТ РАН, Черноголовка



Квантовая суперпозиция — это база

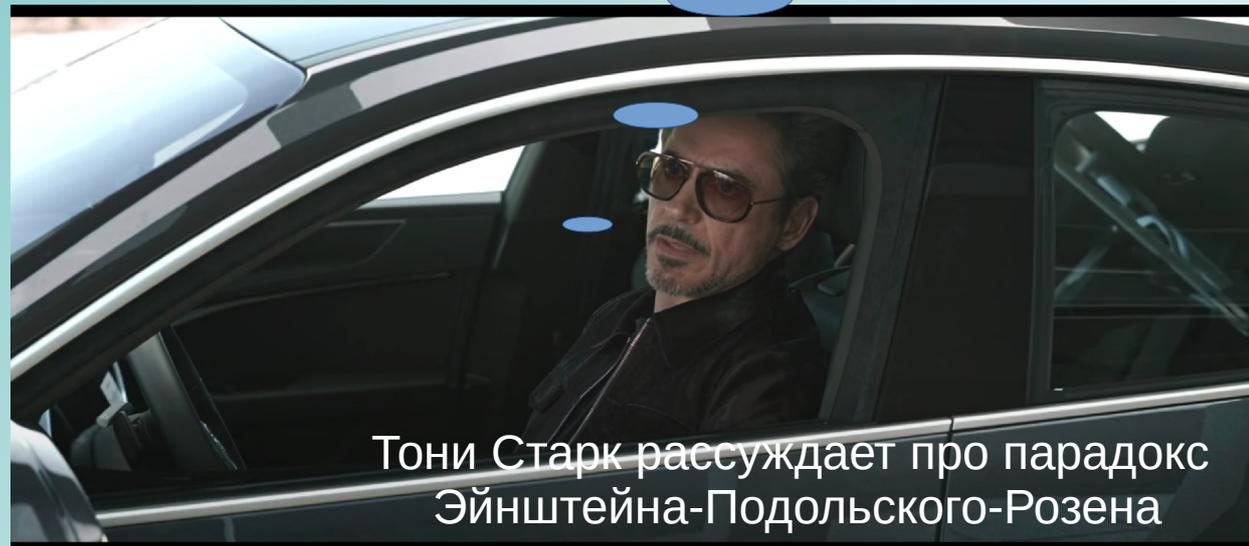


$$|\Psi\rangle = \alpha_1 |\text{cat alive}\rangle \otimes |\text{atom radioactive}\rangle + \alpha_2 |\text{cat dead}\rangle \otimes |\text{atom decayed}\rangle$$

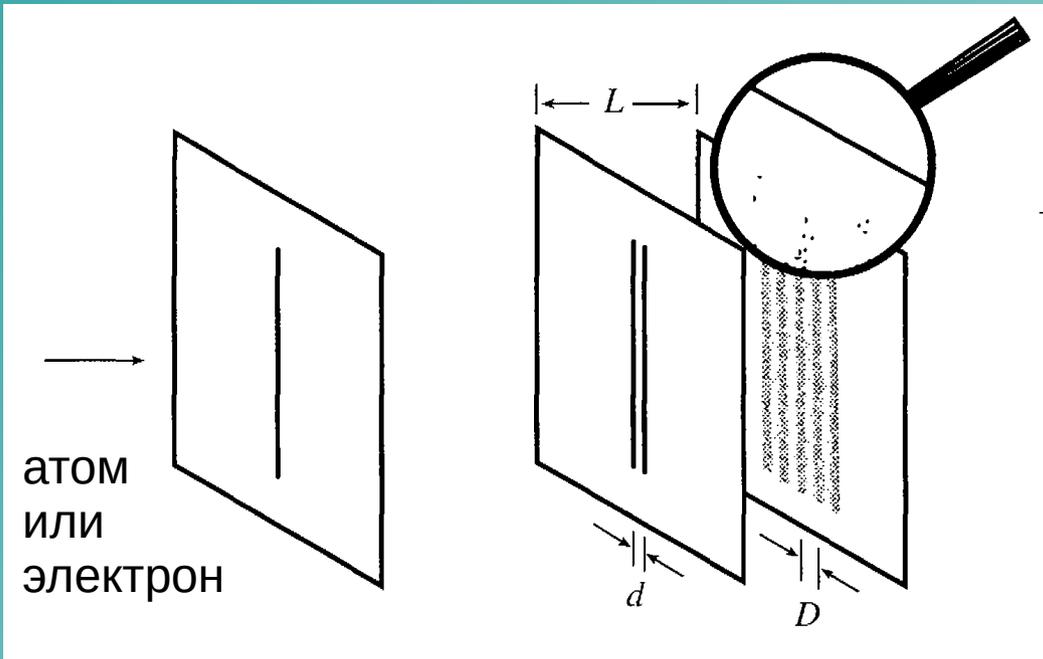
It's the EPR paradox!

$$|\Psi\rangle = \alpha_1 |\Psi_1\rangle + \alpha_2 |\Psi_2\rangle$$

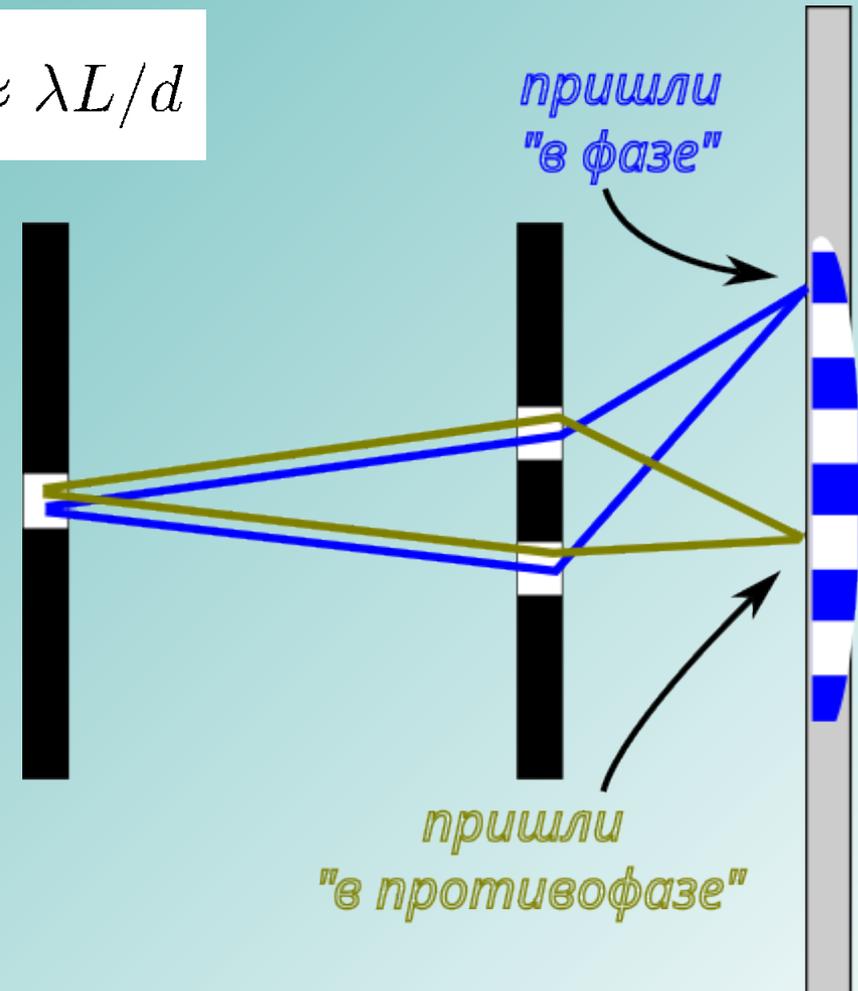
Волновая функция как суперпозиция различных квантовых состояний системы



