RADIO FREQUENCY SIZE EFFECT AT THE LIMITING POINT IN INDIUM

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The curvature of the Fermi surface of indium near the [111] direction is measured by means of the radio-frequency size effect at the limiting point in an inclined magnetic field. The experimental value of the curvature is identical with that which the almost free electron model yields. A study of the amplitude dependence of the size-effect lines yielded the electron mean free path as a function of the temperature. The thickness of the skin layer is determined on the basis of the size-effect line width and its dependence on the frequency.

1. WE report in this article results of a study of the size effect at the limiting point in indium. The experiments, carried out at helium temperatures with apparatus described in [2], consisted of measuring the surface impedance $Z = R + iX$ of single-crystal plates as a function of the magnetic field **H** inclined at a small angle φ to the surface of the metal. The frequency $\omega/2\pi$ of the electromagnetic field was varied in the interval $(1–20) \times 10^6$ Hz.

The conditions of the anomalous skin effect were satisfied in all the experiments. As is well known, the main contribution to the conductivity is made in this case by the so-called effective electrons, which move in the skin layer parallel to the surface. Because of the inclination of the magnetic field, the effective electrons from the vicinity of the limiting point, moving in the sample in elongated helical trajectories along the field, drift into the metal. Singularities, namely the size-effect lines, appear in the impedance of the plates at those values of the magnetic field H_n at which the number of electron revolutions n on the path from one surface of the plate to the other is an integer. The quantity

$$\Delta H = H_1 = H_n - H_{n-1} = \frac{2\pi c}{ed} K^{-1/2} \varphi \qquad (1)$$

is determined by the Gaussian curvature K of the Fermi surface at the limiting point (d is the thickness of the plate).

We have succeeded in observing this effect at magnetic-field directions close to [111]. There is no doubt that the corresponding limiting points belongs to the hexagonal cup on the Fermi surface in the second zone.[2] The exposition and the discussion of the results are divided into three parts: the study of the size-effect line shape as a function of the frequency ω (Sec. 2), the measurement of the radius of curvature of the Fermi surface of indium in the second zone in the vicinity of the [111] direction (Sec. 3), and measurement of the temperature dependence of the mean free path l of the electrons situated in the vicinity of this limiting point (Sec. 4).

2. Figure 1a shows examples of size-effect lines for different ω. The narrowing of the line with increasing ω gives grounds for assuming that, at least at low frequencies, the line width is determined not by the deviation of the sample from an ideal plane-parallel plate (inhomogeneity of thickness, bending, etc.), but by the depth δ at which the electromagnetic field is appreciably attenuated. At 7.5 MHz, the relative line width is $(H_{III} - H_I)/H_{II} \approx 2\delta/d \approx 4\%$, giving a value $\delta \approx 6 \times 10^{-4}$ cm.

Usually the depth of the skin layer is described by means of the quantity

$$\delta_r = \frac{c^2}{4\pi\omega} R = \mathrm{Im}\left(\frac{1}{H(0)} \int_0^\infty H(z)\,dz\right).$$

The results of measurement of the surface impedance,[3] recalculated to 7.5 MHz frequency with the aid of the relation $Z \sim \omega^{2/3}$, yield $\delta_r = 0.7 \times 10^{-4}$ cm. However, the value of δ obtained by us cannot be compared directly with δ_r. Since $Z = -4\pi i\omega c^{-2} E(0)/E'(0)$, the values of Z and δ_r characterize only the relative rate at which the electric field decreases inside the metal near the surface itself. For the same value of Z, the depth at which the field is appreciably different from zero can be different, depending on field distribution. For example, for an exponentially attenuating plane wave in the form

$$E = E(0)\exp\{i\omega t - z(1/\delta_r' + i/\delta_i')\},$$

we obtain at 7.5 MHz, starting from the data of Dheer,[3] a value $\delta_r' = c^2|Z|^2/4\pi\omega X = 1.6\times10^{-4}$ cm. The noticeable deviation of δ_r' from our experimental value offers evidence that the field in the skin layer attenuates much more slowly than exponentially, as indeed would follow from the theory of the anomalous skin effect.[4, 5] The distribution of the field in the skin layer was investigated also by Mina and Khaĭkin[6] at frequencies $\sim10^{10}$ and $\sim 2\times10^{10}$ Hz. The depth of the field in [6], recalculated to 7.5 MHz by means of the formula $\delta \sim \omega^{-1/3}$, agrees with our measurements: δ_r[6] $= (5.3-5.8)\times10^{-4}$ cm.

