Energy spectrum and quasibound Coulomb states of light holes in germanium in a magnetic field

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The effect of the Coulomb field of ionized impurity centers in germanium on the energy spectrum of light holes in the interior of the valence band in a magnetic field is investigated experimentally and theoretically. The Coulomb field produces under each magnetic subband discrete levels that have different states localized in the vicinity of the centers. The distance from these levels to the bottom of the subband was determined experimentally by comparing the measured values of the magnetic fields, at which resonant recombination or resonant photoionization of the holes in p-Ge is observed, with the values calculated under the assumption that the carriers participating in these processes are located on the very bottom of the subbands. An adiabatic approximation for high Landau levels, with energy larger than the Bohr energy of the Coulomb spectrum, is developed for the theoretical analysis of the quasibound Coulomb states in relatively weak magnetic fields. Good agreement between theory and experiment is obtained both for the energies of the quasibound states and for data on the nonparabolicity of the band of the light holes.

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1. INTRODUCTION

Investigations of resonant photoionization[1] and of resonant recombination[2] of carriers in p-Ge in a magnetic field yield information on the energy spectrum of the light holes in the interior of the band. Resonances are observed upon satisfaction of the condition

$$E^{(n,j)}-E_i=\hbar\omega, \tag{1}$$

where $\hbar\omega$ is the energy of the photon or optical phonon, E_i is the energy of one of the bound states of the acceptor, ground or excited, and $E^{(n,j)}$ is the energy of one of the quasibound states numbered j, located inside the continuum under the bottom of the nth magnetic subband. The carrier energy is reckoned from the edge of the band in the absence of a magnetic field (see Fig. 1).

Resonant photoionization is observed following monochromatic or infrared illumination. In these experiments, the dependence of the photocurrent on the magnetic field is oscillatory because of the oscillations of the probability of photoionization of shallow acceptor impurities. At helium temperatures, the transitions take place only from the ground state of the impurity. As for the states to which these transitions go there should exist, generally speaking under each of the magnetic subbands, a series of quasibound states due to the Coulomb interaction of the carrier with the ionized center.[3,4] In the experiments of Refs. 1 and 2, however, only one extremum each was observed. When describing the experimental results this allows us to make do in (1) with a single index for the designation of the quasibound-state energy: $E^{(n,j)}\to E^{(n)}$. Since the levels $E^{(n)}$ are located against the background of a continuous spectrum of low-lying subbands, the holes that land in them go off easily into the band and become free.

For nonmonochromatic infrared illumination, when the rate of generation is a monotonic function of the magnetic field, one observes in the photoconductivity resonant capture of light holes with emission in optical phonons from the levels $E^{(n)}$ to one of the acceptor levels E_i lying below the continuum. These are in essence levels below the zeroth magnetic subband: $E_i=E^{(0,i)}$. Since, however, the valence band is degenerate, they are in a somewhat special position. The quasidiscrete states under the high-lying magnetic subbands $n\neq 0$ of the light holes are made up mainly of the wave functions of the light holes. For light holes, the Larmor energy $\hbar\Omega_l=\hbar eH/m_lc$ is equalized with the effective Rydberg

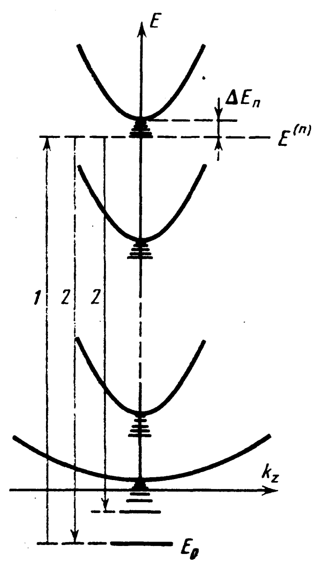

FIG. 1. Simplified level scheme of light holes in the vicinity of a Coulomb center in Ge in a magnetic field. The arrows indicate transitions for processes of resonant photoionization (1) and resonant recombination (2).