Интерференция — эксперимент с двумя щелями



$$D \approx \lambda L/d$$



Посмотрите видео на Veritasium| VertDider

Картинка для частиц
точно как для волн!

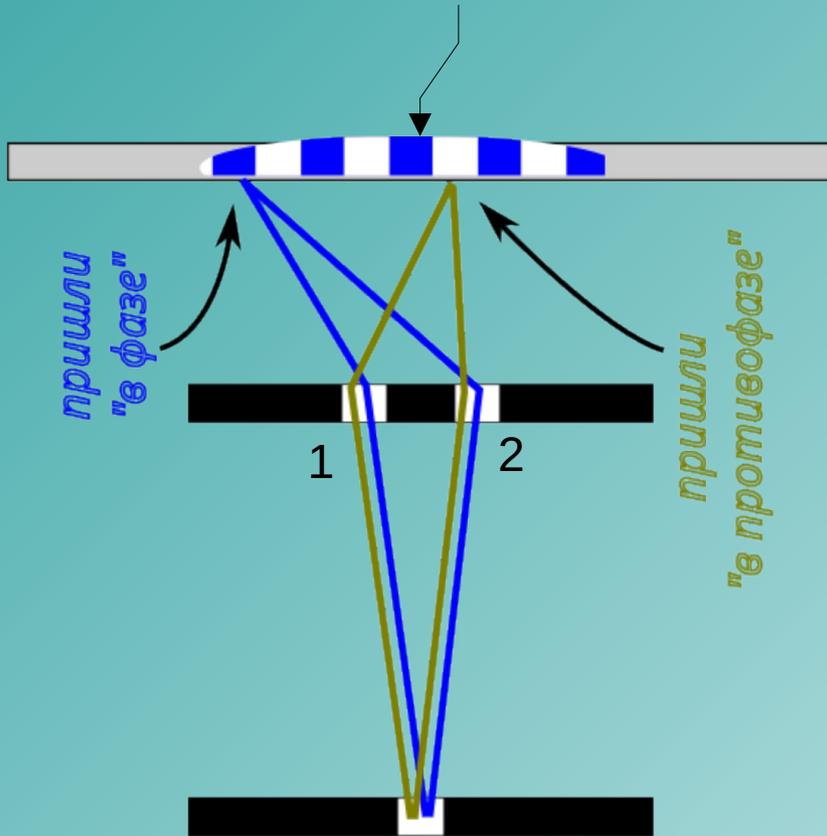
Интерференция — эксперимент с двумя щелями

Интенсивность = плотность вероятности
= квадрат модуля волновой функции

$$|\Psi\rangle^2 = (\alpha_1 \cos(\mathbf{k}_1 \mathbf{r}_1) + \alpha_2 \cos(\mathbf{k}_2 \mathbf{r}_2))^2$$

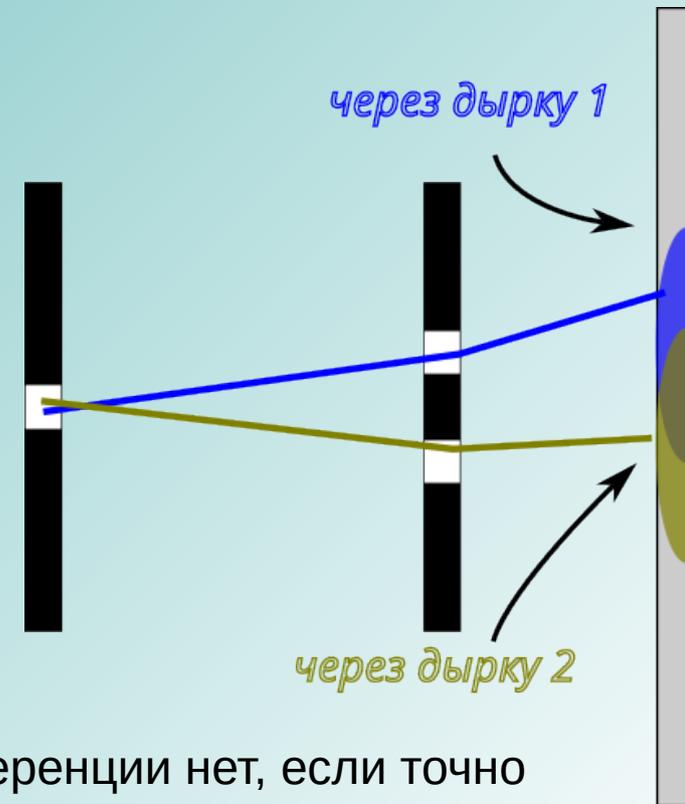
На самом деле так:

$$|\Psi\rangle^2 = |\alpha_1 \exp(i\mathbf{k}_1 \mathbf{r}_1) + \alpha_2 \exp(i\mathbf{k}_2 \mathbf{r}_2)|^2$$



Картинка для частиц точно как для волн?

Ну не совсем....



Никакой интерференции нет, если точно известно по какому пути прошла частица!

