samples by a factor of 1.7 at the same electron density, or by a factor of 5 at the same Fermi energy; (ii) the Si/SiO_2 interface is characterized by a very strong asymmetry of the confining potential in the z-direction. The latter results in a large effective Lorentzian field H^* seen by electrons; the corresponding spin-orbit gap at zero field is $\sim$3.6 K [14]. These effects associated with the broken inversion symmetry of the confining potential are much less pronounced in GaAs/Al(Ga)As heterojunctions and are almost absent for rectangular potential wells.

It is known that the universality class of the symmetry of the 2D system strongly affects its scaling behavior. The above spin-related mechanism should result in the breaking of spin-rotation SU(2) symmetry either completely or partially, and should correspondingly lead either to symplectic or another novel symmetry [20]. This may be important if the relevant energy gap, $\Delta = g^* \mu H^*$ is larger than $\Gamma = h/\tau$, the spin-level broadening. The estimated Δ/Γ -ratio ~ 3 for Si-15A and Si-2Ni, while ~ 1 for the sample Si-39, which does not exhibit the well pronounced M – I transition.

In conclusion, we suggest that the metallic state and M – I transition in the high mobility Si-MOS structures is a consequence of both the spin and the Coulomb interaction effects, where the former are enhanced by the broken inversion symmetry. Apparently, this is in agreement with a very recent observation of the M – I transition in the 2D hole system in Si – SiGe heterostructures [11] which exhibits strong spin-orbit coupling. In this context we also wish to emphasize that the Coulomb interaction was predicted by Finkel'stein [21] to overwhelm the localization trend in the cooperon channel and to lead the drop in resistivity upon lowering T, whereas the spin-related effects favor this drop to start earlier at higher temperatures. Recently it was also proposed that the ground state of the unconventional 2D metallic state is a spin-triplet paired state [22], and that the M – I transition may be a manifestation of non-Fermi-liquid behavior [23].

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Superconductor – insulator transitions and insulators with localized pairs

V F Gantmakher

1. Introduction

Among the various scenarios of superconductor – insulator transitions (SIT) is a specific one proposed by Fisher [1]: a field-induced and field-tuned transition in two-dimensional superconductors. It supposes, at $T = 0$, the existence of delocalized Cooper pairs and localized vortices (superconductor) below the transition, at fields $B < B_c$, and of localized pairs and delocalized vortices (insulator) above it, at $B > B_c$. Several experimental studies [2, 3] apparently support this model describing the experimental data near the transition in terms of scaling relations [1]. As a consequence of the approach of [1 – 3], an insulator should be supposed to exist with localized Cooper pairs and the magnetic field structured into vortices. The properties of such insulators have still not been addressed. However, some experimental observations do point to localized pairs [4]. In particular, negative magnetoresistance in some three-dimensional materials, when superconductivity is alternated with insulating behavior, is one such indication [5]. This negative magnetoresistance results from destruction of the gap in the spectrum of localized electrons by the magnetic field, which affects single-particle tunneling [6].

Still, our knowledge of localized pairs is rather poor. The existing studies leave some doubts. The amorphous Mo – Ge films from [3] display only a 5% increase of resistance for a ten-fold decrease in temperature in the high-magnetic-field limit and behave not like an insulator, but more like a metal with a small quantum correction to the resistance. The measurements with In – O films [2] were made in the quasireentrant region (see below, next section).

Hence, additional experimental observations are desired. In this study, two experiments are described, both related to the problem. First, the existence of the scaling variable [1]

$$x = \frac{(B - B_c)}{T^{1/y}} \tag{1}$$

is re-examined by analyzing magnetotransport in amorphous In_2O_x films (y is the product of two critical exponents from the theory [1]). Our measurements do not reject the possibility of the field-induced phase transition but lead to more general scaling relations.

The problem can be approached from another side, by looking for phenomena which would reveal the existence of localized pairs in insulators. To that purpose, the transport properties of the high-resistance metastable insulating alloy Cd – Sb were studied [7] where localized pairs were supposed to exist. This experiment is described in the last section of the paper.