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energy $R_l = m_l e^4/2\hbar^2\varkappa^2$ ($\varkappa$ is the dielectric constant and m_l is the cyclotron mass of the light holes) in a magnetic field $H \approx 10$ kOe. Therefore the fields (30–110 kOe) used in the experiments are strong enough with respect to this part of the spectrum. The states E_i, however, are made up of wave functions of heavy holes. For this part of the spectrum the fields are still weak in the sense that the Zeeman splitting of the low-lying states are much lower than the impurity ionization energy $|E_0|$.

The energy E_i is known well enough from infrared spectroscopy. Therefore the result of the experiments of Refs. 1 and 2 should be regarded as a definition of the energies $E^{(n)}$. The values of these energies are governed by other factors. First, the spectrum of the holes in the magnetic field, which by itself is complicated enough in the parabolic approximation,[5] is additionally complicated by the fact that the levels $E^{(n)}$ are far from the extremum of the band. Second, the true bottom of the magnetic subband is separated from the quasidiscrete level by a certain distance ΔE_n, which we shall call the Coulomb correction.

We have succeeded in this paper in separating the contributions of these factors and in measuring the value of the Coulomb correction. This called for a theoretical calculation of the spectra of the holes in a magnetic field with allowance for the nonparabolicity of the valence band. We have also calculated the Coulomb correction for comparison with the experimental data. The calculation was made complicated by the fact that the condition of the validity of the adiabatic approximation

$$\beta = \hbar\Omega_l/R_l \gg 1, \tag{2}$$

in which the results of Refs. 3 and 4 were obtained, is actually not well satisfied in the experiment. We therefore consider in the article the conditions for the appearance of quasidiscrete Coulomb levels under high-lying magnetic subbands $n \gg 1$ and show that in the quasiclassical part of the spectrum the adiabatic approximation is valid in weaker fields

$$\beta n \gg 1, \quad \beta \leqslant 1. \tag{3}$$

These results are reported in the second part of the article.

In the third part the experimental results are compared first with the calculation of the energy spectrum in an ideal crystal, and the obtained difference is compared with the calculation of the Coulomb correction. Experiment and theory are in very good agreement, thereby demonstrating the high accuracy of both calculations.

In the last section we report the experimental data on the nonparabolicity of the spectrum of light holes in germanium—the energy dependence of the cross-section areas of the equal-energy surfaces and of the g-factor.

2. DETERMINATION OF THE ENERGY LEVELS OF A COULOMB IMPURITY CENTER IN THE ADIABATIC APPROXIMATION

To determine the energy levels of a hydrogenlike impurity center in an isotropic parabolic band in a magnetic field it is necessary to solve an eigenvalue problem with a Hamiltonian

$$\mathscr{H} = \frac{1}{2m^*}\left(\hat{\mathbf{p}} + \frac{e}{2c}[\mathbf{H}\times\mathbf{r}]\right)^2 - \frac{e^2}{\varkappa r}, \tag{4}$$

where $\mathbf{r}$ is the radius vector, $\hat{\mathbf{p}}$ is the momentum operator, and m^* is the electron effective mass. The Schrödinger equation with the Hamiltonian (4) has in general form no analytic solution, but in strong magnetic fields an adiabatic approximation is possible[4] when the characteristic dimension of the wave function in a plane perpendicular to the magnetic field is much smaller than the characteristic dimension in a direction parallel to the field (the z axis).

In the adiabatic approximation one solves first the problem of electron motion in a plane perpendicular to the field:

$$\left[\frac{\hbar^2}{m^*}\left(-\frac{\partial^2}{\partial\rho^2} - \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{m^2}{\rho^2} + \frac{\rho^2}{4\lambda^4} + \frac{m}{\lambda^2}\right) - \frac{2e^2}{\varkappa(\rho^2+z^2)^{1/2}}\right]$$
$$\times f(\rho, z) = 2W(z) f, \tag{5}$$

where $f\exp(im\varphi)$ is the wave function of the transverse motion, ρ is the radius vector in the plane $z = \text{const}$, φ is the polar angle, m is the magnetic quantum number, and $\lambda = (c\hbar/eH)^{1/2}$ is the magnetic length. This determines the adiabatic potential $W(z)$ as a solution of the eigenvalue equation (5) at a fixed value of the coordinate z along the field. The energy levels E are then determined from the equation for the wave equation $g(z)$ of the longitudinal motion

$$\left[-\frac{\hbar^2}{m^*}\frac{d^2}{dz^2} + 2W(z) - 2E\right] g(z) = 0. \tag{6}$$