Figure 1a shows also that with decreasing ω the line broadens asymmetrically. This gives grounds for assuming that the value of H_n of interest to us should be determined from the left edges of the lines. We have attempted to extrapolate the position of the extrema to $\omega = \infty$ in coordinates that are natural for the conditions of the anomalous skin effect (see Fig. 1b). Straight lines drawn through the experimental points by least squares converge to a value H* which is 4% smaller than H_{II} at 7.5 MHz, and is quite close to the left edge of the line, which is fully distinct at low frequencies. In measurements of K, the quantity $\Delta H = H_1$, which enters in (1), was indeed determined from the value of H_I^*. (In the case of size effects of other types, the parameters of the Fermi surface should also be determined by using the left edges of the lines, and not their middles, as was done in earlier papers.)

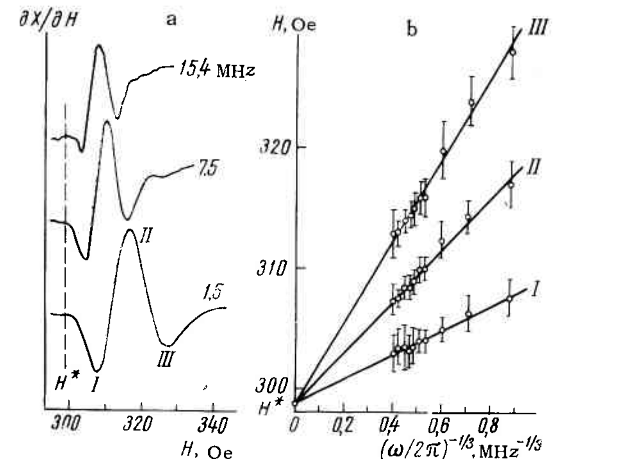

FIG. 1. a–Size effect lines for different generation frequencies; the ordinate scale is arbitrary. The dashed line shows the limiting value of H*, obtained by extrapolation on Fig. 1b; b–dependence of the position of the extrema on the generation frequency. Sample 4', $\varphi = 8°$, alternating electric field E||[111], projection of H on the sample plane is parallel to the [111] axis.

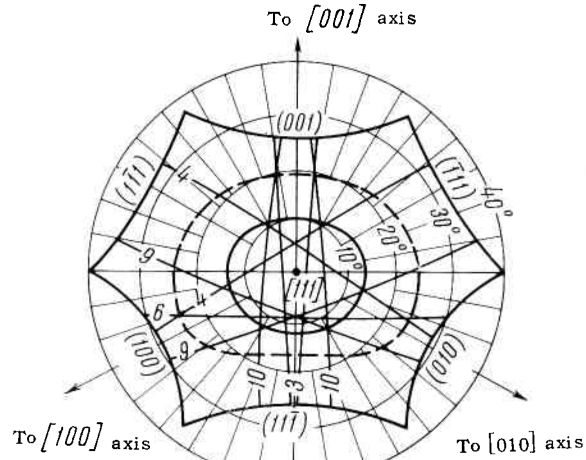

FIG. 2. Stereographic projection of the Fermi sphere, showing: the planes of the used samples (designated by numbers), the lines of intersection of the Fermi sphere by the Brillouin planes (designated by the Miller indices), the region of constant curvature of the Fermi surface (solid line), and the angular dimensions of the region within which the curvature varies by less than 10% (dashed).