2. In – O — scaling relations†

There are two main types for the sets of 2D field-tuned SIT $R(T)$ curves. They differ in the behavior below the transition onset temperature T_{c0} (Fig.1). The first type is more complicated. In fields weak enough, the curves are maximum near T_{c0} and below decrease monotonically, with a positive first derivative, until the resistance R reaches zero (if at all). In fields strong enough, the first derivative is negative everywhere. There is an intermediate field range with two extrema on the curves $R(T)$: a maximum at $T_{max} \approx T_{c0}$ and a minimum at a lower temperature. The latter separates from the maximum and shifts to lower T with decreasing field.

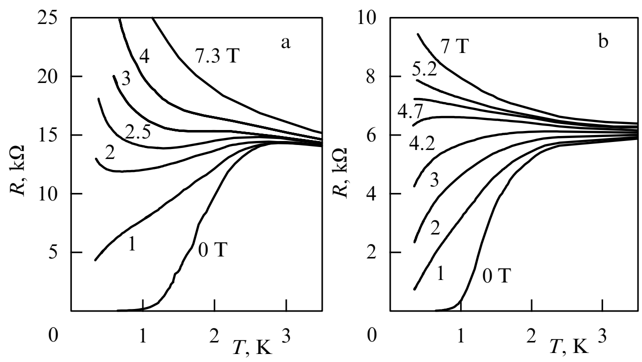

Figure 1. Temperature dependence of the resistance of two different amorphous In – O films, both 200 A thick, in various magnetic fields. Sets (a) and (b) represent two types of films behavior (see text).

The second type is simpler. There is no intermediate field range and no curves with minima at all; the rise in the field results in a shift of the maximum to lower T until it disappears. The authors of [8], based their results obtained with ultrathin films of metals, consider the second type as ideal and attribute the low-temperature minimum (the so-called 'quasire-entrant transition') to inhomogeneities and to one-particle tunneling between superconducting grains.

The amorphous In – O films described in [2] showed behavior of the first type; whereas amorphous Mo – Ge films [3] showed the second type. The scaling variable (1) proved to work in both cases. Both experiments had one feature in common: the films had low T_{c0} values, below 0.3 K. As T_{c0} seems to be the main energy which determines the scale for the problem, one should expect to get similar scaling relations with films of higher T_{c0} in a higher range of temperatures. We checked this in experiments with amorphous In – O films 200 A thick† . The value of T_{c0} in such films depends on the oxygen content and hence on the heat treatment [5, 6]. We encountered films of both types. In Figure 1 examples are presented with two films brought by corresponding heat treatment to close values of T_{c0}. Below we shall present a detailed analysis for a film of the second type only since it is supposed to be more homogeneous [8].

† This experiment was performed together with M Golubkov on films kindly presented by A Frydman and Z Ovadyahu from Jerusalem University.

The analysis in [2, 3] started with determining the values of the critical field B_c and of the critical resistance R_c by using the condition

$$\left.\frac{\partial R}{\partial T}\right|_{B_c} = 0 \tag{2}$$

at the lowest temperature and assuming that the isomagnetic curve at B_c remained of zeroslope till $T = 0$. We did not have a horizontal curve in our fan, Fig. 2. Hence, instead, we marked maxima on the curves $R(T)|_B$ and, utilizing the linear relation between the maximum values R_{max} and their positions, extrapolated the function $R_{max}(T)$ to $T = 0$ and got the limiting R_c value (dashed line in Fig. 2). One of the curves of the fan in Fig. 2, the curve obtained at $B = 5$ T looks like a straight line; it separates curves with a positive second derivative from curves with a negative derivative. Extrapolated to zero temperature (dotted line), it comes practically to the same point R_c. This gives us grounds to claim, although we remain rather far in temperature from the supposed transition, at $T > 0.3\,K$, that we can determine the critical parameters in the state in question of our sample from Fig. 2:

$$B_c = 5\ \mathrm{T}, \qquad R_c = 8\ \mathrm{k\Omega}\,. \tag{3}$$

In essence, we have assumed that not the first but the second derivative is zero at the transition point; the first derivative remains finite and the line which separates two Fisher phases on the (T, R)-plane, if they exist, *has a finite slope*. For this definite film the slope is

$$\gamma \equiv \left.\frac{\partial R}{\partial T}\right|_{B_c} = -0.83\ \frac{\mathrm{k\Omega}}{\mathrm{T}}\,. \tag{4}$$

