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CONDENSED MATTER

Scaling Analysis of the Magnetic Field–Tuned Quantum Transition in Superconducting Amorphous In–O Films[1]

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Abstract—We studied the magnetic field–tuned superconductor–insulator transition (SIT) in amorphous In–O films with different oxygen contents and, hence, different electron densities. Whereas the two-dimensional scaling behavior was confirmed for the states of the film near the zero-field SIT, the SIT scenario changed for the deeper states in the superconducting phase; in addition to the scaling function describing the conductivity of the fluctuation-induced Cooper pairs, the temperature-dependent contribution to the film resistance emerged. This contribution can originate from the conductivity of normal electrons. © *2000 MAIK "Nauka/Interperiodica".*

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Scaling analysis is an important experimental tool for studying quantum phase transitions. For the two-dimensional (2D) disordered superconductors, along with the zero-field superconductor–insulator transition (SIT) driven by the disorder change in a film, a SIT induced by the normal magnetic field also occurs. The scenario of the field-induced 2D quantum SIT was proposed in [1]: at zero temperature, the normal magnetic field alters the state of a disordered film from superconducting at low fields to metallic at the critical field $B = B_c$, with the universal sheet resistance R_c close to $h/4e^2 \simeq 6.4$ kΩ, and to the insulating state at fields $B > B_c$. The SIT was supposed to be continuous, with the correlation length of quantum fluctuations ξ diverging as $\xi \propto (B - B_c)^{-\nu}$, where the critical index $\nu > 1$. At nonzero temperatures, the size of quantum fluctuations is restricted by the dephasing length $L_\phi \propto T^{-1/z}$, where the dynamical critical index z determines the characteristic energy $U \sim \xi^{-z}$ and is expected to be equal to $z = 1$ for SIT. The ratio of these two length parameters defines the scaling variable u such that near the transition point ($T = 0$, B_c) all $R(T, B)$ data should fall on a curve for a universal function of u

$$R(T, B) \equiv R_c r(u), \quad u = (B - B_c)/T^{1/z\nu}. \tag{1}$$

Although small in the scaling region, temperature-dependent corrections with a leading quadratic term are expected to the critical resistance R_c [1, 2].

The above theoretical description is based on the electron-pair localization concept supported by a recent publication [3]. In that paper, it was shown for the 2D superconducting films with sufficiently strong disorder that the region of fluctuation superconductivity, where the localized electron pairs (also called bosons [1] and cooperons [3]) occur, should extend down to zero temperature. In this region, the unpaired electrons are supposed to be localized because of the disorder in the film.

The theory of field-driven 3D quantum SIT has not been developed so far. The idea of considering quantum SIT for the disordered 3D systems in zero magnetic field in terms of charged boson localization [4] was at first not accepted, because the region of fluctuation superconductivity was assumed to be small. In fact, as was shown later in [5], the fluctuation region enlarges as the edge of single-electron localization is approached. This provides an opportunity to apply the scaling relation deduced for the 3D boson localization [6] to the field-induced SIT description

$$R(T, u) \sim T^{-1/z}\tilde{r}(u), \tag{2}$$

where $\tilde{r}(u)$ is a universal function and the scaling variable u is assumed to have the same form as defined by (1). From (1) and (2), it follows that in the vicinity of B_c the $R(B)$ isotherms are straight lines with slopes

$$\frac{\partial R}{\partial B} \propto T^{-(d-2+1/\nu)/z}, \tag{3}$$

where d is the system dimensionality. Since the resistance behaves in (1) and (2) in very different fashions, the problem of film dimensionality is of major importance.

The data obtained in the experimental studies of a-In–O [7], a-Mo–Ge [8], and a-Mo–Si [9] films followed by 2D scaling relation (1), except for the universality of the R_c value, and thus confirmed the existence of quan-

[1] This article was submitted by the authors in English.

tum SIT. The resistance drop observed for a-In–O films at high fields was also explained in the framework of localized bosons [10, 11]. On the other hand, the scaling was found to fail for the ultrathin Bi films. This was interpreted as evidence for the crossover between different flux–flow regimes [12].

In this work, we carried out a detailed study of the scaling relations near the field-induced SIT for different states of an amorphous In–O film. We have found that 2D scaling relation (1) holds for the film states near the zero-field SIT but progressively fails upon departure from it. This failure is manifested by the appearance of the extra temperature-dependent term in the film resistance.

