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RESISTANCE SCALING IN MEDIA WITH FRACTAL-LIKE STRUCTURE IN THE VICINITY OF A METAL–INSULATOR TRANSITION

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Scaling relation between residual R_0 and temperature-dependent R_1 parts of the sample electrical resistance in the vicinity of a metal–insulator transition can be a guide in examining the structure of the material. Three possible values of the exponent ν in the relation $R_1 \propto (R_0)^\nu$ are discussed. $\nu = 1$ is typical for a random mixture of metallic and insulating domains, $\nu = 0$ for granular metals. A special case $\nu = 0.75$ which has been observed recently corresponds to a fractal structure of the insulating phase with the classical size effect governing the conductivity of the metallic channels. Experimental data on Zn–Sb and Al–Ge alloys are presented.

1. INTRODUCTION

TRANSPORT phenomena in inhomogeneous media are usually considered by applying the percolation theory [1]. However, the material science has at its disposal a lot of geometrical patterns which are not of a simple random nature. For instance, granular metals contain in essence a correlated system of rather regularly alternating metallic and insulating regions. An excuse for applying the percolation theory to granular metals lies in possible existence of random holes in the intergrain insulating films or in randomness of thickness of these films. However, the experiment does not qualitatively fit the percolation theory as far as the metal volume fraction is considered [2].

Much more complicated and interesting structures may occur when the solid mixture arises as a result of an aggregation process or of a phase transition in some parts of the sample [3]. If these two phases have different conductivities, for instance, when a metal–insulator transition takes place, one should describe conducting network and the total conductance of the sample. In a medium with a correlated structure an essential difference from the predictions of the percolation theory during the metal-insulator transition may be observed [4]. It arises from the strong space-time correlation among macroscopic volumes which undergo the phase transition.

In this content the term "metal–insulator transition" has two meanings: "point transition" when a small volume becomes dielectric, and "sample transition" when the conductance of the whole sample drops to zero. We shall concern ourselves only with studying the metallic side of the sample transition. The main idea is that while analyzing scaling relations for the sample resistance one can judge what type of the structure is realized in the sample and distinguish, for example, a fractal pattern from a random one.

2. FRACTAL PATTERN OF THE SAMPLE STRUCTURE

We shall discuss an inhomogeneous structure created in the course of the special kind of phase transition, namely, the amorphization of the metastable phase of alloys Zn–Sb and Al–Ge quenched under pressure [5, 6]. The initial state is crystalline and metallic while the final one is amorphous and insulating. The amorphization process could be led slowly by the low temperature annealing. It could be repeatedly interrupted by returning to nitrogen temperatures.

This transition to the amorphous state has the following important features:

1. The specific volume increases significantly during the transition [5].

2. The sample resistance R increases by orders of magnitude while the sample remains metallic at low temperatures, i.e. while it retains $\partial R/\partial T > 0$ [7].

3. The large increase of the resistance is accompanied by a very small decrease of the temperature T_c of the superconducting transition. This point contains the evidence that the sample is macroscopically inhomogeneous in the intermediate states of the transition [7].

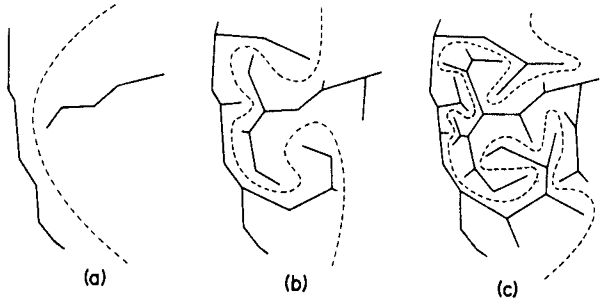

Fig. 1. Two-dimensional cross-sections of the supposed fractal structure of the amorphous phase at various stages of the transition. The dashed lines are traces of the surfaces at which the current paths are disposed.

Basing on the first point of this list and on the resistance data which will be presented below and using arguments of the theory of elasticity a fractal-like model of the intrinsic structure of the sample was proposed in [8]. It supposes the growing insulating amorphous inclusions to be cactus-like with leaves branching many times. These leaves, or sheets, are supposed neither to intersect each other nor to merge (Fig. 1). This maintains the existence of current paths, i.e. conductive channels, at the far-gone stages of the transition, in contrast with the percolation model.

