Interference resonance in two-phonon processes of de-excitation of the acceptors in tellurium

V. F. Gantmakher and I. B. Levinson

Institute of Solid State Physics, Academy of Sciences of the USSR
Institute of Problems of the Technology of Microelectronics and Especially Pure Materials, Academy of Sciences of the USSR

(Submitted 30 October 1986)

If the energy spectrum of a shallow impurity in the semiconductor has a large energy gap in which there is an additional level, the two-phonon de-excitation will exhibit a resonance when this level reaches the midpoint of the energy gap. This behavior probably accounts for the magnetic-impurity resonance in tellurium observed by Von Klitzing3 [Solid State Electr. 21, 223 (1978)].

The first excited state |1⟩ of a shallow hydrogen-like impurity in a semiconductor is separated from the ground state |0⟩ by a large energy gap \( E_{10} \), and is therefore metastable. Because the energy \( E_{10} \) is large, the phonon emitted in the transition |1⟩ → |0⟩ has a wavelength, \( 2\pi/q_a \), much shorter than the radius of the center, \( a \), so that the matrix element of this transition is small in the parameter \( qa \gg 1 \). This metastability accounts for the fact that in strong electric fields \( G \), when the current flow is accompanied by an impact ionization of the impurities and recombination processes, an appreciable number of the impurities is in the excited state |1⟩. The external magnetic field \( H \) changes the impurity spectrum and the band-electron spectrum. If the distance between the Landau levels in the band is the same as the energy gap \( E_{10} \), the number of excited impurities will sharply decrease due to inelastic scattering of the carriers involving a transfer of energy \( E_{10} \) to them from the impurity. This resonance is responsible for the magnetic-impurity oscillations.1,2

Von Klitzing3 detected an unusual “resonance” in the \( H \) dependence of a nonohmic resistance \( R \) (at \( T = 2 \text{ K}, G \approx 1 \text{ V/cm} \)). In tellurium the spectrum of a shallow impurity (bismuth, antimony, arsenic, phosphorus) is not hydrogen-like in the sense that instead of a single excited state, there are two excited states, |1⟩ and |2⟩, which are separated from the ground state |0⟩ by a relatively large energy gap (\( E_{20} \approx 1.1 \text{ meV}, E_{10} \approx 0.4 \text{ meV at } H = 0 \)). Special features of the magnetoresistance \( R(H) \) are always observed4 in such a field \( H \) when \( E_{10} = E_{21} = E_{20} - E_{10} \), i.e., when the level |1⟩ lies halfway between |2⟩ and |0⟩. For example, when bismuth is the impurity and the field \( H \) is directed perpendicular to the crystallographic c axis, the resonance occurs in a field \( H \approx 3.4 \text{ T} \). A unique feature of this resonance condition is that it includes only the energy gaps from the impurity spectrum and that there are no energies other than those associated with the impurity. We will show below that at \( E_{10} = E_{21} \) there occurs a resonant increase of the probability for the transition |2⟩ → |0⟩ with the emission of two phonons, which may account for the special features in the behavior of \( R(H) \) observed by Von Klitzing.