Парадокс с соленоидом. Фаза Ааронова-Бома

Парадокс:

Никакая сила не действует на электрон

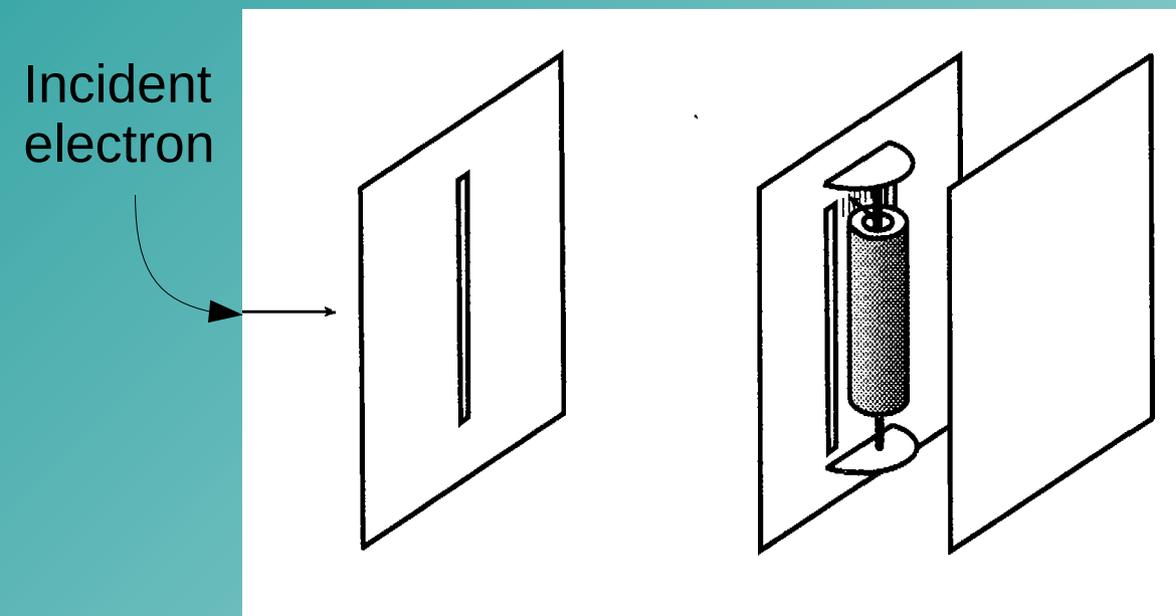
Но сила действует на красный цилиндр, и она зависит от того, с какой стороны от него пролетел электрон.
(закон Фарадея)

Значит, мы можем узнать путь, не разрушив интерференционную картину??! На самом деле нет.

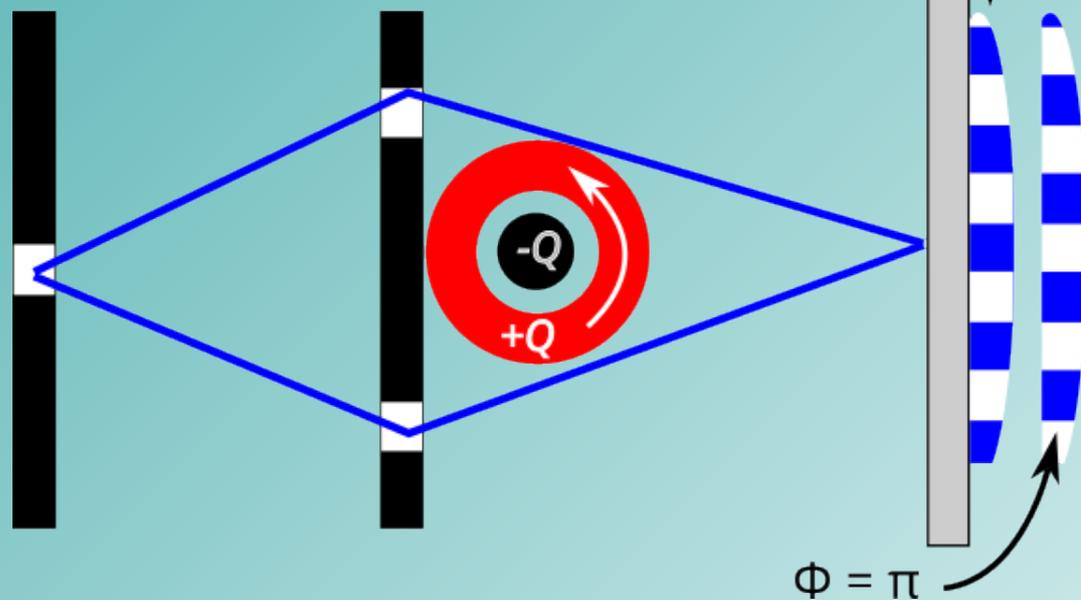
И все-таки можем воздействовать на интерференцию фазой Ааронова-Бома

$$|\Psi\rangle = \alpha_1 |\Psi_1\rangle + e^{i\Phi} \alpha_2 |\Psi_2\rangle$$

$$\Phi = e/\hbar \cdot \int B dx dy$$



$\Phi = 0$



Ферми море. Электрон. Дырка.

