Physica 127B (1984) 75–78
North-Holland, Amsterdam

NONLINEAR EFFECTS IN METALS WITH LONG MEAN FREE PATH OF ELECTRONS

V.F. GANTMAKHER
Institute of Solid State Physics, USSR Academy of Sciences, Chernogolovka, 142432 USSR

A current-induced magnetic field alters the shape of electron trajectories and by means of this changes the conductivity of the metal. Two of the resulting phenomena, soliton-like field penetration at low frequences and nonlinear resonances at microwave frequences in weak magnetic fields, which have been observed experimentally, are briefly discussed. Two others, nonlinear Azbel'–Kaner resonance and the 'current states', are only mentioned.

The effect that large-amplitude electromagnetic waves produce on the properties of the medium in which they are propagating leads to various nonlinear phenomena. In plasma and in semiconductors the electric field of the wave is the main source of nonlinearity. The amplitudes of the electric field E_ω and the magnetic field H_ω of the wave are approximately equal, and the force F_E with which the electric field acts upon an electron is c/v times larger than the magnetic force F_H (v is the velocity of the electron, and c is the speed of light). But in metals the electric field is small,

$$E_\omega/H_\omega \approx c\,|Z| \approx 10^{-4} - 10^{-5}$$

(Z is the impedance of the metal) and the Fermi-velocity of the electrons is large: $v_F/c \approx 10^{-2}$. Therefore $F_H/F_E \approx v_F/c^2\,|Z| \gg 1$ and the magnetic force F_H turns out to be the principal source of nonlinearity. This makes the metal medium in some sense similar to a relativistic plasma.

The influence of the magnetic field is provided by the effect it has on the shape of the electron trajectories. Therefore, the long mean free path l of electrons is a necessary condition for such an influence to exist. The scale of the fields H_ω producing the nonlinear phenomena is determined by the sensitivity of the linear response to the changes of the external magnetic field. These phenomena arise at such magnetic fields where the function $Z(H)$ changes appreciably in a narrow range. For instance, under Azbel'–Kaner resonance conditions, the alternating field sums with the constant one and alters the cyclotron period taking some electrons out of the resonance. The characteristic amplitide H_ω is determined by the width ΔH of the resonance line [1]:

$$H_\omega \approx \Delta H (R/\delta)^{1/2}.$$

The Larmor radius R to the skin-depth δ ratio is included here because the field H_ω acts upon the electron only along a small part of the cyclotron orbit.

Another example, may be a much more known one, is that of the 'current' states which arise due to rectifying of the alternating current in the skin-layer [2, 3]. For rectification to take place it is necessary for the field H_ω to be able to alter the conductivity appreciably. This is the case if the field changes the length of the electron pass inside the skin-layer:

$$(\delta R)^{1/2} \lesssim l, \quad H_\omega \gtrsim H_c = (cp_F/el)(\delta/l). \qquad (1)$$

With $l \approx 1$ mm and $\omega \approx 10^6$–$10^7\ \mathrm{s}^{-1}$ this estimation gives for typical metals $H_c \approx 1$ Oe. The rectified current locks in the bulk. So dc current loops arise in the metal with macroscopic magnetic moments.

We shall now consider two, in essence, quite different phenomena which occur in quite different frequency ranges but which have nevertheless one important feature in common. In both cases the external field in the skin-layer can be ignored compared to the alternating field and the space distribution of the latter is of the main

0378-4363/84/$03.00 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

importance. Let us consider the low frequency case first [4]. Suppose that the normal skin-effect takes place in the metal and that the conductivity and the skin-depth both change appreciably with the magnetic field: $\sigma = \sigma(H)$, $\delta = \delta(H)$. Assume that the external field is absent and that the alternating-field amplitude exceeds H_c significantly. Twice through a period at times $t = 0$, π/ω the alternating field

$$H_{\sim} = H_{\omega} \exp(-kx)\cos(\omega t - kx) \qquad (2)$$

has a node at the metal surface $x = 0$. The node plane travels into the metal with velocity $v_{\omega} \approx \omega\delta$. Near this plane the conductivity is higher than in the remaining part of the skin-layer. As a result, practically all the skin-current is concentrated near this plane. When the amplitude H_{ω} is low the current distribution is exponentially damped at length $\delta(0)$. But with large $H_{\omega} \gg H_c$ current sheets appear instead. They travel inside the metal with velocity v_{ω} and penetrate up to the distance $\delta(H_{\omega}) \gg \delta(0)$.