Next, we eliminate this slope by introducing a function

$$\widetilde{R} = R(T) - \gamma T \tag{5}$$

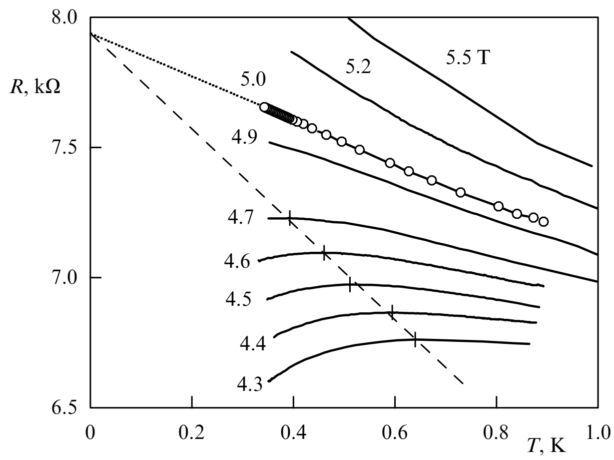

Figure 2. Central part of the set from Fig. 1b represented in detail, with a smaller field step. Experimental points are plotted only on one curve, those at $B = 5.0$ T. The dashed line displays maxima values, $R_{max}(T)$ extrapolated to $T = 0$. The dotted line is linear extrapolation of data for the field $B = 5.0$ T.

and check the scaling properties not of R but of $\widetilde{R}$. Plotting $\widetilde{R}$ against the scaling variable $|B - B_c|/T^{1/y}$ we selected the power y to bring the data to the same curve, the critical field B_c being kept within 1 % of the (3) value (Fig. 3). The adjustment led to the same value of y, $y = 1.3$, which was obtained in [2] and [3]. For additional comparison, the two scaling curves from [3] for $R(T, B)$ normalized by $R_c = 8.1$ T are reproduced in Fig. 3.

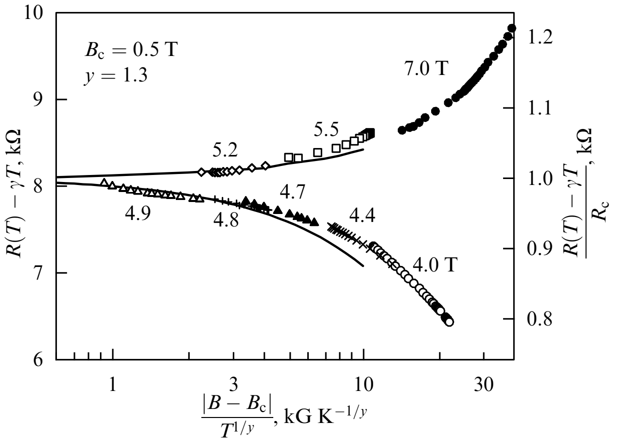

Figure 3. Scaling dependence of the renovated resistance $\widetilde{R}$ for the film presented in Fig. 1b and Fig. 2. Different symbols are used to distinguish the data in different fields. For comparison, the scaling function from Fig. 3 of Ref. [3] normalized by $R_c = 8.1$ kΩ is reproduced by solid lines.

Similar analysis was applied to the data for an amorphous In–O film with the first type of behavior (Fig. 1a). This time we used not the maxima, but the minima on the curves $R(T)|_B$ and got the value R_c by extrapolation of the function $R_{\min}(T)$ to $T = 0$. The main difference consisted in the sign of the slope γ. The deformed function $\widetilde{R}(T, B)$ after using the scaling variable (1) again collapsed onto two branches and even the value of y obtained was practically the same.

These results mean that the scaling relations [1] should either be generalized by replacing the film resistance R by $\widetilde{R}$ from (5) or they are not very specific and hence not very crucial as a criterion of existence of the field-tuned phase transition.