The interaction of an impurity electron with the lattice vibrations is given by
\[ V = V_1 + V_2, \] where \( V_1 \) is a one-phonon interaction, and \( V_2 \) is a two-phonon interaction. The transition \( |2\rangle \rightarrow |0\rangle \) with the emission of two phonons, \( q \) and \( q' \), occurs in the second-order perturbation theory in \( V_1 \) and in the first-order perturbation theory in \( V_2 \). Accordingly, the matrix element of the transition can be written
\[ M_{qq'} = M_{qq'}^{(1)} + M_{qq'}^{(2)}, \] (1)
where
\[ M_{qq'}^{(1)} = \langle 0; q, q'| V_2 |2 \rangle, \] (2)
\[ M_{qq'}^{(2)} = \frac{\langle 0; q, q'| V_1 |1; q \rangle \langle 1; q'| V_1 |2 \rangle}{E_{21} - \hbar s q + i \hbar \tau_1} + \ldots. \] (3)
Here the dots denote the term derived from the existing term through permutation of \( q \) and \( q' \), \( \hbar s q \) is the phonon energy (\( s \) is the velocity of sound), and \( \tau_1 \) is the lifetime of the state \( |1\rangle \); \( \alpha; q, q' \ldots \) is the state of the system "impurity + lattice" when the phonons \( q, q' \ldots \) are in the excited state. The probability for the transition of interest to us is
\[ w_{qq'} = \frac{2\pi}{\hbar} |M_{qq'}^{(1)}|^2 \delta(E_{20} - \hbar s q - \hbar s q'). \] (4)
Since the matrix element \( M^{(1)} \) contains the integral
\[ \int dr \psi^* \psi \psi \phi e^{i(q + q')r}, \] (5)
it is significantly greater than zero when \(|q + q'| \alpha \leq 1\), i.e., when
\[ |q| \approx |q'| \approx E_{20}/2\hbar s. \] (6)
The polar nature of the two-phonon interaction may be another reason that the matrix element \( M^{(1)} \) is large when \(|q + q'| \) are small.\(^5\)

The matrix element \( M^{(2)} \) is large for such phonons when the denominator in (3) is small, i.e., when
\[ \hbar s q \approx E_{21}, \quad \hbar s q' \approx E_{10}, \] (7)
or when the two-phonon transition \( |2\rangle \rightarrow |0\rangle \) through a virtual intermediate state "reduces" to a series of two one-phonon transitions through an actual intermediate state. The square \(|M_{qq'}^{(1)}|^2\) appearing in the transition probability contains the interference term \( M_{qq'}^{(1)} M_{qq'}^{(2)*} + \) c.c., which is large only if both conditions (6) and (7) hold, i.e., when \( E_{21} \approx E_{10} \approx E_{20}/2 \). This term is responsible, in our view, for the particular features of \( R(H) \) observed in Fig. 3.

Relation (6) is valid within \( \Delta q \sim 1/\alpha \) and relation (7) is valid within \( \Delta q \sim 1/\tau s \). Consequently, the resonance width on the energy scale, i.e., the permissible deviation of the position of the level \( |1\rangle \) from the midpoint of the gap \( E_{20} \) is
\[ \Delta E \approx \hbar s / \alpha + \hbar / \tau_1. \] (8)
The lifetime $\tau_1$ of the metastable state $|1\rangle$ is determined by the one-phonon processes. All available calculations of $\tau_1$ are, however, for different type of centers (a hydrogen-like center,\textsuperscript{6} acceptor in a cubic semiconductor,\textsuperscript{7} donor in a multivalley semiconductor\textsuperscript{8}). The time $\tau_1$ can be estimated only crudely by analogy with other semiconductors where it is on the order of $10^{-7}$–$10^{-8}$ s. The first term in (8) is therefore the principal term. Substituting in it $s = 2 \times 10^5$ cm/s and $a = 150$ Å, we find $\Delta E \approx 0.1$ meV. Making use of the plots for the $H$ dependence of the energy gaps $E_{10}$, $E_{20}$, and $E_{21}$ (Fig. 9 in Ref. 4), we find from this value of $\Delta E$ that the resonance line width in the field should be about 0.7 T, which is in order-of-magnitude agreement with the resonance line width of the experimental curves (see e.g., Fig. 6 in Ref. 3).

The interference term in $|M_{qq'}|^2$ cannot be larger than the sum $|M_{qq'}^{(1)}|^2 + |M_{qq'}^{(2)}|^2$. Since $M_{qq'}^{(1)}$ and $M_{qq'}^{(2)}$ are nearly independent of the field, the probability for the two-phonon transitions $w_{qq'}$ at resonance can increase by no more than a factor of two. Since there are also other relaxation channels, it is not surprising that the resistance of tellurium at resonance varies very slightly (Fig. 1 in Ref. 3). The use of a modulation technique, however, makes it possible to detect these resonances with a large sensitivity margin.


Translated by S. J. Amoretty