This phenomenon is illustrated by the following experiment [4]. Two bismuth single-crystal cylinders, both with diameter 18 mm, are placed inside a coil 1 (fig. 1) which creates the alternating field $H_{\sim}$. A planar receiving coil 2 with diameter 4 mm is placed between the cylinders. It records the changes of the magnetic flux through its area, i.e. through the central part of the cylinders. The front part of the pulse at the record corresponds to the passing of the current sheet through the windings of the measuring coil. Note that the sheet here has the form of a cylinder. The back part corresponds to its travelling inside the coil up to its collapse at the axis of the sample.

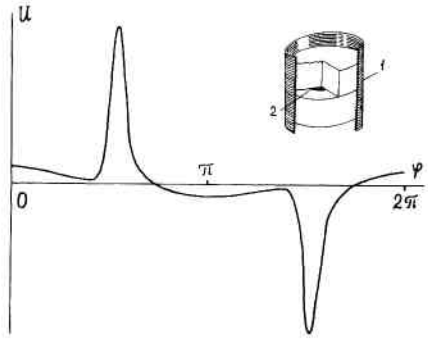

Fig. 1. Voltage changes at the receiving coil during one period of the alternating field.

Let us turn now to microwave frequencies. When the external magnetic field is weak its change leads to resonant transitions between the magnetic surface states and, as a result, to changes of the metal impedance. The spectrum of the surface states is determined by quantizing of the magnetic flux through the segment formed by the metal surface and the nearby part of the classical trajectory of the electron. When the amplitude of the microwave is large and the alternating flux Φ_{ω} is of the order of, or even exceeds, the constant one: $\Phi_{\omega} \gtrsim \Phi$, the fitness of this quantizing condition is questionable. It looks as if the microwave destroys the quantum states.

Experiments performed with tin, indium and bismuth revealed peculiar nonlinear resonances under such conditions [5]. The sample subjected to a large-amplitude microwave field at the frequency $\omega/2\pi \simeq 9$ GHz emits a signal at the frequency 2ω. When the amplitudes H_{ω} are large enough, resonant lines appear in the power ($P_{2\omega}$) versus external field (H) plots, periodically in the inverse field:

$$H_n = H_1/n, \quad n = 1, 2, 3, \cdots . \qquad (3)$$

The H_1 value depends weakly on H_{ω} (see figs 2 and 3). Resonances appear when the amplitude H_{ω} exceeds a threshold $H_{\text{th}}^{(n)}$. The threshold value $H_{\text{th}}^{(n)}$ changes with the resonance number (see Fig. 4). The inequality $H_{\text{th}}^{(n)} > H_n$ is, however, always satisfied. The fields H_n do not coincide with those for linear resonances. Detailed comparison for bismuth with $\boldsymbol{H} \parallel \boldsymbol{C}_1$ is presented in fig. 5. Above and below, the positions of linear resonances at frequencies ω and 2ω are shown, in the middle those of nonlinear resonances at different amplitudes H_{ω}.

