SIZE EFFECT IN INDIUM ON HELICAL TRAJECTORIES IN AN INCLINED MAGNETIC FIELD

V. F. GANTMAKHER and I. P. KRYLOV

Institute of Physics Problems, Academy of Sciences, U.S.S.R.; Institute of Solid State Physics, Academy of Sciences, U.S.S.R.

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A size effect is observed at radio frequencies in high-purity indium single crystals on orbits belonging to the hole surface of the second zone. In a number of cases inclination of a magnetic field by several degrees with respect to the sample surface leads to splitting of the lines. This can be attributed to drift of electrons belonging to extreme noncentral sections of the Fermi surface along the field. In principle this makes it possible to differentiate between central and noncentral orbits on the basis of the behavior of the lines in an inclined field.

MANY effects, related in one manner or another to the motion of the electrons along the magnetic field, were observed in recent studies of the surface impedance in a magnetic field of metallic single crystals with a large electron mean free path. These include cyclotron resonance in a field perpendicular to the surface of a crystal[1], the size effect[2] and the reduction, by a factor of 2, of the period of cyclotron resonance[3] at the limiting points in an inclined magnetic field, and the doubling of the cyclotron resonance lines for Fermi surfaces of the corrugated-cylinder type[4]. This paper is devoted to a related problem—the study of the behavior of the lines of the size effect on extremal trajectories, when the magnetic field is inclined to the surface of the sample. The drift of the electrons along the field, projected on the normal **n** to the surface when the field is inclined, can noticeably shift the lines of the size effect. Since the motion which is actually important in the case of the size effect on the extremal trajectories is that occurring during one-half of the period, from one side of a plane-parallel plate to the other, the drift along the field can come into play even in the case of closed central trajectories on which the average displacement over the entire period is equal to zero.

For example, in the case of a spherical Fermi surface, when there is no drift all along the field on the extremal trajectories, the line shifts with increasing field inclination like $H = H_0 \cos\varphi$ (H_0—position of the line when the field is parallel to the surface of the sample, φ—angle between the surface and the magnetic field). On the other hand, if the Fermi surface is a cylinder with axis **P** parallel to the surface of the sample, then it is easy to show that when the field is inclined to the (**n**, **P**) plane the size-effect line should shift towards the stronger field like $H = H_0/\cos\varphi$. Such a line-shift variation with increasing inclination was observed by us experimentally in tin, with good accuracy, for a line pertaining to the cylindrical part of the fourth Fermi-surface zone. (This surface is well known: the cylinder has a square cross section, and its axis **P** ∥ [001]; see, for example, [5].) The geometry of the experiment was as follows: **n** ∥ [010], the field was rotated in the (100) plane, $0° < |\varphi| < 25°$, and the field at $\varphi = 0$ was parallel to [001].

In both cited cases the line shift varied like the square of the angle at small inclination angles. A linear dependence of the shift on the angle is, however, also possible. For example, in the case of the cylinder whose axis **P** is inclined to the surface at an angle ψ, with the field lying in the (n, P) plane and with φ small, we have*

$$H = H_0 \frac{\cos\psi}{\cos(\psi - \varphi)} \approx H_0(1 - \varphi \operatorname{tg}\psi).$$

All these relations, however, contain little new information concerning the energy spectrum, and their result is more likely negative: at large inclinations of the magnetic field to the sample surface, the position of the size-effect line is not related uniquely to the dimension of the momentum-space orbit in the **n** × **H** direction. This must be kept in mind not only in measurements of the size

*tg = tan.

effect, but, for example, in studies of magneto-acoustic geometrical resonance, when the role of $\mathbf{n}$ is played by the wave vector $\mathbf{q}$ of the sound wave.

More interesting, from our point of view, is the splitting in an inclined field of the size-effect lines from extremal central orbits, which was observed and investigated by us for indium.

Experiment. The experiments were made with a set-up described earlier[3] at a temperature $T = 1.3°$ K. The oscillation frequency f was approximately 3 Mc. The samples were in the form of discs 18 mm in diameter and 0.4—0.5 mm thick. The angle between the direction of the magnetic field and the surface of the sample was regulated by tilting the Dewar relative to the electromagnet; the position at which the field was parallel to the surface was determined from the symmetry of the effect.

