DIMENSIONAL EFFECT IN A METAL IN MULTIPLES OF A CERTAIN MAGNETIC FIELD

V. F. GANTMAKHER

Institute for Physics Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor May 18, 1962


A new dimensional effect has been discovered on measurement of the dependence of the surface impedance of tin on a 1—5 MHz magnetic field at helium temperatures.

A flat sample was placed in a coil of rectangular cross section which was part of an oscillating circuit. A constant magnetic field was applied along the plane of the sample. The frequency of the oscillator \( f \) varied with the magnitude of the field because of variation of the reactance \( X \) of the sample. The dependence of the frequency on the field was measured by a modulation method; the field modulation frequency was 20 cps.

The sample was a single crystal of high-purity tin (about \( 10^{-4}\% \) impurities) grown from the melt in a demountable quartz mold. The sample surface was perpendicular to the [100] axis. The thickness of the plate was 0.39 mm, the electron mean free path reached \((1—3) \times 10^{-1}\) cm at helium temperatures, and the skin-effect depth was \(10^{-4}\) cm at 1—5 MHz.

In a field \( H_0 = 2cp/ed \) (\( p \) is the half-width of the extremal electron orbit in the momentum space along a direction at right angles to the magnetic field and to the sample-surface normal; \( d \) is the plate thickness), when the width of the electron trajectory on the extremal cross section of the Fermi surface becomes equal to the plate thickness, a singularity \(^{[1]}\) appears on the \( X(H) \) curve and this singularity can be used to measure the Fermi surface cross section. Further experiments have shown that singularities on the \( X(H) \) curve appear also in fields that are multiples of \( H_0 \) (we found them up to a field \( 5H_0 \)) when the thickness of the plate is equal respectively to 2, 3, or more widths of the electron trajectory. Figure 1 shows curves on which the singularities are clearly visible in fields of \( 2H_0 \) and \( 3H_0 \).

The reason for the appearance of these singularities in multiple fields is as follows. Electrons in an orbit passing through the skin layer experience a systematic increase in velocity \( Dv \) due
to interaction with the electric field; this governs the participation of such electrons in conduction. As they move along the trajectory, these electrons reach after a time a depth equal to the trajectory width $2p$ and the increase of their velocity reverses sign (for the sake of simplicity we assume that the electric field is at right angles to the magnetic field and that the electron mean free path is much greater than the perimeter of its trajectory). Thus we have in the interior of the metal a current equal in magnitude and opposite in direction to the current in the surface layer. The density of this current is very low since electrons participating in conduction belong to different cross sections of the Fermi surface and therefore move away from the surface to different depths.

However, near the extremal cross sections of the Fermi surface, and correspondingly at the extremal widths of the electron trajectories in the plate, the number of electrons increases strongly and therefore the current density at the depth equal to the width of the extremal trajectory rises sharply. The nature of the dispersion law near the extremal cross section may tend to intensify the effect. This happened in the experiment referred to here: the effect was observed on the very nearly cylindrical part of the Fermi surface [2,3] (the experimental points in Fig. 2 indicate that the Fermi surface diameter in this cross section varies only 2—3%). Therefore all the surface electrons contribute to the effect. Moreover the form of the orbit is such that the curvature of the trajectory is small where the electrons are farthest from the surface and the electrons move parallel to the surface for a relatively long time.

In this experiment the skin layers are on both sides of the surface and the electric field vectors at the two surfaces are in antiphase. In the $2H_0$ field the trajectories of electrons passing through the skin layers touch in the center of the plate and this produces an interaction between the skin layers and a change in the impedance. In the $3H_0$ field the trajectory width is $d/3$ so that the coupling between the skin layers is via a chain of three trajectories; in the $4H_0$ field a chain of four trajectories is involved, and so on. There was no difference between the form of the lines corresponding to the even and odd numbers of consecutive trajectories.

The curves of Fig. 1 show also singularities in fields close to $H_0 + H_1$ and $H_0 + H_2$. These singularities can be explained by assuming that one of the trajectories of a chain which couples the skin layers is replaced by the trajectory of an electron belonging to the Fermi surface of another zone. The first-order effect of the trajectories with

![FIG. 1. Records of the dependence of $dI/dH$ on $H$. The oscillation frequency is 3.26 Mc and the temperature 3.5°K. The high-frequency field is $H_{\perp} \parallel [001]$. The angles between the constant field $H$ and the [001] axis are indicated at the curves. The ordinate scale on the right of the vertical dashed line is magnified by a factor of 9 compared with the left-hand side.]

![FIG. 2. Cross sections of the fourth hole zone in tin according to the model of nearly free electrons. a—Experimental results from [1] and from dimensional-effect measurements [1] which agreed well with each other.]

![FIG. 1. Records of the dependence of $dI/dH$ on $H$. The oscillation frequency is 3.26 Mc and the temperature 3.5°K. The high-frequency field is $H_{\perp} \parallel [001]$. The angles between the constant field $H$ and the [001] axis are indicated at the curves. The ordinate scale on the right of the vertical dashed line is magnified by a factor of 9 compared with the left-hand side.]

![FIG. 2. Cross sections of the fourth hole zone in tin according to the model of nearly free electrons. a—Experimental results from [1] and from dimensional-effect measurements [1] which agreed well with each other.]

smaller widths is clearly visible in the left-hand parts of the curves in Fig. 1 (the fields $H_1$ and $H_2$).

Local penetrations of the electromagnetic field into the interior of a metal were predicted theoretically by Azbel’; however, he discussed only the high-frequency case when the electrons that contribute to this effect take part in cyclotron resonance. Therefore Azbel’s theory is not directly applicable to our experiments although the phenomenon dealt with by him and our effect are very similar.

The author is deeply grateful to Yu. V. Sharvin for constant guidance, to M. S. Khaikin and R. T. Mina for useful discussions, and to A. I. Shal’nikov for his interest.

3 M. S. Khaikin, JETP 43, 59 (1962), this issue p. 42.

Translated by A. Tybulewicz