Though it seems that an adequate model to describe this phenomenon must be a quantum one we attempted to explain the observed nonlinear resonances in terms of classical trajectories. For this purpose we need trajectories which are little affected by the field H_{ω}. This calls attention to the node planes $H_{\sim} = 0$. Consider

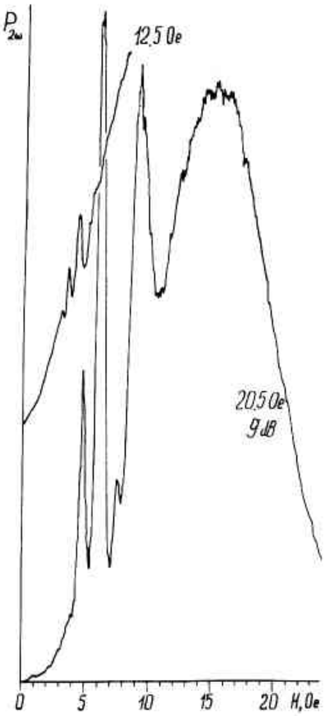

Fig. 2. Nonlinear resonance in indium (normal to the sample surface $\boldsymbol{n} \parallel [011]$, $T = 3.4$ K). The curves are marked by H_ω values and by the relative sensitivity values of the receiving circuit. The curve $H_\omega = 12.5$ Oe is displaced upward.

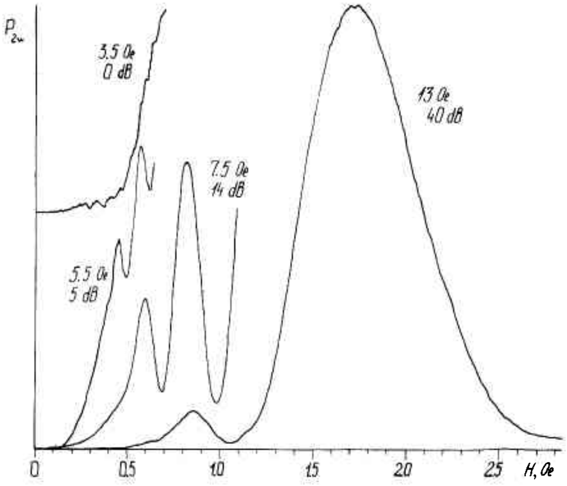

Fig. 3. Nonlinear resonances in bismuth ($\boldsymbol{n} \parallel \boldsymbol{C}_3$, $\boldsymbol{H} \parallel \boldsymbol{C}_1$, $T = 1.5$ K). The curve $H_\omega = 3.5$ Oe is displaced upward.

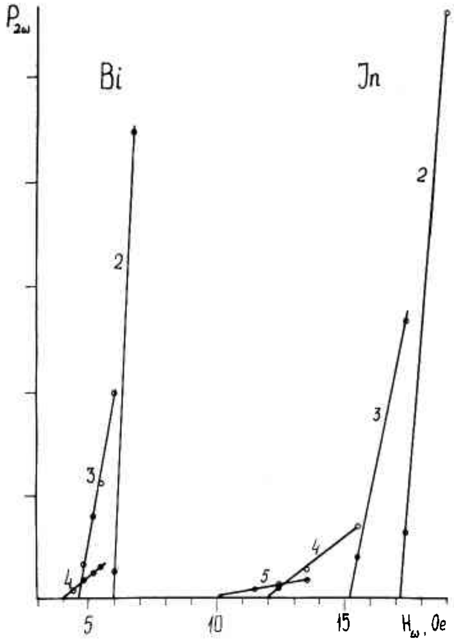

Fig. 4. The dependence of the amplitudes of resonance lines on H_ω. The straight lines are marked by the number n of the corresponding resonance.

