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TEMPERATURE DEPENDENCE OF ELECTRON MEAN FREE PATH IN BISMUTH

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We undertook measurements of the amplitudes of the lines of the radio-frequency size effect [1] in bismuth for the purpose of obtaining data on the temperature-dependent part of the reciprocal of the electron mean free path. We processed plots of the derivative of the imaginary part of the surface impedance of plates of thickness d = 0.6 mm in the frequency range 5 - 14 MHz.

The lines of the size effect from the electronic "ellipsoids" were observed for samples of practically all orientations, provided the magnetic field made an angle less than about 80° with the long semiaxis p_3 of the ellipsoid, and provided the angle between the polarization vector $\vec{E}$ of the high-frequency electric field and the projection of $\vec{p}_3$ on the surface of the sample is sufficiently small. Typical plots, as well as a description of the experimental procedure, are given in [1]. The lengths p_1 and p_2 of the minor semiaxes of the ellipsoids, obtained from these measurements, agree within the limits of errors with the data of [2].

We were unable to see lines from the hole surface even for a sample whose normal $\vec{n}$ was parallel to the bisector axis and with polarization parallel to the C_3 axis, when the electron surfaces produced no size-effect lines and it was therefore possible to increase the sensitivity of the setup. Lines from two electron ellipsoids were obtained with the same sample at a different polarization. It can therefore be stated that the amplitudes of the lines from the electron and hole surfaces differ by a factor of at least 150. Although the parameters of the Fermi surface should affect the line amplitudes, this ratio is apparently governed essentially by the difference in the mean free path.

Assuming that the line amplitude $A^{(i)}$ is given by

$$A^{(i)} \sim \exp(-\xi/\ell^{(i)}), \tag{1}$$

where ξ is the path along the trajectory from one surface of the plate to the other and ℓ is the mean free path, we get

$$\alpha = \ell^{el}/\ell^{h} \approx 1 + \frac{\ell^{el}}{\xi} \ln(A^{el}/A^{h}). \tag{2}$$

Since $\xi \approx \pi d/2 \approx 1$ mm, we get $\alpha \approx 3.5$ for $\ell^{el} = 0.5$ mm and $\alpha = 2$ for $\ell^{el} = 0.2$ mm. (Incidentally, ℓ^{el} is certainly larger than 0.2 mm, since the lines from the electron ellipsoids were observed, with a large sensitivity margin, for a sample 1.2 mm thick made of the same material).

The ratio of the two corresponding electron and hole mobility tensor components was measured by Friedman [3] and found to be 2.5 at 2°K. However, it is difficult to compare our data with [3] directly, since the mobility tensor components obtained in [3] from the static conductivity are determined by the contribution of the carriers from the entire Fermi surface in some paritcular valley, whereas in the size effect only scattering in a narrow region on the Fermi surface, near the extremal orbit, is significant.

The temperature dependence of the line amplitude was investigated for samples of different orientations in the temperature interval 1.3 - 4.2°K in a field $\vec{H} \perp \vec{n}$. Assuming that

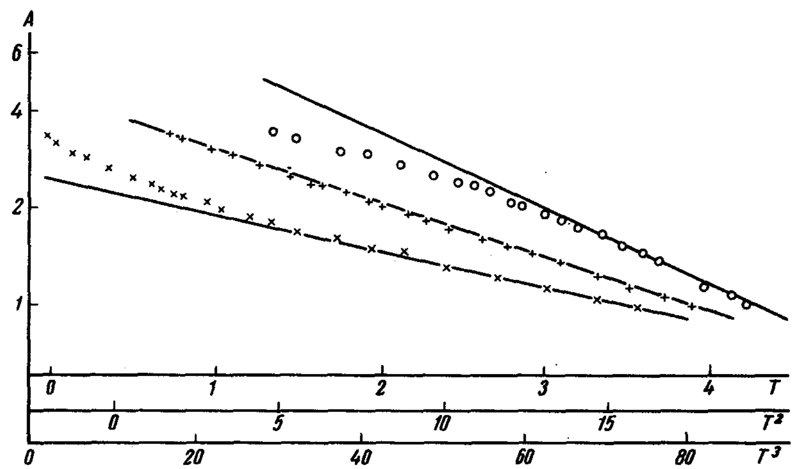
$$1/\ell = 1/\ell_0 + 1/\ell_T(T) = 1/\ell_0 + \beta^{-1}T^{\gamma} \tag{3}$$