The total wave function is

$$\Phi(\rho, z, \varphi) = g(z) f(\rho, z) e^{im\varphi}. \tag{7}$$

Equation (5) in general form has likewise no analytic solution. If the Coulomb interaction with the center is weak, then

$$W(z) = \hbar\Omega\left(N + \frac{1}{2}\right) - \frac{e^2}{\varkappa}\int_0^\infty \frac{d\rho\,\rho\,(f_{n_\rho,m}(\rho))^2}{(\rho^2+z^2)^{1/2}}$$
$$= \hbar\Omega\left(N + \frac{1}{2}\right) + \delta W(z). \tag{8}$$

Here N is the number of the Landau level, $n_\rho = N - (|m| + m)/2$ is the radial quantum number ($n_\rho > 0$, so that $m \leqslant N$), $f_{n_\rho,m}$ is the normalized wave function:

$$f_{n_\rho,m} = \lambda^{-1}\exp\left(-\frac{\rho^2}{4\lambda^2}\right)\left(\frac{\rho^2}{2\lambda^2}\right)^{|m|/2} F\left(-n_\rho, |m|+1, \frac{\rho^2}{2\lambda^2}\right)\frac{1}{m!}$$
$$\times\left[\frac{(|m|+n_\rho)!}{n_\rho!}\right]^{1/2},$$

where $F(\alpha,\gamma,z)$ is a confluent hypergeometric function.

The energy spectrum in ultraquantum fields was determined in Ref. 4 at $m = 0$ in the case when inequality (2) is valid. Using the method of joining[4] the solutions of Eq. (4) which are known at $|z| \gg \lambda$ and at $z \ll a_B$, where $a_B = \hbar^2\varkappa/m^*e^2$ is the Bohr radius of the electron, we can obtain the spectrum at $m \neq 0$:

$$E = \hbar\Omega(N + 1/2) - m^*e^4/2\hbar^2\nu^2. \tag{10}$$

The value of ν is determined by solving the transcendental equation

$$2C+\psi(1-\nu)+\frac{1}{2\nu}-\frac{1}{2}\ln\frac{\beta\nu^2}{2}+\frac{1}{2}\psi(n_\rho+|m|+1)=0. \quad (11)$$

Here C is the Euler constant, $\psi(x) = d\ln\Gamma(x)/dx$, $\Gamma(x)$ is the Euler gamma function.

We shall show that for impurity levels under Landau levels with large numbers N the adiabatic approximation is valid if the less stringent criterion (3) is satisfied. In the quasi-classical approximation $N\gg 1$ the electron wave function is of the form

$$f_{n_\rho,m}=(\pi\rho\lambda)^{-1/2}\left[2Em^*\lambda^2-m-\frac{m^2\lambda^2}{\rho^2}-\frac{\rho^2}{4\lambda^2}\right]^{-1/4}$$
$$\times\sin\left\{\int_{\rho_1}^{\rho}d\rho\left[2Em^*\lambda^2-m-\frac{m^2\lambda^2}{\rho^2}-\frac{\rho^2}{4\lambda^2}\right]^{1/2}\right\}. \quad (12)$$

(For example, at $m = N$ it is a wave packet near the Larmor radius, with width of the order λ.) The turning points ρ_1 and ρ_2 are given by

$$\rho_{1,2}^2=2\lambda^2\{2N-m+1\mp[(2N+1)(2N-2m+1)]^{1/2}\}. \quad (13)$$

The potential $\delta W(z)$ is equal to

$$\delta W(z)=-\frac{2e^2}{\pi\varkappa(\rho_2^2+z^2)^{1/2}}K\left(\left(\frac{\rho_2^2-\rho_1^2}{\rho_2^2+z^2}\right)^{1/2}\right), \quad (14)$$

where $K(x)$ is a complete elliptic integral of the first kind. From Eq. (6) with the potential (14) we determine the energy levels. This can be done, e.g., using the variational principle.

