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NONLINEAR REFLECTION OF AN ELECTROMAGNETIC WAVE FROM BISMUTH WITH SUPERSONIC DRIFT OF CARRIERS

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It has been shown experimentally that the supersonic Hall drift of carriers in bismuth in magnetic fields $H = 20$–100 kOe stimulates nonlinear reflection of 9.2 GHz electromagnetic wave: the second harmonic power increases by a factor of 10^2 to 10^3. According to the model proposed this effect is related to amplification by the supersonic drift of the hypersonic wave generated at the surface of the crystal.

INTRODUCTION

WHEN THE ELECTRON FLUX in the crystal attains the velocity of sound s there arises an acoustoelectronic instability leading to the phonon generation (see review [1], for example). The degenerate electron gas flux can usually be described by a shift of the Fermi surface as a whole in the **k**-space. The electron distribution function is then ($\hbar\mathbf{kv} \ll \epsilon_F$)

$$f(\mathbf{k}) = \left[\exp\frac{\epsilon(\mathbf{k}) - \hbar\mathbf{kv} - \epsilon_F}{T} + 1\right]^{-1}. \qquad (1)$$

Let the drift velocity **v** entering into (1) be directed along the x axis and, for simplicity, $T = 0$. When a threshold $v = s$ is attained, then, according to the energy and momentum conservation laws, the transitions $(k_x, k_\perp) \to (-k_x, k_\perp)$ can occur on the spherical Fermi surface $\epsilon_F = \hbar^2 k^2/2m$, followed by phonon emission with the wave vector $q \approx 2k_x$. Inasmuch as $k_x^2 + k_\perp^2 = k_F^2$, the **q**-vector values satisfy the inequality

$$0 \leqslant q \leqslant 2k_F. \qquad (2)$$

In bismuth, comprising $n = 3 \cdot 10^{17}\ \text{cm}^{-3}$ electrons and the same number of holes, $v \geqslant s$ can most readily be realized in the crossed fields $\mathbf{E} \perp \mathbf{H}$ at low temperatures, when the scattering rate $1/\tau$ is small, and the dissipative flux along **E** is by $\Omega\tau$ times smaller than the Hall flux across **E** and **H** (Ω is the cyclotron frequency). In this case the drift Hall velocity for the carriers of either the sign is $v = c(E/H)$ (c is the velocity of light) attaining sound velocity

$$cE/H = s \qquad (3)$$

leads to a kink in the V–I characteristic, first observed in [2], to the appearance of acoustic noise [3] and to the possibility of sound amplification [4].

With $v < s$ the function (1) is accompanied in a stationary state by a shifted phonon distribution

$$\phi(\mathbf{q}) = [\exp(\hbar\mathbf{q}(\mathbf{s} - \mathbf{v})/T) - 1]^{-1}. \qquad (4)$$

With $v > s$ the distribution (4) becomes impossible. Instead, spatially non-uniform distribution of phonon modes (2) which increases exponentially along the drift direction has to be established. The final stationary function $\phi(\mathbf{q})$ depends on the dissipation processes in the phonon system.

The Fermi surface is anisotropic in bismuth. The minimal k_F value (the smallest half-axis of electronic ellipsoids) lying in the plane (C_1, C_2) is such that the corresponding ultimate frequency of the slow transversal wave with $\mathbf{q} \parallel C_2$, $\omega = qs = 2k_F s$ equals $1.45 \cdot 10^{10}$ Hz. Up till now, the sound amplification has been realized only at frequencies lower than $2 \cdot 10^8$ Hz. The present communication describes the experiment enabling one, although indirectly but convincingly enough, to observe sound amplification at a frequency $0.9 \cdot 10^{10}$ Hz.

EXPERIMENT

A bismuth sample, placed in a constant magnetic field **H**, was subjected to a double electromagnetic action (Fig. 1). The pulse of voltage U produced a carrier drift in the sample in the direction normal to its surface. Also, the sample was irradiated by the electromagnetic field of frequency $\omega/2\pi = 0.92 \cdot 10^{10}$ Hz. The electromagnetic emanation from the sample was measured at a second-harmonic frequency 2ω. The duration of the voltage pulse t_E and the microwave one t_ω could be varied independently within $0.5\,\mu\text{s} \leqslant t_E \leqslant 20\,\mu\text{s}$ and $2\,\mu\text{s} \leqslant t_\omega \leqslant 10\,\mu\text{s}$, the shift Δt could have not only different value, but different sign too. The voltage U was varied within $0 \leqslant U \leqslant 60$ V. With the distance between the contacts on the sample being