Импульсное пространство

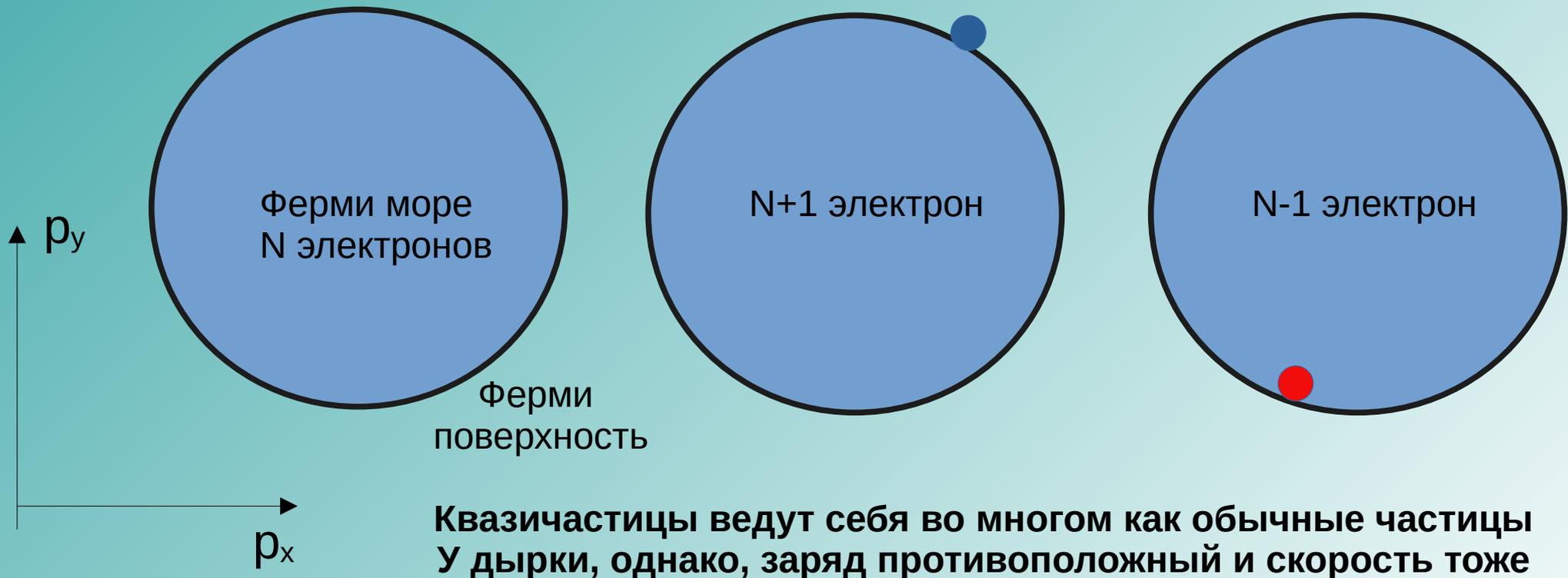
Основное состояние

«Возбужденные» состояния с одной квазичастицей

Очень много электронов
но ни одной квазичастицы

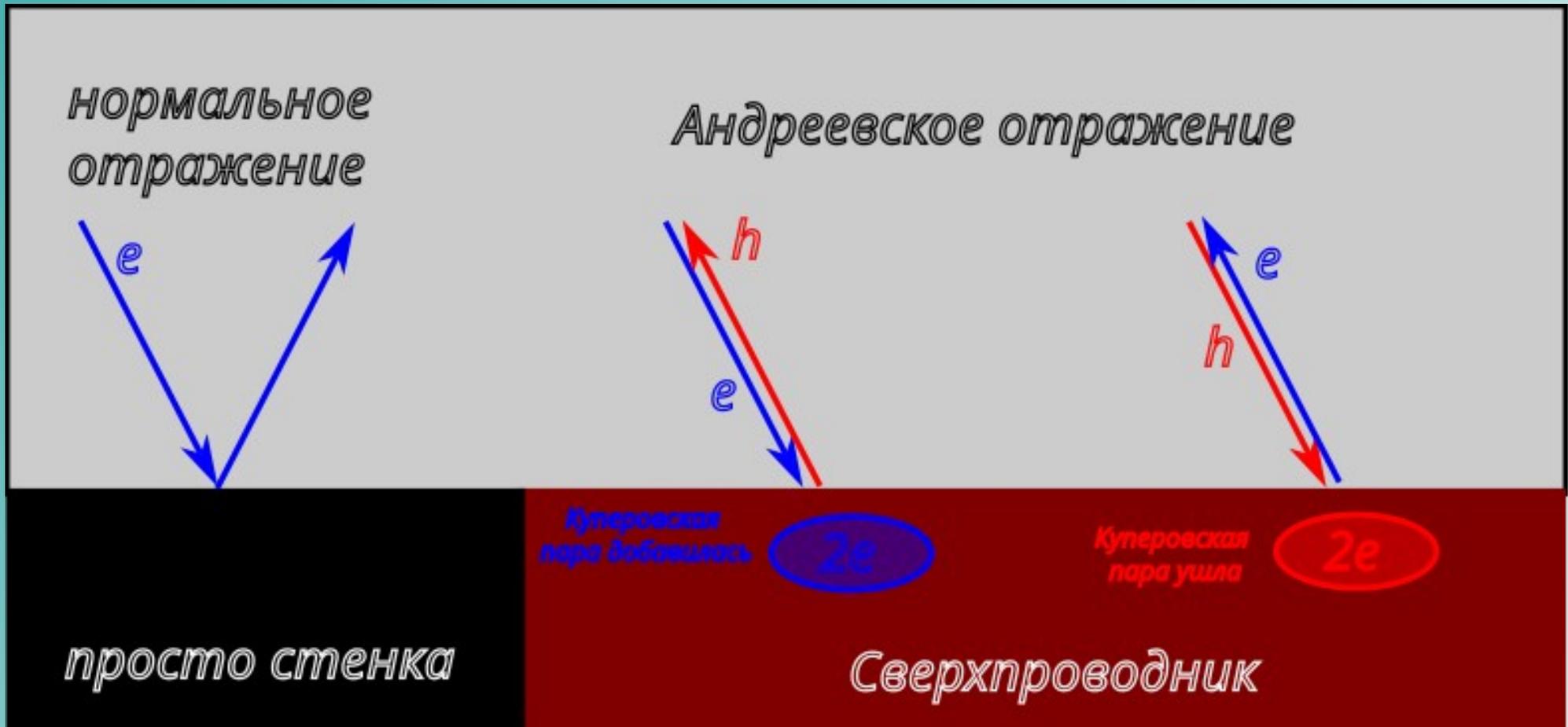
квази-электрон

квази-дырка



Суперпозиция электрона и дырки: $|\Psi\rangle = \alpha_e |N + 1\rangle + \alpha_h |N - 1\rangle$