Both the size effect itself and the splitting of the lines during inclination were observed for several samples with different orientations. One example of the recorded result is shown in Fig. 1.

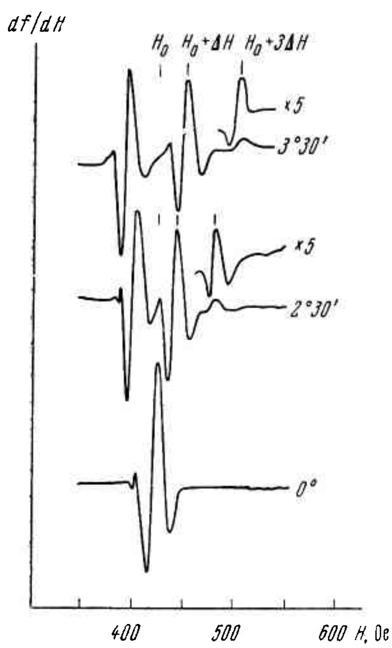

FIG. 1. Size-effect line vs. the inclination of the field, $\mathbf{n} \parallel [010]$, accurate to 1°, d = 0.40 mm. The high-frequency electric-field vector is $\mathbf{E} \parallel [100]$, the angle between [001] and the projection of $\mathbf{H}$ on the plane of the sample is 3°. Weak lines are plotted in addition with a magnification of 5x.

(The quantity $\partial f/\partial H$, recorded during the time of the experiment with a two-coordinate automatic plotter as a function of H, is proportional to $-\partial X/\partial H$, which is the derivative of the imaginary part of the surface impedance.) The split lines shift symmetrically in both directions, and the magnitude of the shift depends linearly on the angle of inclination. Experiments with samples having normal directions that differed by several degrees have shown that it is just the inclination of the field relative to the surface of the sample which is important, and not the change in the angle between the field and the crystallographic axes.

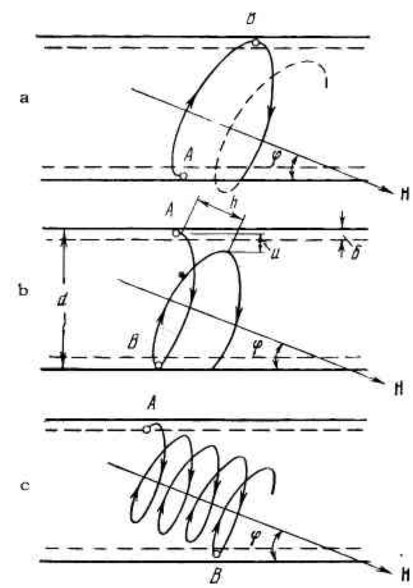

FIG. 2. Trajectories of a central-section electron in an inclined field; A and B – points of first and second passages of the electron through the skin layers, respectively.

Discussion. To explain the causes of the line splitting let us turn to Fig. 2. Inasmuch as the skin layers are present on both sides of the plate, for the size-effect line to appear it is sufficient that the electron execute merely half a revolution, going from one skin layer to the other; it is necessary only that in both skin layers the trajectory of the electron be tangent to the surface of the sample, that is, that the electron be "effective." Assume that the extremal orbit, which gives the size-effect line, is not central, and let the electron drift along this orbit with an average velocity $\bar{v}_H$. Then inclination of the field leads to the appearance of a drift-velocity component $\bar{v}_H\varphi$, directed normal to the surface of the sample. This component can either be directed opposite to the main motion of the electron along the loop (Fig. 2a), or else be in the same direction and superimposed on it (Fig. 2b, c). It is easy to show that when the loops of the helix are tangent to the surfaces of the plate, the thickness of the plate d is connected with the diameter of the helix D and the pitch h by the formula

$$d = D\cos\varphi + (n - 1/2)h\sin\varphi, \quad n = 0, 1, 2, \ldots$$

From this we get for small φ:

$$H = H_0[1 + (n - 1/2)h\varphi], \quad n = 0, 1, 2, \ldots$$

It is clear from Fig. 2 that the amplitudes of the first two lines are approximately equal, and should then rapidly decrease, owing to the increase in the path length λ from one surface to the other. The third line was already difficult to record and was not seen in all cases. Assuming that the amplitude of the line is $A \sim e^{-\lambda/l}$ (l—mean free path of the electron), and that $\lambda_n = \pi d\,|n - 1/2|$, we obtain a mean free path $l \approx 0.5$ mm from the ratio of the amplitudes of the second and third lines on Fig. 1.

