Trajectory effects in the nonlinear microwave response of tin in a weak magnetic field

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Tin exhibits a nonlinear rf conductivity at liquid-helium temperature, apparently because the alternating magnetic field affects the motion of electrons along sliding trajectories in the skin layer.

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The only nonlinear effects which have been observed previously for normal metals in the microwave range have been in bismuth, where the number of carriers and the effective masses are low.[1,2] We have now managed to detect a nonlinear microwave response of tin, in which the number of electrons per atom is on the order of one. The experimental apparatus is similar to that described in Ref. 2: A sample in a bimodal resonator is subjected to a large-amplitude microwave field at the frequency $\omega/2\pi = 9.2$ GHz. The sample emits a signal at 2ω with a power $P_{2\omega}$ which depends on the static magnetic field H applied parallel to the surface of the sample. (The samples are disks 18 mm in diameter and about 0.5 mm thick.)

The most important difference between the apparatus used in the present experiments and that of Ref. 2 is that here the sample is not immersed in liquid helium but is instead cooled through the use of a heat-transfer gas. This modification makes it possible to work with a low noise level at temperatures above the λ point. The heating of the sample with respect to the helium bath does not exceed 0.15 K, as can be seen by monitoring the superconducting transition in tin.

Figures 1 and 2 show the experimental results at $T = 3.8$ K for samples with a

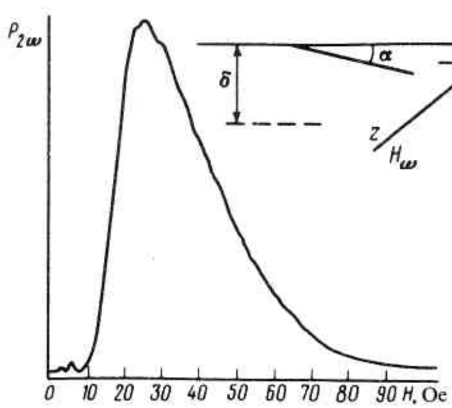

FIG. 1. The power $P_{2\omega}$ versus H; the amplitude of the exciting wave is $H_{\sim} = 12$ Oe.

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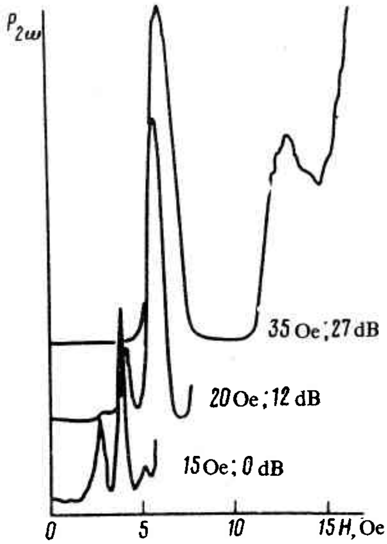

FIG. 2. The power $P_{2\omega}(H)$. The curves are labeled with $H_\sim$ and with the relative sensitivity of the detection system.

normal n||[001] in a field H||[100]. The signal $P_{2\omega}(H)$ (Fig. 1) has a broad maximum at 20–40 Oe and fine structure at weaker fields. This structure is shown in more detail in Fig. 2. It depends on the amplitude $H_\sim$ of the microwave field H_ω. As $H_\sim$ is gradually increased, resonant peaks appear at weak fields; they are subsequently dwarfed and

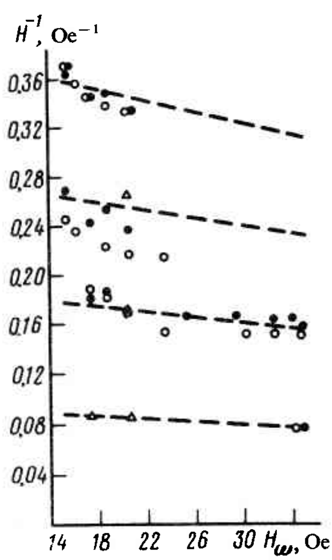

FIG. 3. Shift of the resonant lines upon a change in the amplitude of the incident wave. The different symbols correspond to different samples; the dashed lines show approximations of the experimental points by the formula $H^{-1} = H_1^{-1} n$.

swamped by resonances at stronger fields. A total of four peaks are observed. Although they can easily be shifted by increasing $H_\sim$, a periodic dependence of these peaks on the reciprocal of the field can be extracted quite accurately (Fig. 3):

$$H_n = H_1/n, \qquad H_1 \approx 12\,\text{Oe}, \qquad n = 1, 2, 3, \ldots \tag{1}$$

This signal is very sensitive to the quality of the surface finish and is seen only for samples having a mirror-finish surface. Lowering the temperature from 4.3 to 3.8 K intensifies the signal by a factor of several units.