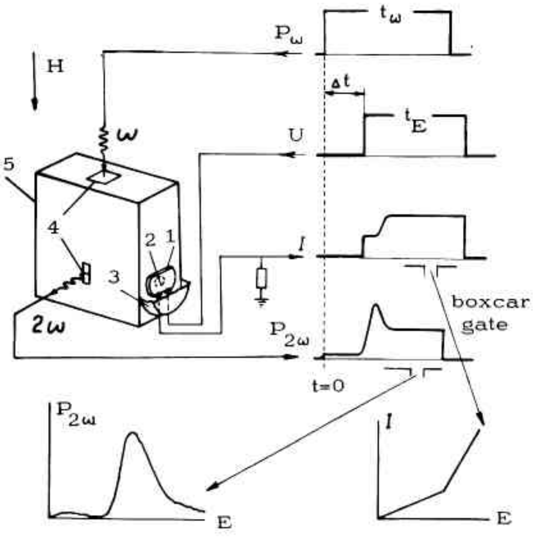

Fig. 1. Schematic drawing of the experiment (1 is the sample, 2 is the opening in the cavity wall, 3 is the brass screen, 4 are the slits into the waveguides, 5 is the cavity itself).

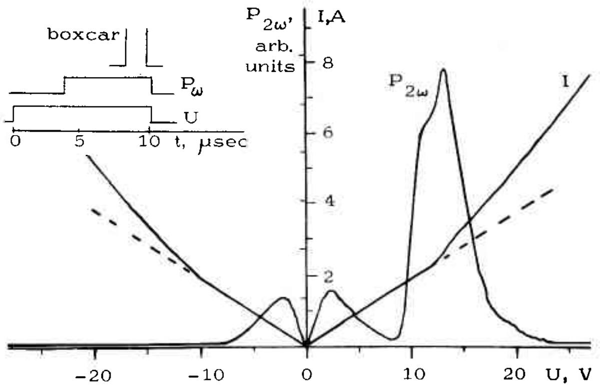

Fig. 2. V–I characteristic and the second harmonic power against the voltage at the sample. Both curves are recorded under the stationary conditions (see the insert). $H = 45$ kOe.

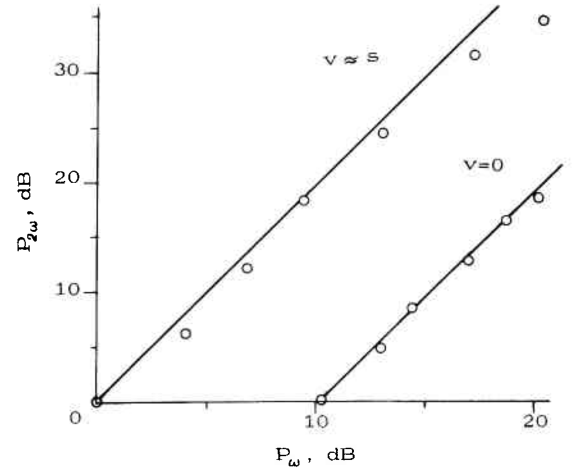

Fig. 3. Second harmonic power without the current through the sample ($v = 0$) and with the current correponding to maximum generation power in Fig. 2 ($v \approx s$). $H = 45$ kOe.

2.5 mm, this enabled to produce the field E up to 240 V cm^{-1}. The power released in the sample reached 500 W. The pulse repetition frequency was 16 Hz.

The shape of the current I pulse and of the second harmonic $P_{2\omega}$ pulse could be observed both on the oscilloscope and/or recorded on the two-coordinate recorder by means of a boxcar. Also, having fixed the boxcar electron gate position and changing U, one could record the dependence of the second harmonic power $P_{2\omega}$ on E and the V–I characteristic.

The sample 1 was retained against the opening 2 in the wall of the rectangular cavity 5 and closed from the outside with a brass screen 3 (Fig. 1). The cavity was tuned to the frequency ω (TE_{101} mode). Owing to a great magnitude of the $P_{2\omega}$ signal it was unnecessary to tune the cavity to the 2ω frequency. The description of the microwave circuits of the measurement system is given elsewhere [5].

The sample was placed in liquid helium at $T = 4.2$ K (further temperature decrease did not affect the results), the magnetic field was produced by a superconducting solenoid and could reach 80 kOe. Under these conditions at ω- and 2ω frequencies the magnetosonic waves [6] with the directions of the magnetic and electric fields $\mathbf{H}_\omega \parallel \mathbf{H}_{2\omega} \parallel 0z$, $\mathbf{E}_\omega \parallel \mathbf{E}_{2\omega} \parallel 0y$ propogate along the normal $\mathbf{n} \parallel 0x$ in bismuth.