The development of the "cactus" structures can be described by a parameter d which has the meaning of the mean distance between the leaves and between the sidebranching lines. According to the arguments proposed by Esipov [8] the current paths are located at the surface with fractal dimension three which separates two "cactuses"; from the mathematical point of view the current paths are similar to trajectories of brownian particles, i.e. random-walk trajectories [9]. The fractal dimension of such a trajectory is two: its length is proportional to squared radius K of the domain it occupies.

So, as far as electrical resistance is concerned, the conductive channels are conductive brownian trajectories. Now d becomes the step length of the random walk and, with the distance between the contacts L being constant, we have the channel length $\lambda \propto d^{-1}$.

It is natural to suppose that the cross-section of the channel is d^2. The simplest way to introduce this is to use the lattice representation: when the random walk occurs on a cubic lattice with a period d the cross-section of the cell d^2 is a natural cross-section of the trace. The development of the fractal pattern given on Fig. 1 corresponds to decrease of d. This, in turn, means that instead of a lattice with a fixed period we deal with a series of lattices with gradually decreasing periods. Here is, again, a remarkable contrast with the classical percolation theory.

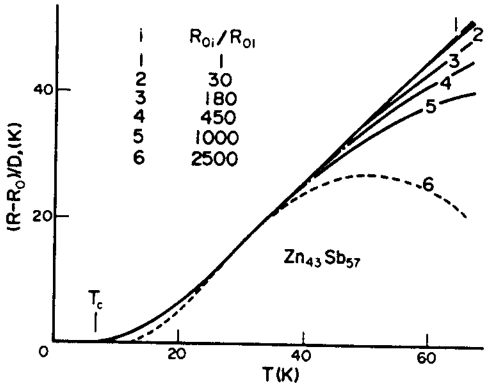

Fig. 2. Scaled temperature dependence of the Zn–Sb alloy resistance in the sample states 1–5 – solid line. The same for the sample in the state 6 – dashed line.

3. Zn–Sb DATA

The experiment with the Zn–Sb alloy and its possible interpretation were discussed in detail in [7, 8]. Here we'll first summarize briefly the results.

Figure 2 demonstrates the low-temperature dependence of the sample resistance during the amorphization. It is drawn with the use of a simple rescaling. For each experimental curve the residual resistance $R_0 = R(T = 7.1\,\mathrm{K})$ was subtracted first; then the ordinates $R(T) - R_0$ were divided by the slope coefficient $D = (\partial R/\partial T)_{T=30\,\mathrm{K}}$. Note that after these two manipulations: the shift of the origin and the scaling, all the curves coincide not only at two points but within a rather large temperature interval. The further the amorphous phase evolves the less is the region where the temperature dependence is universal. However, even with R_0 increased by three orders of magnitude the region remains rather large. It is only at the last stage of the transition that the scaling procedure fails to reproduce the universal function (curve 6 in Fig. 2).

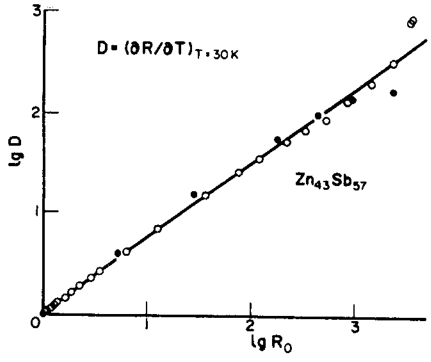

Fig. 3. Double-logarithmic plot of the derivative $\partial R/\partial T$ dependence on the residual resistance R_0 for two Zn–Sb samples. Full circles – data obtained from the curves presented in Fig. 2. Straight line corresponds to the exponent $\alpha = 0.75$.

In Fig. 3 the slope coefficient D is plotted against the residual resistance R_0 chosen to be a measure of amorphization.