We note that up to 15 MHz the quasi-static condition $\omega \ll 2\pi\varphi v/d$ (v–electron velocity at the limiting point) remains practically unchanged. During the time that the electron travels from one surface of the plate to the other, the phase of the field on the surface changes only by 0.1π at 15 MHz. Violation of the quasi-static condition should probably influence the line shape with further increase of ω.[7]

3. To determine K in the vicinity of the [111] direction, we chose a set of samples with different orientations. In Fig. 2 the planes of the samples are shown in stereographic projection. Inasmuch as samples 3, 4, 6, 9, and 10 were already used in [2], we have retained the numbering of that paper. The reproducibility of the result was verified with an additional sample 4′, of thickness $d = 0.30_1$ mm, grown in the same orientation as sample 4. In the experiments we measured only the change in the inclination angle, and the absolute value of φ was determined by extrapolation to $\Delta H = 0$. The accuracy with which φ was determined could be greatly improved because the size effect was observed upon inclination in opposite direction. The minimum value of φ at which the effects could still be observed was $\approx 4°$.

By rotating the magnetic field around the normal **n** to the plane of the surface of the sample at fixed inclination angle φ, it is possible to investigate the variation of K as the turning point moves over the Fermi surface (see Fig. 2). An example of the dependence of ΔH on the angle of rotation κ

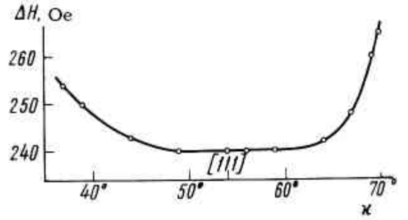

FIG. 3. Period of the size-effect oscillations at the limiting point vs. the rotation of H about the normal to the surface of sample 4. The frequency is 3 McS, $\varphi = 6°15'$, $\chi = 0$ at H || [100]; alternating electric field E || [111], T = 1.6°K.

is shown in Fig. 3, where it can be clearly seen that the value of K does not change in the vicinity of the direction [111]. The region of constant curvature on the Fermi surface is marked on Fig. 2 by the solid line. Inside this region $K^{-1/2} = (1.11 \pm 0.02)p_0$, where $p_0 = 2\pi\hbar/a = 1.45_5 \times 10^{-19}$ g-cm/sec is the limiting momentum of the Brillouin zone in the direction of [100] (a—lattice constant at helium temperatures). The error in the curvature measurement is ~ 2% and consists of the error in the measurement of φ (~ 0.5%), H (~ 0.5%), and d (~ 0.8%). Within the limits of these errors, the results are equal for all samples. We emphasize once more that we have substituted in (1) the quantity ΔH^*, and not the quantity ΔH determined, for example, by the center of the line. The result practically coincides with the radius of the Fermi sphere in the free-electron approximation, $p_f = 1.10p_0$.

Knowledge of the radius of curvature of the Fermi surface and of the effective mass[6] allows us to determine the Fermi velocity at the center of the hexagonal cup: $v = 1.10 \times 10^8$ cm/sec.[1)]

The increase of ΔH on the edges of the region of existence of the size effect (Fig. 3) offers evidence that the initial Fermi sphere is distorted on approaching the limit of the Brillouin zone. We note that in these sections the size effect no longer yields an exact method of measuring the curvature of the surface at the limiting point. Indeed, as can be seen from Fig. 4, the effective orbit Q which is closest to the limiting point has finite angular dimensions. The experimentally observed period of ΔH is indeed determined by this orbit. In formula (1) we have neglected the difference between the parameters of the orbit Q and the parameters of the limiting point. However, strictly speaking, in place of $(K^{-1/2})_{\lim}$ one should substitute in (1) the product $m^* \bar{v}_H$ of the effective mass by the average value of the component of the velocity along the field for the orbit Q. For a spherical surface, this leads only to replacement of the factor φ by $(\sin 2\varphi)/2$ in (1),[8] which is negligible if $\varphi \lesssim 10°$. On the other hand, if the curvature of the surface varies along the orbit Q, then the interpretation of the experimental data becomes much more complicated. We have therefore confined ourselves to approximately indicating in Fig. 2, by means of a dashed line, the angular region inside which K varies within 10%.