It was most convenient to work at such constant-field values when a standing wave was established in the sample. (Note, that because of the external brass screen the electric field at the opposite surfaces of the sample was different.) The experiments were done with the samples of different form (rod, platelet, disc) and orientation. The experimental curves presented were obtained using a disc of thickness $d = 0.4$ mm, $\mathbf{n} \perp C_2$, $\angle(\mathbf{n}, \langle C_1) = 6^\circ$, $\mathbf{H} \parallel \mathbf{H}_\omega \parallel C_2$, $\mathbf{I} \parallel \mathbf{E}_\omega \perp \mathbf{H}$.

THE SECOND HARMONIC GENERATION

The main result is that in the region of the kink in the V–I characteristic related to acoustoelectronic instability [2] strong generation of the second harmonic takes place in spite of the relatively low level of the initial irradiation at the frequency ω. A typical example is presented in Fig. 2 wherein the positive direction of E corresponds to the carrier drift deep down from the surface irradiated by the microwave. With reversal of the drift direction the $P_{2\omega}$ peak vanishes, the V–I characteristic being practically unaltered. In the peak region the relationship between the amplitudes $P_{2\omega}(E^+)/P_{2\omega}(E^-) \approx 20$–$30$ dB (see Fig. 3; $P_{2\omega}(E^-) \approx P_{2\omega}(E = 0)$). The peak may be vanished by reversal of the magnetic field in the solenoid as well.

Figure 2 indicates that an increase of the power $P_{2\omega}$ was also observed in weak fields E. We shall not discuss this phenomenon here.

DISCUSSION

The $P_{2\omega}$ peak position with respect to scale E, correlating with (3), and the importance of the drift direction show unambiguously that the amplification of the sound wave, running from the irradiated surface, takes place in the sample. Therefore the described experimental result should be regarded as a consequence of the following processes:

(a) the electromagnetic wave incident onto the surface generates a sound wave of frequency ω running from the surface;

(b) the sound wave is amplified by the drift flux of electrons and holes and, simultaneously, transformed to higher harmonics, in particular, to second one;

(c) the 2ω-frequency sound wave is again transformed to electromagnetic one at the sample surface.

We shall denote the powers of the initial and amplified sound waves of frequency ω as Q_0 and Q_1, the power of the second sound harmonic as Q_2, η and β will stand for the corresponding transformation coefficients, αd for the sound amplification coefficient at a length d. Making use of a quadratic character of the transformation $Q_1 \to Q_2$ that follows from Fig. 3, we obtain

$$P_{2\omega} = \eta Q_2 = \eta\beta Q_1^2 = \eta\beta(\alpha d)^2 Q_0^2 = \eta^3\beta(\alpha d)^2 P_\omega^2. \tag{5}$$

There are several mechanisms of the surface excitation of sound (see [7], for example) and it is unlikely that any of these can be preferred with confidence under our experimental conditions. We shall only note that the experimentally measured coefficients of transformation $P_\omega \to Q_0$ of the electromagnetic power to the sonic one at frequencies of about 10^{10} Hz amounted to $\eta = Q_0/P_\omega \approx 10^{-5}$ [7]. Though these experiments were conducted with other than bismuth materials and in the absence of the magnetic field, this value can be employed for numerical estimates.

An approximate estimate for the β value can be obtained by assuming that $\beta^{-1} = Q_{1\max}$, where $Q_{1\max} = \alpha d\eta P_{\omega\max}$ is the power magnitude at which deviation from a quadratic dependence ensues (see Fig. 3). Then despite some uncertainty due to the lack of match between the sample and the cavity at a frequency 2ω, one can obtain from (5) the estimation $\alpha d \approx 40$–60 dB. It is quite a reasonable value that agrees fairly well with what can be obtained theoretically. For example, according to [8] the sound amplification coefficient per 1 cm sample length with $\Omega\tau \gg 1$, $\Omega \gg \omega$ and $qR \gg 1$ (R being the cyclotron radius) equals

$$\alpha = \alpha_0 \left(\frac{\omega}{\Omega}\right)^2 \frac{\mu}{\mu^2 + \mu_0^2}, \qquad \mu = \frac{v}{s} - 1, \tag{6}$$

where

$$\alpha_0 = 2nD^2\tau/\rho m s^5, \qquad \mu_0 = \tfrac{1}{3}(\Omega\tau)^{-1} \left(\frac{\omega}{\Omega}\right)\left(\frac{v_F}{s}\right)^2, \tag{7}$$

D is the deformation potential constant, ρ is the density of bismuth. This expression is unlikely to be directly applicable in our case. For instance, it does not allow for quantization in the magnetic field. For qualitative estimations, however, it might be employed. The factor $\mu/(\mu^2 + \text{const.})$ that enters into (6) is typical for such expressions [1, 9]. It leads to a maximum in dependence $\alpha(v)$. The consequence of this is the maximum on the $P_{2\omega}(E)$ curve (see Fig. 2).