The experimental results demonstrated in Figs. 2 and 3 can be described by the relation

$$R(d, T) = R_0(d) + R_1(d)f(T) = R_0(d) + R_1(d^*)\left[\frac{R_0(d)}{R_0(d^*)}\right]^{\alpha} f(T). \qquad (1)$$

Here d is the amorphization parameter and d^* is its initial value, factorization for $R(T) - R_0$ is a consequence of the universal temperature dependence illustrated by Fig. 2, the exponent $\alpha = 0.75$:

$$D \propto R_0^{0.75}. \qquad (2)$$

Let's represent the resistance R as a product of a resistivity ρ by a geometrical factor ϕ (the latter equals L/S (length/cross section) for a homogeneous cylindrical sample). Then

$$R = \rho(d, T)\phi(d) = [\rho_0(d) + \rho_1(d)f(d)]\phi(d). \qquad (3)$$

From (2) and (3) it follows

$$\rho_1(d) \propto [\rho_0(d)]^{\alpha}[\phi(d)]^{\alpha-1}. \qquad (4)$$

So not only the factor ϕ but also the resistivity "knows" about the parameter d. As d is the width of the conductive channels, this may be explained with the help of the classical size effect in the resistance. Let's introduce l_0 – the mean free path in the bulk material determined by the scattering by point defects and the temperature dependent part l_T of the bulk mean free path $l(T)$: $l_T^{-1} = [l(T)]^{-1} - l_0^{-1}$. The usual expression for resistivity is

$$\rho = \frac{p_F}{ne^2}\frac{1}{l} = \frac{p_F}{ne^2}\left(\frac{1}{l_0} + \frac{1}{l_T}\right) \qquad (5)$$

(n is the carrier density, p_F is the Fermi momentum). For a wire with diameter $d \ll l_0$ the mean free path l_0 can be, within a rather good accuracy, replaced in (5) by d [10]. Then, with whatever relation between l_T and d, one has

$$\rho_0(d) \propto d^{-1}, \quad \rho_1(d) \propto d^0. \qquad (6)$$

Combining this with (4) we finally obtain:

$$\phi(d) \propto d^{\alpha/(\alpha-1)} = d^{-3}, \qquad (7)$$

where the right exponent corresponds to experimental value $\alpha = 0.75$.

Equation (7) fits the fractal model perfectly. If the concentration of initial leaf nuclei is $C\,\mathrm{cm}^{-3}$ then each "independent" fractal shown in Fig. 1 has the space size of $C^{-1/3}$ and the length of a trajectory situated along its boundary is $C^{-2/3}d^{-1}$. For the whole system, where $SC^{2/3}$ trajectories are parallelly connected each being a chain of $LC^{1/3}$ links with length $C^{-2/3}d^{-1}$ and with cross-section d^{-2}, we shall obtain

$$\phi = (L/S)(Cd^3)^{-1}. \qquad (8)$$

Note, that resistivity itself has sense only at "large scale" when $L \gg C^{-1/3}$. When $L < C^{-1/3}$, the dependence ρ on L must exist. It has not been experimentally studied yet.

Let us make some numerical estimates. The whole range of variation of the resistance observed at the metallic side of the transition (four orders of magnitude) corresponds to tenfold decrease of the parameter d. For the size effect to take place the larger limit d^* of the interval where the thickness d varies (and (1) holds) must be smaller than the mean free path for bulk material l. The latter can scarcely be larger than $(1–2) \times 10^{-5}$ cm. So we have

$$1000\,\text{Å} \simeq d^* \geqslant d \simeq 100\,\text{Å}.$$

The lower limit $d_{\min} \simeq 100$ Å corresponds to curves 5, 6 in Fig. 2. Using a usual estimate for the carrier density in the metallic phase and the initial apparent resistivity value $R(S/L) \approx 50\,\mu\Omega\,\mathrm{cm}$ [6] one gets from (3) and (8) that $C(d^*)^3 \simeq 1–10$, i.e. $C \simeq 10^{15}–10^{16}\,\mathrm{cm}^{-3}$. Both estimates, $d_{\min}$ and C, sound quite reasonable.

4. Al–Ge DATA

The experiment with the Al–Ge alloy was performed similarly to those with Zn–Sb. The differences were only quantitative (see the Table). However, the results are quite different, as it can be seen from the comparison of Figs. 4 and 5 with Figs. 2 and 3. Note that Fig. 5 demonstrates the exponent $\alpha = 1$ instead of $\alpha = 0.75$.