3. Cd–Sb — pair tunneling [7]

The main process which determines the low-temperature conductance in insulators is tunneling. Suppose that we have localized Cooper pairs. For an electron of a pair to hop, it has to 'pay off' the binding energy Δ. This introduces an additional exponential factor into the conductivity

$$\sigma_1(T) \approx \sigma_n \exp\left(-\frac{\Delta}{T}\right), \tag{6}$$

where σ_n is the expected conductivity at a temperature T in the absence of pairing. A high enough magnetic field is assumed to destroy the pairing. The phenomenon can be recognized by negative magnetoresistance. The field eliminates the gap and makes the resistance $\exp(\Delta/T)$ times smaller. In principle, measuring the ratio $\beta(T) = \sigma_n(T)/\sigma_1(T)$ affords the possibility of extracting the gap Δ. Such a negative magnetoresistance caused by one-particle tunneling is well known in granular superconductors, both in films [9, 10] and bulk [11].

We attempted to measure the function $\beta(T)$ on metastable Cd–Sb alloy in a high-resistance state which was certainly insulating and still had negative magnetoresistance originating in superconductivity [12]. A sample of $Cd_{47}Sb_{53}$ alloy had a rod-like form, with all dimensions amounting to several millimeters. The sample was transformed into a metallic phase, which is a superconductor with $T_{c0} \approx 4.5$ K, in a high-pressure chamber and, being retained in this state, was quenched to liquid-nitrogen temperature. After cooling, the sample was clamped in a holder by two pairs of gold wires with pointed ends and placed into a cryostat. The low-temperature transport measurements were alternated with heating of the sample to room temperature. At room temperature, the material slowly transformed into a disordered-insulator state. The transformation was monitored by resistance measurements and could be interrupted by returning to liquid-nitrogen temperature. The experimental procedure is described in detail in [12].

The sample resistance R is in essence the averaged value, because over the course of the transformation the sample becomes inhomogeneous. According to Ref. [13], below T_{c0} the sample looks like a mixture of different 'weakly-superconducting' elements, such as tunnel junctions, constrictions, thin wires, etc. The density and scales of these elements change during transformation, leading to evolution of the sample behavior below T_{c0} [11, 12]. We shall deal with the state in which the average sample resistance R has increased by five orders of magnitude over the initial, where only a weak kink has remained on the curve $R(T)$ at $T = T_{c0}$ and the curve has negative derivative everywhere. The T_{c0} value, while evolving to this state, has slightly decreased to ~ 3.8 K.

The low temperature measurements were performed in an Oxford TLM-400 dilution refrigerator with a base temperature of 25 mK. A four-terminal lock-in technique at a frequency of 10 Hz was used. The ac current was 1 nA and corresponded to the linear regime. The low-temperature part of the curve $R(T)$ is shown in Fig. 4.

The experimental dependences of $\alpha(T, B) = R(T, B)/R(T, B = 4\,\text{T})$ on the magnetic field at different temperatures are presented in Fig. 5. The magnetoresistance is negative and saturates in magnetic fields exceeding 2 T,

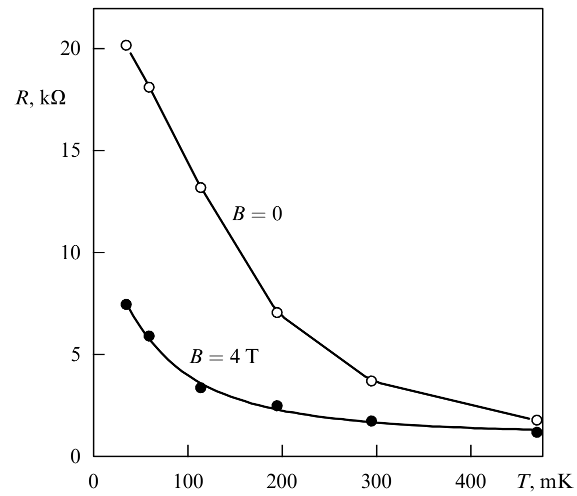

Figure 4. Low-temperature dependence of the resistance of the Cd–Sb sample in the state under investigation illustrating its insulating behavior and negative magnetoresistance.