Let us first discuss the resonant peaks at weak fields. The strong, and unpredictable, dependence of the height of these peaks on the surface finish of the sample indicates that these peaks are formed by so-called sliding electrons, which are reflected from the surface at small angles in a specular manner. The positions of the resonances, however, are not those which would result from resonant transitions between quantum magnetic surface levels[3] and which are observed in the surface impedance in the linear case.[4,5] The probable explanation is that the alternating magnetic field disrupts the quantum states constructed in Ref. 3, since the instantaneous field values in the skin layer are one above the resonant values most of the time. In view of the complexity of the corresponding quantum-mechanical problem, we have accordingly attempted to explain the observed nonlinear resonances in terms of classical trajectories, trajectories which go outside the strong alternating magnetic field.

We direct the Ox axis along the inward normal to the surface, and we assume that the alternating magnetic field in the skin layer is parallel to the external field: $\mathbf{H}_\omega \| \mathbf{H} \| Oz$ (see the inset in Fig. 1). We consider electrons which are moving in the (x, y) plane in directions perpendicular to the magnetic field $\mathbf{H}$ and $\mathbf{H}_\omega$. We assume that these electrons start from the $x = 0$ surface at a time $(t = 0)$ at which the field $\mathbf{H}_\omega$ has a node at that surface. Those electrons which move into the interior in phase with this node will escape from the skin layer without sensing the strong magnetic field. These are electrons which are moving at an angle $\alpha = v/v_F \approx 10^{-2}$ from the surface, where v_F is the Fermi velocity, and v is the velocity at which the node of the alternating field moves into the interior. If $H_\omega = H_\sim \exp(-k_1 x)\cos(\omega t - k_2 x)$, for example, we have $v = \omega/k_2$, regardless of the value of k_1. Requiring that these electrons—now moving outside the skin layer in the external field H—return to the skin layer after an integer number of periods end, we find a condition[4] on H:

$$H_n = \frac{1}{n}\left(\frac{c\omega^2 p_F \delta}{e v_F^2}\right)\frac{1}{n}, \qquad n = 1, 2, 3, \ldots \tag{2}$$

($\delta \approx k_2^{-1}$ is the skin thickness, and p_F is the Fermi momentum). In field (2) this group of electrons escapes from the skin layer after reflection, moving at a node of the magnetic field of the wave.

There is a tendency for the part of the skin current carried by this group of electrons to be increased by a focusing of the beam of trajectories near that which is moving away from the surface at the angle α. This is essentially focusing by an alternating-sign field, as is used for beam stabilization in linear accelerators. An electron which leaves the surface at the time $t = 0$ at an angle close to but not equal to α

experiences a force

$$F \approx \frac{e}{v} v_F \frac{\partial H}{\partial x} (x - x_0), \qquad x_0 = v_F at, \qquad |\partial H/\partial x| \simeq H_\sim/\delta \, , \tag{3}$$

whose sign depends on the sign of $\partial H/\partial x$ and that of the horizontal velocity component $v_y(|v_y| \approx v_F)$. The electrons subjected to this force execute harmonic oscillations around the trajectory α with a period

$$\omega_f = (\Omega_\sim v_F/\delta)^{1/2}, \qquad \Omega_\sim = eH_\sim/mc \, . \tag{4}$$

It is not difficult to see that for our field amplitudes $H_\sim$ we have $\omega_f \simeq \omega$ in order of magnitude, so that during the time interval in which the electrons are escaping from the skin they have ample time to concentrate around the trajectory α. This focusing occurs twice per period, at time intervals π/ω; there is an alternate focusing of electrons with $v_y > 0$ and $v_y < 0$. One of these groups of electrons is the group of sliding electrons, which undergo resonances in fields (1). Once per period they should produce a spike in the skin current, giving rise to the second harmonic.

We will close with a few words on possible causes of the broad maximum at fields $H \approx 20$–30 Oe. Khaikin[4] has pointed out yet another characteristic value of the static magnetic field:

$$H_m = \frac{8}{\pi^2} (c\omega^2 p_F \delta / e v_F^2), \tag{5}$$

which is the value at which time time spent by an electron in the skin is equal to half a period, $\Delta t \approx \pi/\omega$. The field is $H_m \approx 2.5 H_1$, in approximate agreement with the position of the broad maximum with respect to the H_1 resonance. It would be quite natural for the maximum near the field H_m to be broad, since there is essentially no resonance here. Perturbation-theory calculations[6] also indicate second-harmonic generation in this field range.[6]

Unfortunately, the literature reveals no reliable interpretation of the spectrum of quantum surface states for tin. The best test of these suggestions regarding the nature of the observed resonances would thus be experiments with metals for which such spectra are available.

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