The characteristic scale relative to z changes with the ratio of m to N. If $N\gg 1|m|$ we can use the asymptotic form of the potential ($\rho_2\gg\rho_1$):

$$\delta W(z)=\frac{2}{\pi a_B}\ln\frac{(z^2+\rho_1^2)^{1/2}}{4\rho_2}, \quad (15)$$

from which it follows that the characteristic scale of the longitudinal motion of the electron at $m = 0$ is of the order of $(\rho_2 a_B)^{1/2}$. The search for the ground state using the single-parameter wave function

$$g(z)=(\alpha/\pi)^{1/4}\exp(-\alpha z^2/2) \quad (16)$$

yields for the binding energy and the optimal value of α the expressions

$$E=\hbar\Omega\left(N+\frac{1}{2}\right)+\frac{\hbar^2}{m^*a_B\rho_2\pi}\left[\ln\frac{\pi a_B}{256\rho_2}+1-C\right],$$
$$\alpha=\frac{4}{\pi\rho_2 a_B}. \quad (17)$$

It follows from (17) that when the inequality (3) is satisfied the distance between the bottom of the Landau level and the energy level of the bound state is small compared with the distance $\hbar\Omega$ to the preceding Landau level.

For the case $n_\rho\ll|m|$ (i.e., $|m|\gg N$ at $m<0$ and $(N-m)/N\ll 1$ at $m>0$) the asymptotic expression for the potential is of the same form as for the harmonic oscillator:

$$\delta W(z)=\frac{e^2}{\varkappa\rho_2}\left(-1+\frac{z^2}{2\rho_2^2}\right). \quad (18)$$

The energy levels

$$E=\hbar\Omega\left(N+\frac{1}{2}\right)-\frac{e^2}{\varkappa\rho_2}+\frac{\hbar^2(l+1/2)}{m^*(a_B\rho_2^3)^{1/2}} \qquad (l=0,1,2,\ldots) \quad (19)$$

at $\beta N\gg 1$ are also far from the preceding Landau level. The characteristic scale in z is of the order $(\rho_2^3 a_B)^{1/4}$. It is of interest to note that in both limiting cases the largest contribution to the correction to a spectrum unperturbed by a Coulomb potential is given by the shift of the bottom of the potential well. We consider now the levels of a shallow impurity at the edge of a fourfold degenerate valence band. Owing to the large difference between the masses of the light (m_l) and heavy (m_h) holes, the magnetic fields can be strong for the light holes and weak for the heavy ones if the following inequalities hold

$$(m_h e^2/\hbar^2\varkappa)^2 > eH/c\hbar > (m_l e^2/\hbar^2\varkappa)^2. \quad (20)$$

In such fields the action of the magnetic field can be regarded for the ground state of the acceptor as a perturbation, whereas a series of diamagnetic states which are adjacent to the bottom of the subband as to the dissociation edge, has already been formed under the magnetic subbands of the light holes.

The energy of a diamagnetic exciton made up of an electron and a light hole is calculated in Ref. 6. If the electron mass in the equations of this reference are allowed to tend to infinity we obtain the energy of the states of a light hole bound on an acceptor in magnetic fields in which the inequality (2) is valid. Here, too, for levels with large number $n\gg 1$, the adiabatic approximation is valid not only for $\beta\gg 1$ but also for the weaker inequality $\beta n\gg 1$. To determine the level energies of a light hole on an acceptor at $\beta n\gg 1$ we must average the potential on the wave functions of the transverse motion in a magnetic field.