and using (1), we get

$$\ln A(T) = C - \xi\beta^{-1}T^{\gamma}; \qquad C = \ln A(0). \tag{4}$$

Reduction of the experimental data by means of (4) led in all cases to a value $\gamma = 2$ (see the figure) in lieu of the cubic dependence observed in other metals [1, 4]. The numerical value of β fluctuated in different samples in the range 1.1 - 1.5 cm-deg^2 (the fact that the electron trajectory is ellipsoidal was taken into account in the calculation of β).

Plots of logarithm of relative amplitude $A(T)/A(4.2°)$ of the size-effect lines, in the form of functions of T (o), T^2(+), and T^3(×). Sample thickness d = 0.54 mm, frequency f = 9.6 MHz, $H \| C_2$, $|H| \sim 1.2$ Oe, polarization $E \perp B$.



Generally speaking, formula (1) is inexact. Since the electron can execute many revolutions along the trajectory, we should write for the line amplitude

$$A \sim \sum_{n=1}^{\infty} \exp(-\xi n/\ell) = e^{-\xi/\ell}(1 - e^{-\xi/\ell})^{-1}. \tag{5}$$

When ℓ_0 is small, however, Eq. (5) goes over into (1), regardless of the value of ℓ_T, and the change of the phase of the electromagnetic field on the surface of the metal is negligible as a result of the smallness of ℓ_0 (cf. [5]). The contribution to the effect is actually determined by a single passage of the electron through the "receiving" skin layer, as in the case of a limiting point [6]. As a control, we processed the experimental data in accord with formula (5) using values 0.5 and 1 mm for ℓ_0, but this did not change the exponent of the temperature, and only increased slightly the numerical coefficient β. It is curious that, inasmuch as the quantity $\exp(-\xi/\ell_0)$ enters in the constant C of formula (4), measurements of ℓ_T can be made when $\ell_T >> \ell_0$.

The degree of T^2 is typical of electron-electron scattering processes, but the measured coefficient β is smaller by one order of magnitude than expected from theoretical estimates. At the same time, the peculiarities of the Fermi surface of bismuth should lead to the relation $\ell_{e.ph} \sim T^{-2}$ in the electron-phonon interaction, in lieu of a cubic dependence. Indeed, the probability of electron-phonon scattering is

$$1/\ell_{e,ph} \sim \int d^3q \frac{q}{\exp\left(\frac{qs}{kT}\right) - 1} \delta(\epsilon(p) - \epsilon(p') \pm qs)\delta(p - p' \pm q), \quad (6)$$

where $\vec{p}$ and $\vec{p}'$ are the electron momenta before and after scattering, $\vec{q}$ is the phonon momentum, s is the speed of sound, the factor q in the numerator is due to the square of the matrix element, and the δ functions ensure satisfaction of the energy and momentum conservation laws. At low temperatures usually $|\vec{p}| >> q$, and since $v >> s$ the integration in (6) reduces to integration over the plane $\vec{v}\cdot\vec{q} = 0$ ($\vec{v}$ - velocity of scattered electron). In the case of bismuth, the momentum of the thermal phonon is $q \approx kT/s \approx p_1$ at $T = 0.5°K$ and $kT/s \approx p_3$ at 6°K, so that in the temperature interval of interest to us we have

$$p_1,\ p_2 < kT/s < p_3. \quad (7)$$

Since the phonon momentum is relatively large and the condition $v >> s$ is satisfied as before, the scattering phonon transfers the electron from the given state to any other point of the same ellipsoid. The integration region in (6) can be regarded, as a consequence of (7), as an infinite cylinder with axis $\vec{q}\,||\,\vec{p}_3$. Replacement of the two-dimensional region of integration by a one-dimensional one decreases the degree of T in the $\ell_{e,ph}(T)$ dependence by unity.

A similar dependence, $\ell_T \sim T^{-2}$ was observed in [3] in the measurement of the static conductivity. This is natural, for by virtue of (7) a single collision with a phonon leads to scattering of the electron through a large angle.

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