From the natural condition for the existence of this effect, namely $u \gg \delta$ (u—displacement of the electron into the interior of the metal during one revolution, δ—depth of skin layer), we can estimate the minimum ratio of the average velocities of the electron along and transverse to the magnetic field needed in our experiments, namely $k = \bar{v}_H/\bar{v}_\perp > 0.1$. This condition is certainly satisfied in our case. From the magnitude of the splitting we find that $k = \Delta H/\varphi H_0 \approx 1$.

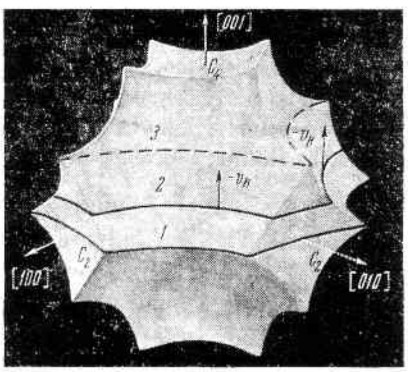

FIG. 3. Fermi surface of indium in the second zone, in accordance with the model of the almost-free electrons.

At first glance so high a value of k is surprising for an extremal orbit which is certainly far from the limiting point. However, it can be readily explained by taking into account the specific form of the Fermi surface of indium in the second zone (see Fig. 3; the existence of this surface, constructed in accordance with the model of almost free electrons, is confirmed both by the magnetoacoustic measurements of Rayne[6] and by our own measurements). Let, for example, the normal to the surface of the sample be directed along one of the C_2 axes, and let the magnetic field H be directed along the C_4 axis, so that the quantities measured are the maximum dimension of the orbit along the second C_2 axis. Then, owing to the concavity of the "square cups," we get in addition to orbit 1 also the extremal non-central orbits 2 and 3, with very large average velocity along the field. (The difference between the real trajectories and the very simple helix shown in Fig. 2 is immaterial in our reasoning.) The orbit 1 (and 3) passing near the corners on the surface makes apparently no contribution to the effect, while orbit 2 is precisely the one which produces the line that splits when the field is inclined. Figure 3 shows also that when the field is rotated in the plane of the sample, orbit 2 should soon disappear, whereas orbit 1, to the contrary, will leave the corners and will start to make an ever increasing contribution to the conductivity. Orbit 1 is the central one, and the resultant displacement along the field, during the time of motion along this orbit from one surface to the other, is equal to zero, accurate to φ^2 for all field directions. Consequently, when the field is rotated in the plane of the sample, the lines that split with inclination should give way to lines that do not split. This happens indeed (see Fig. 4), thus confirming the correctness of our proposed explanation.

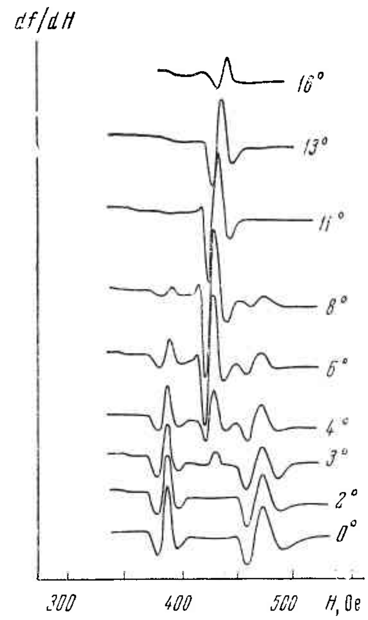

FIG. 4. Plot of the size-effect lines against the angle of rotation of the field in the plane of a sample, at a fixed inclination $\varphi = 5°$. $\mathbf{n} \parallel [001]$ (inclination within 3°), d = 0.40 mm, $\mathbf{E} \parallel [100]$. On the right of each curve is indicated the angle between the [010] axis and the projection of **H** on the plane of the sample.

Thus, the splitting of the size-effect lines on helical trajectories by inclination of the magnetic field makes possible the following:

1) Estimate the mean free path of the individual group of the electrons, similar to what was done in [2] for electrons near the limiting points.

2) Distinguish between central and noncentral orbits, an important factor in the study of the shape of the Fermi surface.

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