We present the explicit form of such a state function with number n and projection of the angular momentum M for a degenerate valence band ($n = 1,0,1,\ldots$, and $M = \pm 1/2, \pm 3/2,\ldots$.)[6]:

$$\Phi_{n,M}(\xi,\varphi,\sigma)=\sum_{\mu=-3/2}^{3/2}\frac{C_{n,\mu}(\sigma)e^{i(M-\mu)\varphi}\xi^{|M-\mu|}e^{-\xi/2}}{|M-\mu|!}$$
$$\times\left[\frac{(n_\rho(\mu)+|M-\mu|)!}{2\pi n_\rho(\mu)!}\right]^{1/2}F(-n_\rho(\mu),|M-\mu|+1,\xi)\chi_\mu. \quad (21)$$

The index σ describes here four possible states of the hole in the degenerate valence band for one and the same n. At $\sigma = 1/2, -1/2$ the state pertains respectively to series a and b of the light-hole levels. The magnetic subbands are numbered here just as in Ref. 6. The expression

$$n_\rho(\mu)=n+M-1/2-(|M-\mu|+M-\mu)/2 \quad (22)$$

was obtained with allowance for the fact that the hole has a positive charge. From (22) follows a natural limitation on the projections of the angular momentum:

$$M\geqslant -n+1/2. \quad (23)$$

(We recall that in a simple band this inequality takes for an electron the form $m\leqslant N$). The remaining symbols in the

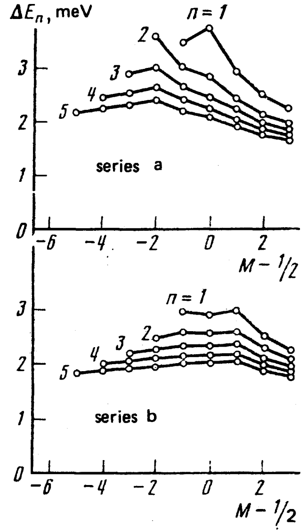

FIG. 2. Dependence of the binding energy ΔE_n on the projections of the angular momentum M at different values of the number n. $H = 60$ kOe, $\mathbf{H}\|[111]$.

expression for $C_{n,\mu}(\sigma)$ are given in Ref. 6. Now not only the potential energy but also the longitudinal mass depend on the number of the level n and the series in the Schrödinger equation (6) for the motion along the magnetic field. Therefore $\Delta E_n(H)$ is calculated by a variational method. The test wave function was chosen in the form of a product of the wave function (21), which satisfies the Schrödinger equation in the absence of an impurity-center potential, and the one-parameter function (16).

The dependence of the binding energy of the metastable states on the projections of the angular momentum M that are allowed in accord with (23), obtained by this method, is shown in Fig. 2.

3. QUASIBOUND COULOMB STATES OF LIGHT HOLES IN A MAGNETIC FIELD

To determine ΔE_n we compared the experimentally obtained energy spectrum of the valence-band states that participate in the resonant photoionization or recombination with the theoretically calculated spectrum of the Landau levels of the light holes in the valence band in the absence of Coulomb interaction with the impurities. The energy spectrum of germanium in a magnetic field was calculated by the method proposed in Refs. 7 and 8 and refined in accord with the results of Ref. 9. The germanium band parameters needed for the calculations were taken from Ref. 10.

In the present paper we use in the main the data obtained in Refs. 1 and 2. We carried out in addition experiments in magnetic fields that are stronger than those in Refs. 1 and 2, using a superconducting solenoid that produced a field up to 110 kOe. In addition, the optical range of the apparatus was extended in these experiments into the region of longer wavelengths, up to 30 μm, so that resonant-photoionization experiments could be performed at photon energies lower than in Ref. 1.

Figure 3 shows by way of example the dependence of the photocurrent on H^{-1} at various wavelength of the mono-

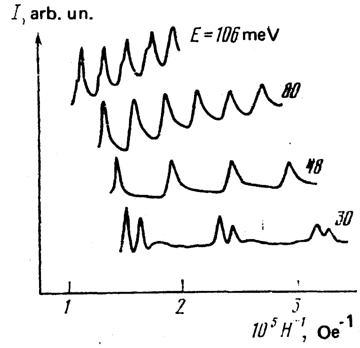

FIG. 3. Dependence of the photocurrent J on H^{-1} for a germanium sample doped with gallium. $N_{Ga} = 2\times10^{14}$ cm^{-3}, $T = 1.6$ K, $\mathbf{H}\|[111]$. The curves are marked on the right by the values of the energy at which photocarriers are produced in the band.

chromatic illumination for the case $\mathbf{H}\|[111]$. On the right of the curves are indicated the values $E = \hbar\omega - |E_0|$ reckoned from the top of the valence band at $H = 0$. Such experiments, performed for different orientations of the magnetic field relative to the crystal axes in the energy range $20 \leqslant E \leqslant 140$ MeV, can yield a set of values of resonant fields H_n at different energies. Experiments on resonant recombination in the magnetic field of light holes, with emission of optical phonons,[2] yield analogous data lying in a narrower energy interval (from 26 to 35 meV), but having high accuracy (not worse than 1%).