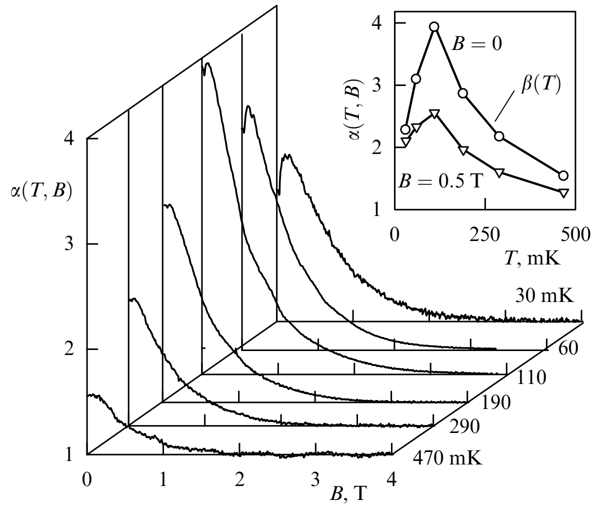

Figure 5. Normalized magnetoresistance $\alpha(T, B)$ of the Cd – Sb sample as a function of B at different temperatures. Inset: α as a function of T at two values of B; $\alpha(T = 9) \equiv \beta(T)$.

bringing the value of α to unity. So, the resistance values at a field of 4 T, used for normalizing, are taken from the field-independent region. Note that $\beta(T) = \alpha(T, 0)$. In the temperature range between 490 and 190 mK the value of $\beta(T)$ increases with decreasing temperature, in qualitative agreement with Eqn (6) (inset to Fig. 5). This corresponds to what has been observed previously at higher temperatures [12]. However, in the low-temperature limit the behavior of $\beta(T)$ changes drastically. It is maximum at a temperature of about 100 mK and then decreases with further lowering of the temperature. In weak magnetic fields an initial increase appears on the dependence. However, at any fixed magnetic field $B < 2$ T the value of α behaves similarly (see inset to Fig. 5). This contrasts sharply with the temperature dependence of the normal-state resistance which is monotonous in the range of temperatures used (see Fig. 4).

The observed decrease of the ratio β with decreasing temperature unambiguously indicates that at low temperatures the conductivity σ_1, which originates from single-particle tunneling, is shunted by the conductivity σ_2 of another kind:

$$\sigma = \sigma_1(T) + \sigma_2 \, . \tag{7}$$

We believe that σ_2 is due to incoherent pair tunneling (coherent, i.e. Josephson, pair tunneling is supposed to be absent in this insulating state; probably the maximum of $\alpha(B)$ at $B \approx 0.1$ T at the lowest temperatures designates the destruction of the remnants of the coherent scattering by the magnetic fields). The single-particle tunneling current i_1 is described in the first-order approximation by the barrier transparency t: $i_1 \propto t \exp(-\Delta/T)$. It is proportional to the product of two small factors, one of which is temperature dependent. Since the Cooper pairs are at the Fermi level, the two electrons forming a pair do not need to be excited above the gap for simultaneous tunneling. Hence $i_2 \propto t^2$, without the exponential temperature-dependent factor. When the temperature is sufficiently low, so that

$$t > \exp\left(-\frac{\Delta}{T}\right), \quad \text{i.e.} \quad T < \frac{\Delta}{|\ln t|} \, , \tag{8}$$

the single-particle tunneling is frozen out, and the two-particle tunneling current comes into play.

The two particles bound into a pair in the initial state may come to be unbound in the final state. Such a process of pair tunneling looks similar to the two-particle contribution to the tunnel current through a superconductor — the normal-metal junction (SIN junction) [14]. The latter may prove to be very important in high-resistance granular superconductors.

4. Conclusion

Both experiments described above can be interpreted as confirmation of the existence of localized pairs. But they do not give any information about how this localization is realized. The one-particle localization radius ξ may turn out to be either larger, or smaller than the coherence length ξ_{sc}. The case $\xi \gg \xi_{sc}$ is an extreme limit of granular superconductors, with only one pair in a grain. The opposite case $\xi \ll \xi_{sc}$ is assumed, for instance, in the model of localized bipolarons [15]. To distinguish these two possibilities, other experiments are required.

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Superconductor – insulator transition in the disordered Bose condensate: a discussion of the mode-coupling approach

A Gold

1. Introduction

In the last 20 years the Anderson transition [1] in disordered Fermi systems has been studied extensively. The Anderson transition is a disorder induced metal – insulator transition in a non-interacting electron gas at temperature zero. It is widely believed that a disordered non-interacting Fermi gas can be described by the scaling theory [2]: according to this theory one expects that in two dimensions and at temperature zero a metallic phase does not exist, due to weak-localization corrections, and for vanishing temperature the static conductivity should scale to zero. For the disordered interacting