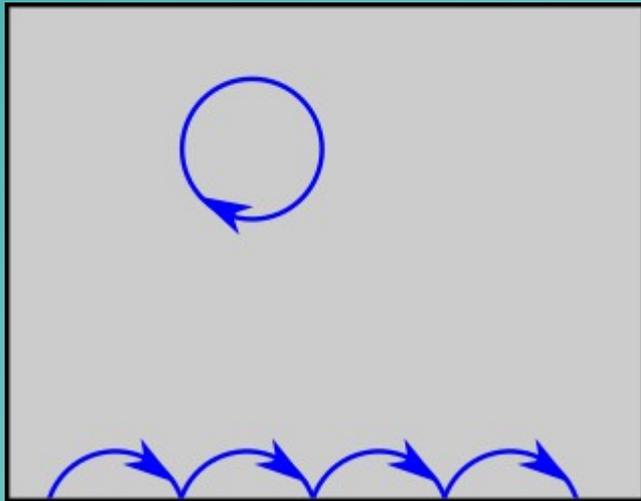
Электрон. Дырка. Андреевское отражение



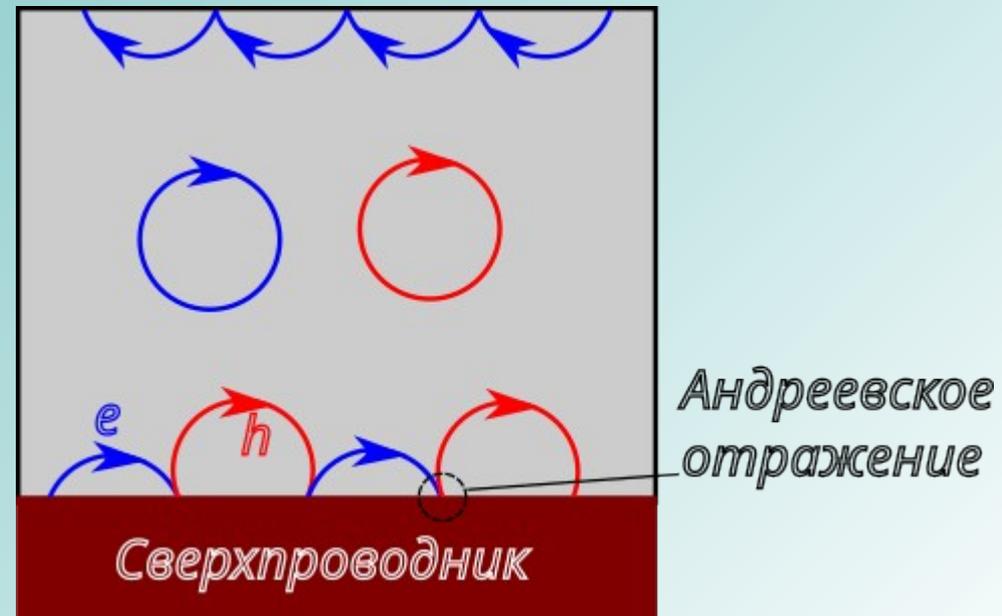
Андреевское отражение в магнитном поле

Двумерный случай, электроны в перпендикулярном магнитном поле

Нормальное отражение,
«скачущие» орбиты краевые каналы



Андреевское отражение,
«скачущие» орбиты
Андреевские краевые каналы

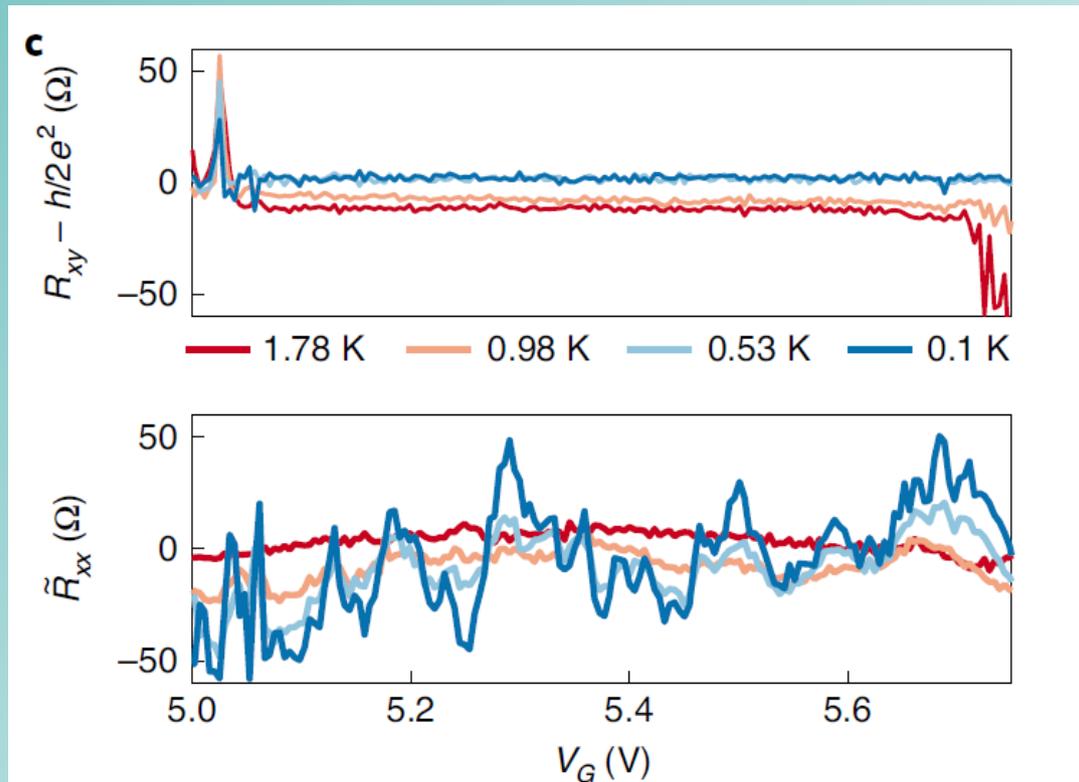
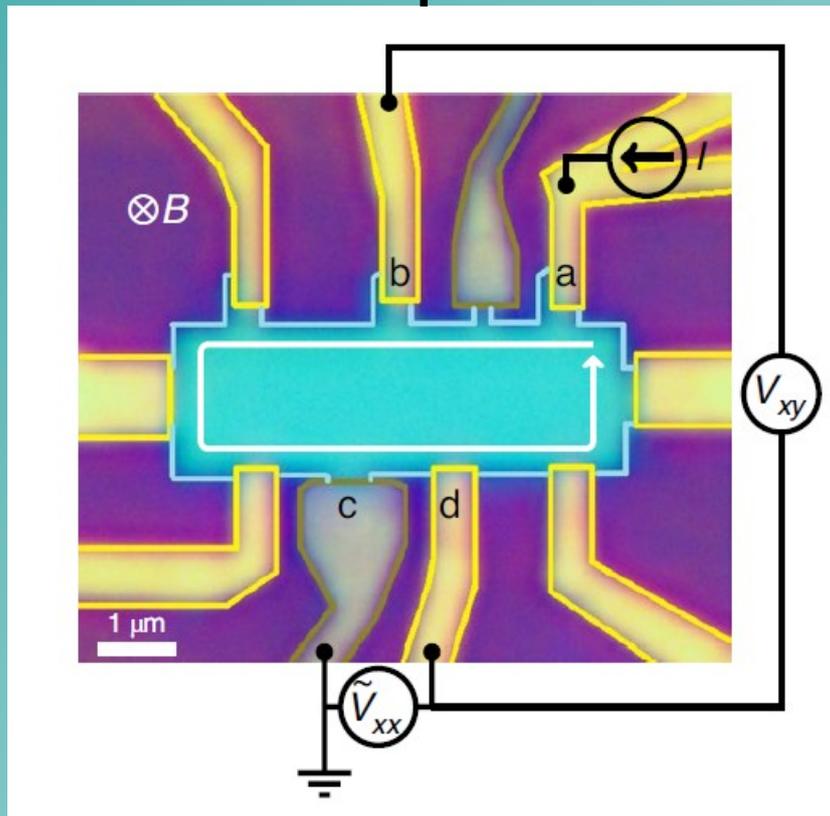


Recent data for semi-super in Quantum Hall

Early experiments with 2DEGs
(small B-fields)
Eroms et al, PRL 2005
Batov et al., PRB 2007