Figure 4 shows the entire aggregate of the experimental

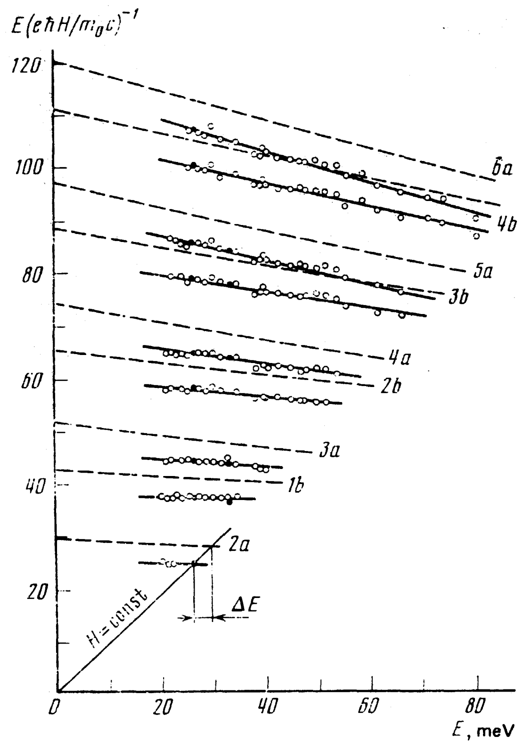

FIG. 4. Positions of extrema, in a scale proportional to H^{-1}, at different values of E. Dashed lines—calculated spectrum of light hole without allowance for the Coulomb field of the impurity. $\mathbf{H}\|[100]$. Here and in Figs. 6 and 7 below the dark and light circles were obtained respectively in experiments on resonant recombination and resonant photoionization of light holes.

data, where the values of H_n^{-1} are given for different energies in dimensionless units $E/\hbar\Omega_0$ ($\Omega_0 = eH/m_0c$, m_0 is the free-electron mass). The set of experimental values of resonant fields corresponding to one value of E (e.g., obtained from one curve in experiments on resonant photoionization) is plotted in these coordinates by a set of points on a vertical straight line. The results of the theoretical calculation without allowance for the Coulomb correction are shown in the same figure by dashed lines. As can be seen from the figure, the experimentally determined positions of the levels that participate in the resonant transitions turn out to be significantly shifted into the region of lower energies relative to the Landau levels of the light holes of the unperturbed band, for the same values of the magnetic fields.

The energy shift should depend on the magnetic field. It must therefore be determined by comparing the experimental and theoretical values of the energies E for one and the same field H. These values are obtained from the intersection of the experimental (solid) and theoretical (dashed) curves with the lines $H = \text{const}$, which pass through the origin. By way of example, Fig. 4 shows the procedure for determining this shift for the Landau level numbered 2a.

The measured values of ΔE_n obtained by reducing the data on resonant recombination of light holes from the level $E \approx 26.7$ meV to the ground state of the acceptor for $\mathbf{H}\parallel[111]$ are shown in Fig. 5. The numbers n to which the measured values of ΔE_n pertain are indicated above the points. Similar data were obtained also for other orientations. In all cases, transitions were observed with participation of only one state under each Landau level.