One can act semi-empirically too when analysing the αd value. Putting $\alpha d \approx 17.5$dB/cm · 0.04 cm = 0.7 dB [4] that relates to a frequency of 10^8 Hz and making use of the dependence $\alpha_{\max} = \alpha(\mu_0) \propto \omega$ from (6), we obtain that at our frequency one has to expect the amplification coefficient $\alpha d \approx 70$ dB. These estimates also enable one to evaluate the sonic energy flux $Q_{1\max}$ that appears to be of the order of 10^2 W cm^{-2}. This corresponds to the oscillation amplitude of about 10^{-9} cm It is not surprising that at such amplitudes harmonics are intensively generated. The energy transfer to harmonics is facilitated by lack of the sound velocity dispersion.

TRANSITIONAL PROCESSES

Due to low velocity of sound the stationary function of phonon distribution $\phi(\mathbf{q})$ is established comparatively slowly. Being highly non-equilibrium it affects, in its turn, the function f leading to long transitional electrical processes. The process of changes of the V–I characteristic in bismuth when $v > s$ was studied in detail in [10]. It was shown that the onset of the stationary nonlinear regime occurs in times $\tau_{nl} \approx 10^{-6}$ s. We have also observed in the present experiment several transitional processes during the establishment of the sound wave amplification regime.

Figure 4 shows the amplitude of the second harmonic power $P_{2\omega}$ vs the time. This dependence appears more complicated than the corresponding dc one [10]. It is characterised by two parameters i.e. by the time of increase, τ_1, and by the maximum width, τ_2. The data of Fig. 3 relate to the stationary regime. As suggested by Fig. 4 the generation power

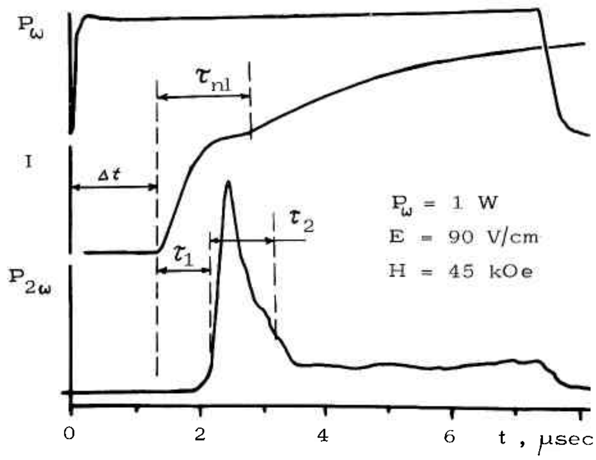

Fig. 4. Time dependence of the dc through the sample and second harmonic generation power.

can be by one order of magnitude stronger in the transitional region. Note the decrease of the generation before it goes to the stationary regime. It is likely to be due to increase of the phonon–phonon scattering. Indeed, as the stationary function of the phonon distribution $\phi(\mathbf{q})$ is established the phonons are accumulated in the crystal. This suggestion however, needs to be checked.

In conclusion, we have experimentally observed a new mechanism of the electromagnetic second harmonic generation, that is, through the sound generation as an intermediate step. A new method of hypersound generation in bismuth has simultaneously been realized – by means of irradiating it by the electromagnetic wave in the presence of a supersonic Hall carrier drift in the sample.

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REFERENCES

1. V.I. Pustovoit. *Sov. Phys. -Uspechi,* **12**, 105 (1969).
2. L. Esaki. *Phys. Rev. Lett.* **8**, 4 (1962).
3. K. Walter. *Phys. Rev. Lett.* **15**, 706 (1966).
4. K. Walter. *Phys. Rev. Lett.* **16**, 642 (1966).
5. V.F. Gantmakher, G.I. Leviev & M.R. Trunin. *Sov. Phys. -JETP* **55**, 931 (1982).
6. V.S. Edel'man. *Sov. Phys. -Uspechi,* **20**, 819 (1977).
7. A.N. Vasil'ev & Yu. P. Gaydukov. *Sov. Phys. -Uspechi,* **26**, 952 (1983).
8. H.N. Spector. *Phys. Rev.* **131**, 2512 (1963).
9. H.N. Spector. *Solid State Phys.* (Edited by F. Seitz and D. Turnbull) **19**, 291 (1966).
10. T. Yamada. *J. Phys. Soc. Jap.* **20**, 1647 (1965).