Graphene-hBN stack with MoRe superconductor

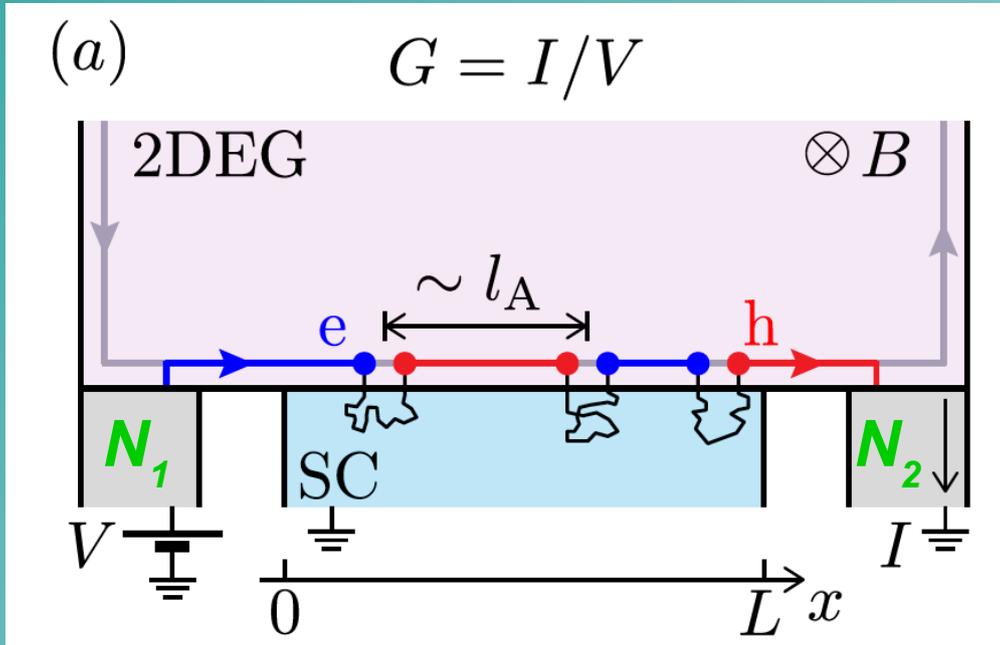
$$B = 3 \text{ T } \nu = 2$$



Zhao et al., Nature Physics 2020

Relatively small and random non-local response

Recent theory of Andreev scattering in Quantum Hall

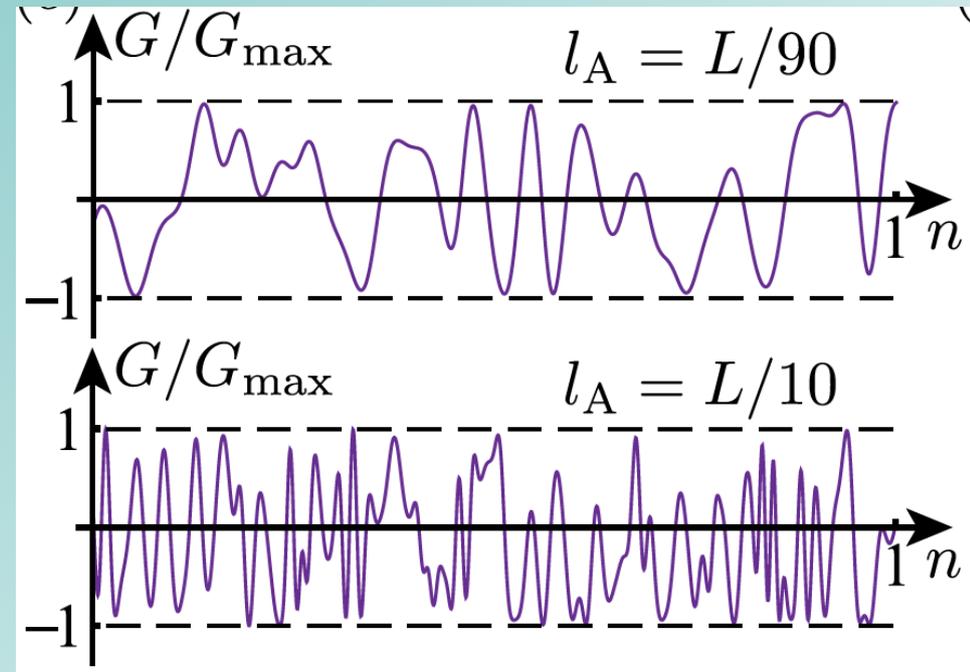


$$G \equiv G_{21} = 2e^2/h (T_{21}^{ee} - T_{21}^{he})$$

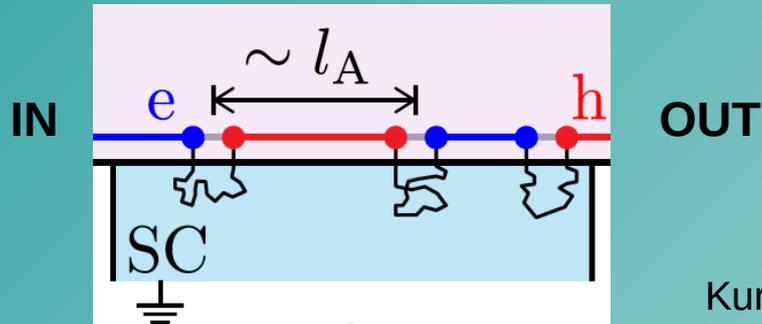
$$\langle G \rangle = G_Q e^{-2L/l_A}. \quad (12)$$

At $L \gg l_A$, conductance G is distributed uniformly in the interval $[-G_Q, G_Q]$ with $\langle G \rangle = 0$ and variance $\langle\langle G^2 \rangle\rangle = G_Q^2/3$. Thus, the conductance fluctuations pattern is sign-alternating and evenly distributed between positive and negative values, see Fig. 1(c).

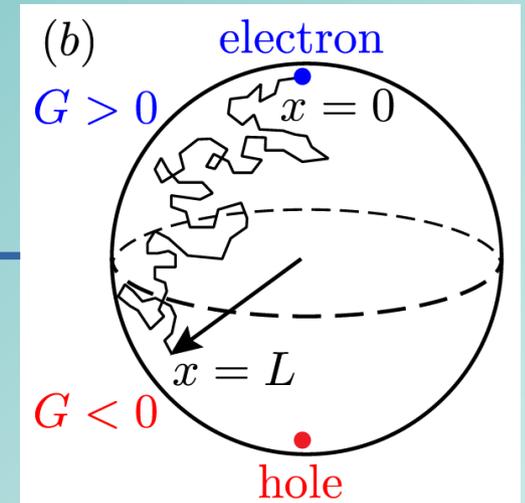
Kurilovich et al., arXiv 2022



Edge propagation as a rotation in e-h basis



Kurilovich et al., arXiv 2022



The change of the wave function across each element is small. Therefore, we can describe the wave function evolution by a differential equation:

$$i \frac{\partial}{\partial x} \begin{pmatrix} a_e(x) \\ a_h(x) \end{pmatrix} = \begin{pmatrix} -\vartheta(x) & \alpha^*(x) \\ \alpha(x) & \vartheta(x) \end{pmatrix} \begin{pmatrix} a_e(x) \\ a_h(x) \end{pmatrix}. \quad (10)$$

Here $a_e(x)$ and $a_h(x)$ are the electron and hole components of the quasiparticle wave function, respectively.