Exponent 0.75 in equation (2) is the result of the combination of a fractal structure of the insulating phase and a rather long mean free path l. If, instead, $l \ll d_{\min}$ holds one will find

$$R_1(d) \propto R_0(d) \equiv [R_0(d)]^1. \qquad (9)$$

Similarity of the processes in the both alloys [5] gives reason to suggest that the structures in both are the same and that the relation between l and $d_{\min}$ is the main source of the difference.

The relation (9) is far less informative than relation (2). It holds also during a uniform transition when the changes in the resistivity are controlled by the number of carriers n in (5) as well as in the case when the resistance changes are due only to the alternation in the shape of the conductive channels, i.e. of the factor ϕ. The latter case apparently takes place

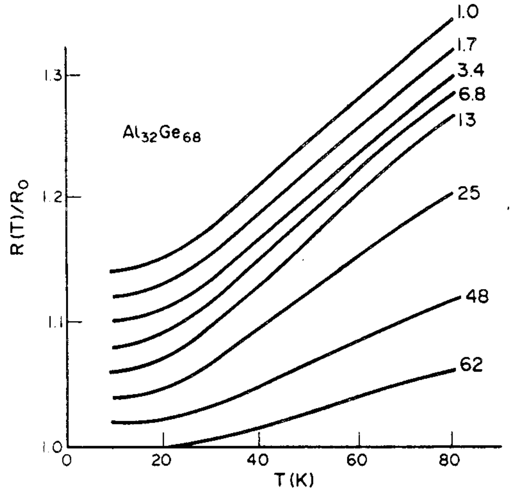

Fig. 4. Temperature variation of the Al–Ge sample resistance at different stages of amorphization. Each next curve is shifted 0.02 upward for clarity. At the right of the curves the resistance ratios R_{0i}/R_{01} are indicated.

during approaching the transition in a random mixture of two phases. Indeed, when approaching the threshold, what alters is the size of the cells of the conductive backbone in the infinite cluster. However, the links between the nodes of this backbone always contain so called red bonds, regions with minimal possible cross-sections [11, 12]. So the contribution of the size effect to the link resistivity does not alter during approaching the transition and only the factor ϕ changes.

Table 1.

	Zn–Sb	Al–Ge
Initial "apparent" resistivity $R_{01}S/L$, $\mu\Omega$ cm	50	100
Range of resistance changes at the metallic side of the transition R_0/R_{01}	10^4	10^2
Shift of the superconducting transition temperature $\partial T_c/\partial \ln R_0$, K	0.16	0.7*
Temperature at which the sample transition occurs, approximately, K	190	300

*When R_0/R_{01} became 10, the transition splitted into two demonstrating existence of odd phases in the intermediate state.

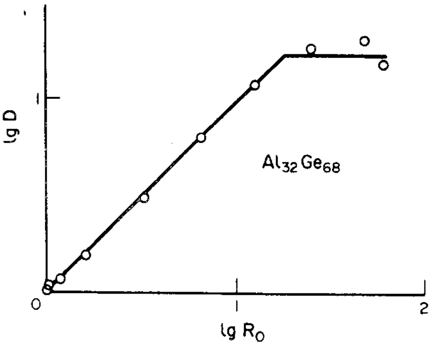

Fig. 5. The same as in Fig. 3 for a Al–Ge sample.

Insulating films in an ideal granular metal are connected in series with metal volumes. As the tunnel resistance of the films does not depend on T one gets

$$R_1(d) = \text{const} \propto [R_0(d)]^0. \qquad (10)$$

In Fig. 5 apart from the well defined region where (9) holds there is a region where the data can be interpreted as satisfying (10). Probably, this reflects the final stage of the sample transition when d becomes too small and the insulating sheets merge. Then the fractal structure transforms into the granular one.

5. CONCLUSION

In conclusion, the scaling relations between the parts of the resistance in the vicinity of the metal–insulator transition contain information about the macrostructure of the sample. Equations (9) and (10) express the two limiting cases which correspond to a random mixture of phases and to a granular metal. The exponent α in (1) can be changed by the dc size effect. The Zn–Sb experimental data give such an example.

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