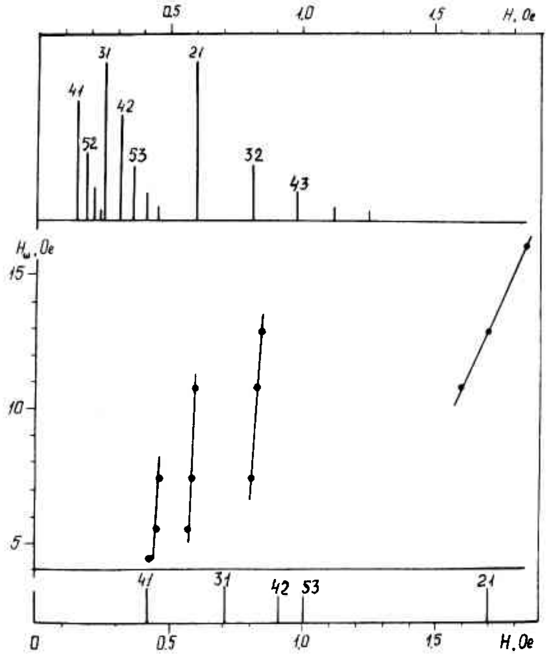

Fig. 5. Comparison between positions of the linear and non-linear resonances in bismuth.

electrons which start from the surface at a time when the node is there. Those electrons which move into the interior at an angle $\alpha = v_\omega/v_F$, i.e. in phase with the node plane, escape from the skin layer without sensing the strong magnetic field. Requiring that these electrons – now moving outside the skin layer in the external field $\boldsymbol{H}$ – return to the skin layer at the same phase of the alternating field, we get a condition on H:

$$H_n = \frac{mc}{e}\frac{\alpha}{\pi}\frac{\omega}{n} = \frac{1}{n}\left(\frac{\hbar c\omega^2 k_F \delta}{\pi e v_F^2}\right), \quad n = 1, 2, 3, \cdots. \tag{4}$$

This gives the periodicity (3) in the inverse field which has been found in the experiment.

The contribution of these resonant trajectories is enhanced by a focusing effect. The field (2) produces the force

$$F = \frac{e}{c} v_F \frac{\partial H}{\partial x}(x - x_0), \quad x_0 = \alpha t v_F \tag{5}$$

which acts upon the electron that has started from the surface at a time $t = 0$ at an angle $\varphi \neq \alpha$. The equation for motion in the direction $\boldsymbol{u}$ normal to the resonant trajectory has a form ($u = x - x_0$)

$$\frac{d^2u}{d\tau^2} + b^2 e^{-2\tau} u = 0, \quad 2\tau = x_0/\delta = \omega t, \quad b^2 = \frac{4}{\pi}\frac{H_\omega}{H_1}. \tag{6}$$

It follows from its general solution that when

$$H_\omega = H_{\omega 0} = \frac{\pi}{4} b_s^2 H_1, \tag{7}$$

b_s being the zero of the Bessel function ($s = 1, 2, \cdots; b_1 = 2.405$), the beam is transformed into a parallel one. This means that the electrons which started along nonresonant trajectories are transfered by the field into the resonant one. Eq. (7) leads to the existence of the threshold $H_{\omega 0}$. Above the threshold the focusing condition (7) is complied with only at a part of the specimen surface because of the nonuniformity of the magnetic field in the resonant cavity. The focused beam has to be formed at the depth where the alternating field amplitude is equal to the external field: $H_\omega \exp(-kx) = H$. This leads to an increase of the threshold field $H_{\omega 0}$ comparatively to that given by the expression (7) and to its dependence on the number n of the resonance. This dependence was demonstrated on fig. 4.

So it seems that the classical model explains the experimental facts quite satisfactorily, whereas in the linear case quantum considerations were necessary.

In conclusion, the aim of this brief review is to draw attention to the comparatively new and rather peculiar domain of solid state electrodynamics which apparently has no close resemblance in plasma physics.

References

[1] V.F. Gantmakher, G.I. Leviev and M.R. Trunin, Sov. Phys. JETP 55 (1982) 1607.

[2] V.T. Dolgopolov, Sov. Phys. JETP 41 (1975) 173; Sov. Phys. Usp. 23 (1980) 134.

[3] V.T. Dolgopolov and P.N. Chuprov. Solid State Comm. 48 (1983) 165.

[4] V.T. Dolgopolov and P.N. Chuprov, Sov. Phys. JETP 56 (1982) 1325.

[5] V.F. Gantmakher, G.I. Leviev and M.R. Trunin, Sov. Phys. JETP 58 (4) (1983).