Andreev and normal scattering in the proximity region:

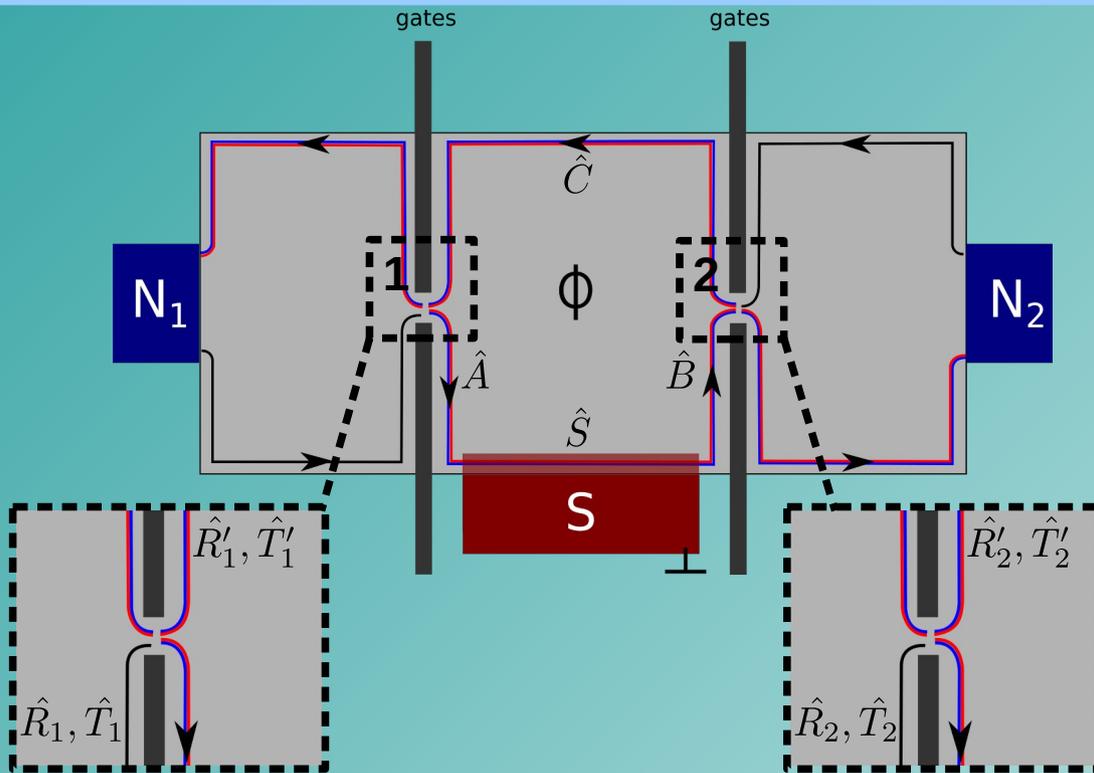
1. random rotation in the e-h basis (like a q-bit on a Bloch sphere)

2. conserve the total number of quasiparticles

$$\begin{pmatrix} a_e \\ a_h \end{pmatrix}_{out} = \hat{S} \begin{pmatrix} a_e \\ a_h \end{pmatrix}_{in}$$

$$\hat{S} = \begin{pmatrix} t_{ee} & t_{eh} \\ -t_{eh}^* & t_{ee}^* \end{pmatrix}; \quad |t_{ee}|^2 + |t_{eh}|^2 = 1$$

Calculate the interference



one-full-turn matrix

$$\hat{M}_0 = \hat{R}'_1 \hat{C} \hat{R}_2 \hat{B} \hat{S} \hat{A}$$

propagation matrices from N_1 to N_2 and from N_1 to N_1

$$(1 \rightarrow 2) = \hat{T}_2 \hat{B} \hat{S} \hat{A} \left(1 + \hat{M}_0 + \hat{M}_0^2 + \dots \right) \hat{T}_1$$

$$\begin{aligned} (1 \rightarrow 1) &= \hat{R}_1 + \hat{T}'_1 \left(\hat{R}'_1 \right)^{-1} \left(\hat{M}_0 + \hat{M}_0^2 + \dots \right) \hat{T}_1 = \\ &= \frac{1}{\hat{R}_1} \hat{R}_1 + \hat{T}'_1 \left(\hat{R}'_1 \right)^{-1} \left(1 + \hat{M}_0 + \hat{M}_0^2 + \dots \right) \hat{T}_1 \end{aligned}$$

Interference of all turns

$$\left(1 + \hat{M}_0 + \hat{M}_0^2 + \dots\right) = \left(1 - \hat{M}_0\right)^{-1} \Rightarrow$$

$$\left(1 - \hat{M}_0\right)^{-1} = \frac{1}{1 + R_1 R_2 - 2\sqrt{R_1 R_2} |t_{ee}| \cos(\Phi + \arg(t_{ee}))}.$$

$$\cdot \begin{pmatrix} 1 - |r'_1 r_2| t_{ee}^* e^{-i\Phi} & -|r'_1 r_2| t_{eh} e^{i(\Phi - 2\phi_A)} \\ -|r'_1 r_2| t_{eh}^* e^{-i(\Phi - 2\phi_A)} & 1 - |r'_1 r_2 t_{ee}| e^{i\Phi} \end{pmatrix}$$

Block matrices and conductance matrix

The full quasiparticle scattering amplitudes are obtained by taking a sum over all possible trajectories inside the interferometer and are contained in the blocks:

$$(1 \hat{\rightarrow} 1) = \frac{1}{R_1} \hat{R}_1 \left[1 - T_1 \hat{T}_1^{-1} (1 - \hat{M}_0)^{-1} \hat{T}_1 \right]$$

$$(1 \hat{\rightarrow} 2) = \hat{T}_2 \hat{B} \hat{S} \hat{A} (1 - \hat{M}_0)^{-1} \hat{T}_1$$

$$(2 \hat{\rightarrow} 1) = \hat{T}'_1 \hat{R}'_1{}^{-1} (1 - \hat{M}_0)^{-1} \hat{R}'_1 \hat{C} \hat{T}'_2$$

$$(2 \hat{\rightarrow} 2) = \frac{1}{R_2} \hat{R}'_2 \left[1 - T_2 \hat{T}'_2{}^{-1} (1 - \hat{M}'_0)^{-1} \hat{T}'_2 \right], \quad (3d)$$

with

$$(i \hat{\rightarrow} j) \equiv \begin{pmatrix} s_{ji}^{ee} & s_{ji}^{eh} \\ s_{ji}^{he} & s_{ji}^{hh} \end{pmatrix},$$

$$G_{ij} \equiv \frac{dI_i}{dV_j} = \frac{2e^2}{h} (\delta_{ij} - T_{ij}^{ee} - T_{ij}^{eh})$$

where, in the notations of Ref. [3], $s_{ij}^{\alpha\beta}$ represents a scattering amplitude of a quasiparticle of the type β from terminal N_j to a quasiparticle of the type α in the terminal N_i ($\alpha, \beta \in e, h$). Also I defined above $\hat{M}'_0 = \hat{C}^{-1} \hat{R}'_1{}^{-1} \hat{M}_0 \hat{R}'_1 \hat{C}$ and used the identity $(1 - \hat{M}_0)^{-1} = (1 + \hat{M}_0 + \hat{M}_0^2 + \dots)$. Without the loss of generality, the phases accumulated by a quasiparticle before entering the interferometer and after leaving it are assumed zero and corresponding evolution is given by identity matrices.