In a simple band, from among the series of levels produced under each magnetic subband in the presence of the impurity Coulomb field, the preferred level is the one with angular momentum $m = N$ ($N > 0$), inasmuch as for all the remaining levels with $m < N$ transitions are possible into the continuous spectrum of magnetic subbands with smaller N, with conservation of the energy and momentum. Therefore the levels with $m < N$ are quasistationary, whereas those with $n = N$ are sharp with a width governed by some extraneous perturbation. In the case of a compound band with a degeneracy point, transitions from a level of a light hole with $M_{\min}$ into a heavy-hole Landau subband with smaller number is possible even within the framework of the problem that takes only the impurity potential into account. However, the level with $M_{\min}$ remains the sharpest, inasmuch for a transition into a light-hole subband with a smaller number n must be changed by $\Delta n \ll n$, whereas a transition into a heavy-hole subband involves a much larger number change of n, which lowers substantially the transition probability. It is therefore natural to assume that the observed line is due to a transition with participation of a level with $M = M_{\min} = -n + 1/2$.

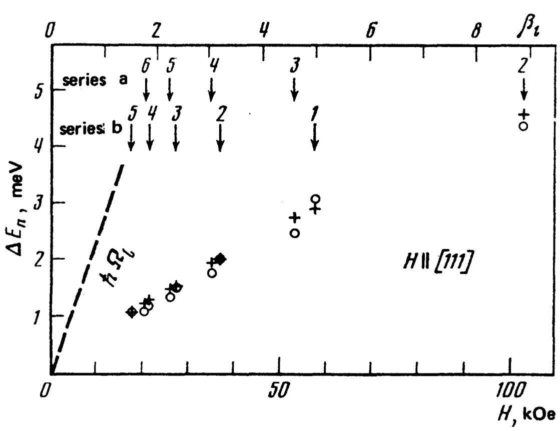

FIG. 5. Results of measurements of the energy ΔE_n, obtained in experiments on resonant recombination of light holes from the level $E \approx 26.7$ meV (light circles). The crosses represent the results of a variational calculation of ΔE_n.

Figure 5 shows the results of the calculation of ΔE_n for these states. We note that there is good agreement between the experimental and theoretical data. From the line width observed in experiments on resonant recombination of light holes,[2] we can determine the lower limit of the lifetime of this state, namely $\tau \gtrsim 2\cdot 10^{-12}$ sec.

4. INVESTIGATION OF NONPARABOLICITY OF LIGHT-HOLE BANDS IN GERMANIUM

The experimental results of Ref. 1 and 2 and of the present paper can be treated also in a quasiclassical approximation. This makes it possible to use them to study the nonparabolicity of energy bands. Indeed, we express the energy $E^{(N)}$ in the form

$$E^{(N)} = E_N \pm {}^1/_2 \mu_B g^* H - \Delta E_N. \tag{24}$$

Here $\mu_B = e\hbar/2m_0c$ is the Bohr magneton, g^* is the effective g-factor, and N is the quasiclassical number of the magnetic subband and is determined together with the energy E_N from the condition of quantization of the area $S(E_N)$ of the quasiclassical orbit of the carrier in k space:

$$S(E_N) = 2\pi eH(N+\gamma)/c\hbar, \quad |\gamma| < 1. \tag{25}$$

N differs from n for the levels of series a, namely $N = n - 1$, but $N = n$ for the series b. We note that the function $S(E)$ is linear only in a parabolic band.

In our experiments the value of $E^{(N)}$ is fixed by the resonance condition (1). Expanding $S(E)$ in a series near this quantity:

$$S(E^{(N)}) \approx S(E_N) + (\partial S/\partial E)(\pm {}^1/_2 \mu_B g^* H - \Delta E_N)$$

and using (25), we find after simple transformations that the resonance positions, in reciprocal-field scale, are

$$H_N^{-1} = \frac{2\pi e}{c\hbar S}\left[N + \gamma \pm \frac{1}{4} g^* \frac{m_l}{m_0} - \frac{\Delta E_N}{\hbar\Omega_l}\right]. \tag{26}$$

In the derivation of (26) we used the definition of the cyclotron mass

$$m_c = (\hbar^2/2\pi)\,\partial S/\partial E.$$

It follows from the equation obtained that the resonant fields should be periodic in H^{-1}, with a period

$$\Delta(H^{-1}) = 2\pi e/c\hbar S, \tag{27}$$

determined by the area of the extremal intersection of the equal-energy surface $E = \hbar\omega - |E_i|$ in k space with a plane perpendicular to the magnetic field. Since the $\Delta E_N(H)$ de-