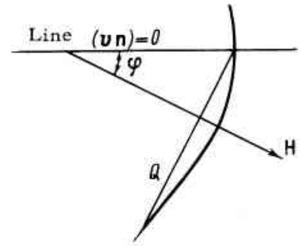

FIG. 4

4. We have also studied the dependence of the size-effect line amplitude A on the temperature T, the angle of inclination φ, and the number n. We have thus repeated the same set of measurements previously made on tin.[1, 9] It has turned out that for indium, in accord with the theory,[1] but in contradistinction to the case of tin, $A_n \sim 1/n$ at all temperatures and at all angles.

For all fixed n, the relation $A_n \sim e^{-d/l_{ph}}$ should be satisfied. This makes it possible to use the size effect at the limiting point to study the electron mean free path. First, by fixing T, we can determine from $A(\varphi)$ the value of the mean free path at the given temperature. Such measurements were made at T = 1.3°K. A numerical reduction of the results, similar to that made in [1], yielded $l \approx 0.5$ mm.

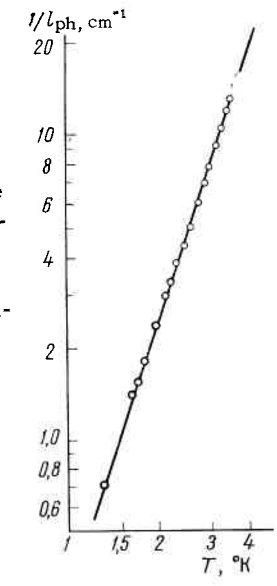

FIG. 5. Temperature dependence of the mean free path. Sample 6, frequency 1.5 MHz, $\varphi = 9°$, E || [111], H || (110). Error in the determination of $1/l_{ph}$ does not exceed the dimension of the circles.

[1)]The somewhat different value given by Mina and Khaikin[6] was based on the preliminary results of our measurements of $K^{-1/2}$.

By fixing φ, we can determine the temperature dependence of the amplitude of the size effect.[9] The function $\varphi d^{-1} \ln A$ is equal to $1/l(T)$, accurate to a constant term. By extrapolating the limiting value of this function to $T = 0$, we obtain the temperature dependence of the quantity $1/l_{ph} = 1/l(T) - 1/l(0)$, shown in Fig. 5 on a logarithmic scale. The straight line drawn through the experimental points corresponds to a power-law dependence, $1/l_{ph} \sim T^3$. This shows that even a single collision with a phonon makes the electron ineffective.[9] We see from Fig. 5 that at 1.3° K the phonons make a negligible contribution to the electron scattering.

We note that the same mean free path (0.5 mm at 1.3° K) was obtained in the same samples for the electrons of the extremal Fermi-surface section of the second zone, close to the central section.[10] This indicates that the electron, being in states close to the sharp edges of the Fermi surface,[2] does not experience any anomalous large scattering by impurities or defects. Our results for $l(0)$ coincides with the averaged mean free path $l'(0)$, which can be calculated from the residual dc resistivity ρ_0 of indium of the same purity[11] with the aid of the well known formula

$$\rho = p_f / Ne^2 l' \tag{2}$$

(N is the number of electrons per cm^3, determined using three free electrons per atom).

The obtained agreement $l'(0) \approx l(0)$ would seem to indicate the possible use of formula (2). However, if we attempt to calculate l'_{ph} by substituting in (2) the temperature-dependent part of the electric resistivity, then we get at $T = 1.3°$ K $l'_{ph} \approx 5\, l_{ph}$; at the same time, from the difference in the power-law dependences of the size-effect amplitude (T^3) and the electric resistivity (T^5) it follows that the number of elementary interactions with the phonons, which determines the electron-phonon mean free path, differs for these processes, so that the relation should be $l'_{ph}/l_{ph} \sim (\Theta/T)^2 \sim 5 \times 10^3$ (Θ is the Debye temperature). Similar discrepancies were noted in tin.[9]

At the same time, it should be noted that the theoretical estimate $l_{ph}^{theor} \sim (v\hbar/k\Theta)(\Theta/T)^3$ [12] is in good agreement both with the results of experiments for indium, and with the experimentally obtained relations between the electron-phonon free paths for tin[9] and indium ($l_{ph}^{Sn} \approx 5\, l_{ph}^{In}$ at 2° K).

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