$$T_{ij}^{\alpha\beta} = |s_{ij}^{\alpha\beta}|^2$$

Backscattering ($1 \rightarrow 1$)

$$\begin{aligned}(1 \hat{\rightarrow} 1) &= \frac{1}{R_1} \hat{R}_1 + \hat{T}'_1 \left(\hat{R}'_1 \right)^{-1} \left(1 - \hat{M}_0 \right)^{-1} \hat{T}_1 = \\ &= \frac{1}{R_1} \hat{R}_1 + \frac{T_1}{R_1} \hat{R}_1 \left(\hat{T}_1 \right)^{-1} \left(1 - \hat{M}_0 \right)^{-1} \hat{T}_1\end{aligned}$$

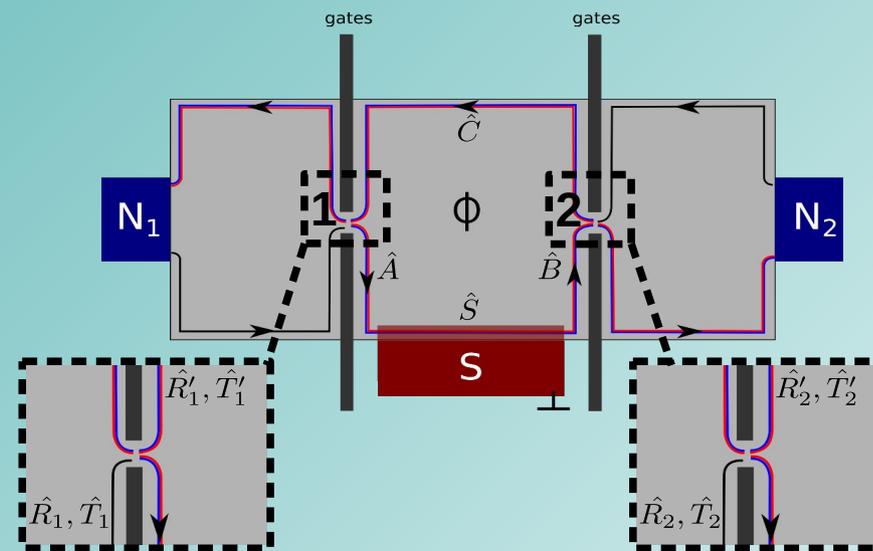
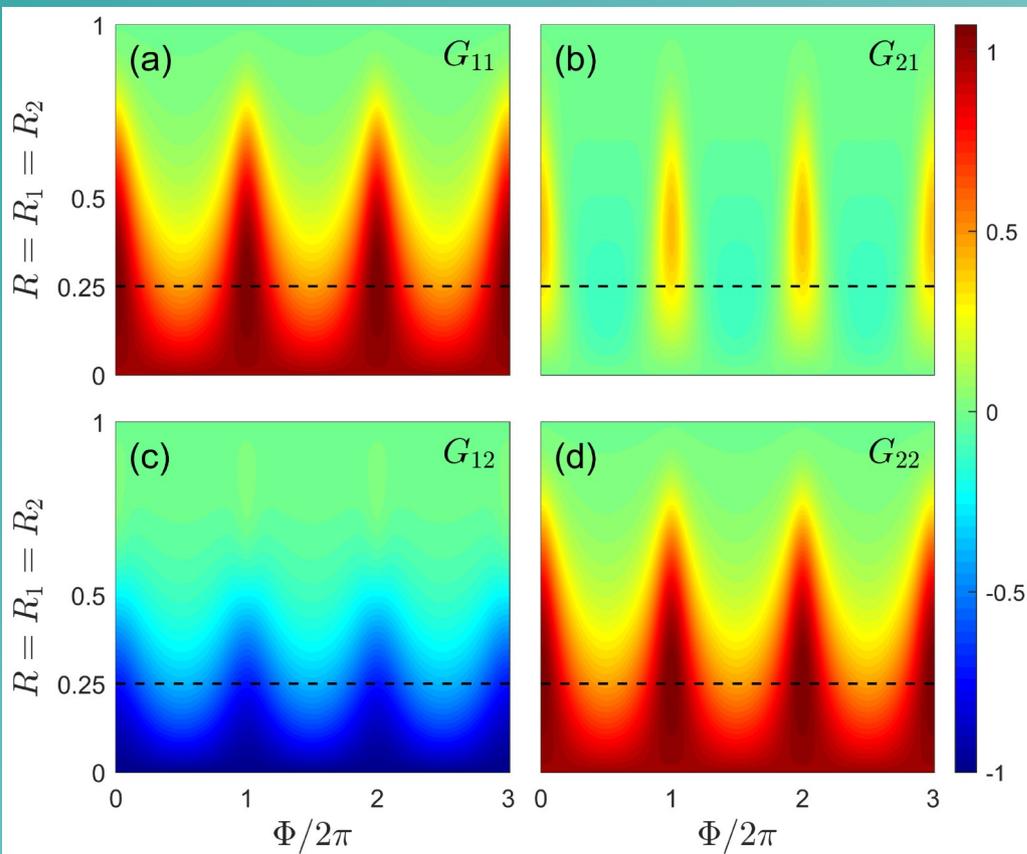
normal backscattering amplitude:

$$(1 \hat{\rightarrow} 1)_{11} = \frac{r_1}{R_1} \left[1 - \frac{T_1 (1 - \sqrt{R_1 R_2} |t_{ee}| e^{-i\Phi})}{1 + R_1 R_2 - 2\sqrt{R_1 R_2} |t_{ee}| \cos(\Phi)} \right]$$

Andreev backscattering amplitude:

$$(1 \hat{\rightarrow} 1)_{12} = -\frac{r_1}{R_1} \frac{T_1 \sqrt{R_1 R_2} t_{eh}}{1 + R_1 R_2 - 2\sqrt{R_1 R_2} |t_{ee}| \cos(\Phi)} e^{i(\Phi - 2\phi_A - 2\arg(t_{ee}))}$$

Some results



Main features:

- Diagonal conductance may exceed quantized value
- Non-local conductance may change the sign
- A-B oscillation clearly visible