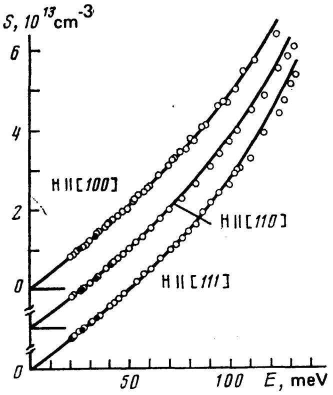

FIG. 6. Plots of $S(E)$ for three different orientations of H relative to the crystal axes.

pendence obtained by us is close to linear (see Fig. 5), the presence of the term $\Delta E_N(H)/\hbar\Omega_l \approx$const in (26) leads only to a shift of the extrema (this is precisely the shift seen in Fig. 4) and does not change their periodicity in the reciprocal-field scale. Thus, the presence of quasibound Coulomb states in a magnetic field does not distort the $S(E)$ dependence obtained in experiment in accord with (27) from the values of the photocurrent oscillation periods at different values of the characteristic energy.

The aggregate of all our experimental data on the $S(E)$ dependence at three different orientation of the magnetic field relative to the crystal axes is shown in Fig. 6. The solid line in the same figure show plots of $S(E)$ based on the theoretical spectrum of the Landau levels in the valence band. As seen from the figure, the experimentally measured and calculated values of S for light hole are in good agreement in the entire investigated energy interval.

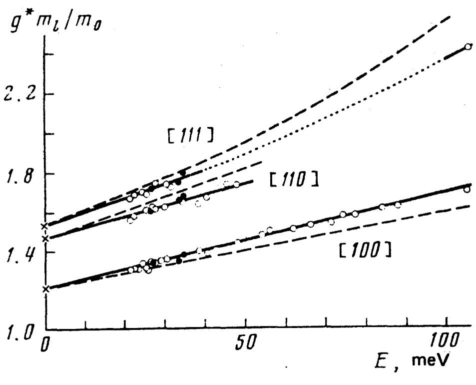

FIG. 7. Dependence of g^*m_l/m_0 on E. Dashed lines—calculation results. The points at $E = 0$ were obtained from the results of Ref. 5.

The described experiments make it possible to measure, besides the $S(E)$ dependences, also the value of the g factor and its dependence on the energy. It follows from (26) that the spin splitting of the Landau levels leads to the appearance of two periodic series of extrema, shifted relative to one another on the H^{-1} scale by an amount $1/2g^*m_l/m_0$ in units of the period. Two such series are seen, for example, on the upper and lower curves in Fig. 3. The energy dependences of g^*m_l/m_0, measured from the relative shift of the series of the extrema, are shown for different orientations of the magnetic field in Fig. 7 together with the calculation results. It can be seen that at small E the discrepancies between the theory and experiment are small, but they increase with increasing energy.

Measurements of the g factor can in principle be subject to a systematic error due to the quasiclassical Coulomb states. Indeed, the experimental points on Fig. 5, corresponding to the levels of series b, are noticeably shifted upwards relative to the points corresponding to series a. At $\mathbf{H}\|[100]$ one observes the inverse relation: ΔE_N for the levels of series a depends on H more strongly than for series b. As seen from (26), the different dependence of $\Delta E_N(H)$ for the two series influences the measured value of the g factor. However the jumps observed in Fig. 5 are probably due to very small (of the order of 0.1 meV at characteristic energies of several dozen meV) errors in the calculation of the energy spectrum. At this accuracy, allowance for the parameter q in the calculation of the energy spectrum of the valence band can become substantial, or else the addition of the terms discarded from the matrix Hamiltonian in Ref. 7.

At any rate, in the energy region $E \approx 100$ meV the largest discrepancy between theory and experiment is certainly not connected with the influence of the Coulomb correction. In fact, with increasing E the systematic error due to this correction should decrease, inasmuch as at the same values of H the numbers N increase with increasing E, and consequently the energy ΔE_N itself decreases.

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