The experiments were performed with 200-Å-thick a-In–O films grown by electron-gun evaporation of a high-purity In_2O_3 target onto a glass substrate.[2] This material proved to be very useful for investigations of the transport properties near the SIT [7, 10, 13–15]. Oxygen deficiency with respect to the fully stoichiometric insulating In_2O_3 compound causes the film conductivity. By changing the oxygen content, one can cover the range from a superconducting material to an insulator with activated conductance [14]. The methods for the reversible change of the film state are described in detail in [10]. To reinforce the superconducting properties of our films, we used heating in vacuum to a temperature of 70–110°C until the sample resistance became saturated. To shift the state in the opposite direction, the film was exposed to air at room temperature. Since the film remains amorphous during these manipulations, it is natural to assume that the treatment used results mainly in a change in the total carrier concentration n and that there is a certain critical concentration n_c corresponding to the zero-field SIT.

The low-temperature measurements were carried out using a four-terminal lock-in technique at a frequency of 10 Hz on two experimental setups: a He^3-cryostat down to 0.35 K or an Oxford TLM-400 dilution refrigerator in the temperature range 1.2 K–30 mK. The ac current was equal to 1 nA and corresponded to the linear response regime. The aspect ratio for the samples was close to unity.

We investigated three different homogeneous states of the same a-In–O film.[3] We characterize the sample state by its room-temperature resistance R_r. Assuming that the disorder is approximately the same for all states, we have for the carrier density $n \propto 1/R_r$; i.e., the smaller R_r, the deeper the state in the superconducting phase and, hence, the larger the value of B_c. The parameters of the states investigated are listed in the table. State 1 is closest to the zero-field SIT, and state 3 is the deepest in the superconducting phase.

Parameters of the states studied for the sample

State	R_r, kΩ	R_c, kΩ	B_c, T	α, K^{-1}
1	3.4	7.8	2.2	0
2	3.1	8	5.3	–0.1
3	3.0	9.2	7.2	–0.6

The sets of isomagnetic curves $R(T)$ for all states studied are depicted in Fig. 1. For each set, the curves can be roughly divided into two groups according to the sign of the second derivative: the positive (negative) sign corresponds to the insulating (superconducting) behavior. In what follows, the isomagnetic curve $R_c(T)$ designating the boundary between superconductor and insulator and corresponding to the boundary metallic state at $T = 0$ is referred to as the separatrix. Whereas it is easy to identify a horizontal separatrix for state 1 in accordance with (1), the fan and separatrix are "tilted" for states 2 and 3; i.e., each of the curves in the lower part of the fan has a maximum at a temperature T_{max} that shifts with B. To determine the separatrix $R_c(T)$, one should extrapolate the maximum position to $T = 0$. To do this, it is desirable to know the extrapolation law, because the accessible temperature range is restricted.

The absence of a horizontal separatrix for states 2 and 3 can also be established from the behavior of isotherms $R(B)$ (Fig. 2). As seen from Fig. 2, the isotherms for state 1 intersect at the same point (B_c, R_c), whereas those for state 3 form an envelope.

To determine B_c and R_c for states 2 and 3, we use the simplest linear extrapolation to $T = 0$ for the functions $R(T_{max})$ and $B(T_{max})$ (see Fig. 4). The open symbols correspond to the maximum positions on the isomagnetic curves (Fig. 1), and the filled symbols represent the data obtained from the intersections of the consecutive isotherms[4] (Fig. 2); if two consecutive isotherms for close temperatures T_1 and T_2 intersect at a point (B_i, R_i), the isomagnetic curve for the field B_i reaches its maximum $\approx R_i$ at $T_{max} \approx (T_1 + T_2)/2$. As seen from Fig. 4, the $B(T_{max})$ dependence is weak, so we believe that the linear extrapolation would suffice to determine B_c. By contrast, the accuracy of determination of R_c is poor.

The derivative $\partial R/\partial B$ near B_c is shown as a function of temperature in Fig. 3. Within the experimental accuracy, the exponents turned out to be identical for the film states 1 and 3, in agreement with the results

[2] The films were kindly presented by A. Frydman and Z. Ovadyahu from Jerusalem University.

[3] Observation of the so-called quasi-reentrant states for the field-driven SIT was reported in [7, 15, 16] and explained by the inhomogeneities and single-particle tunneling between superconducting grains [16]. This interpretation was supported in our experiments by the fact that the quasi-reentrant behavior observed for some film states disappeared upon the annealing of the sample in vacuum for several additional hours after its resistance had been saturated. We do not discuss quasi-reentrant states in this paper.

[4] A similar extrapolation procedure for determining R_c was employed in [17], where the metal–insulator transition in a 2D electron system was studied and the carrier density was used as a driving parameter.

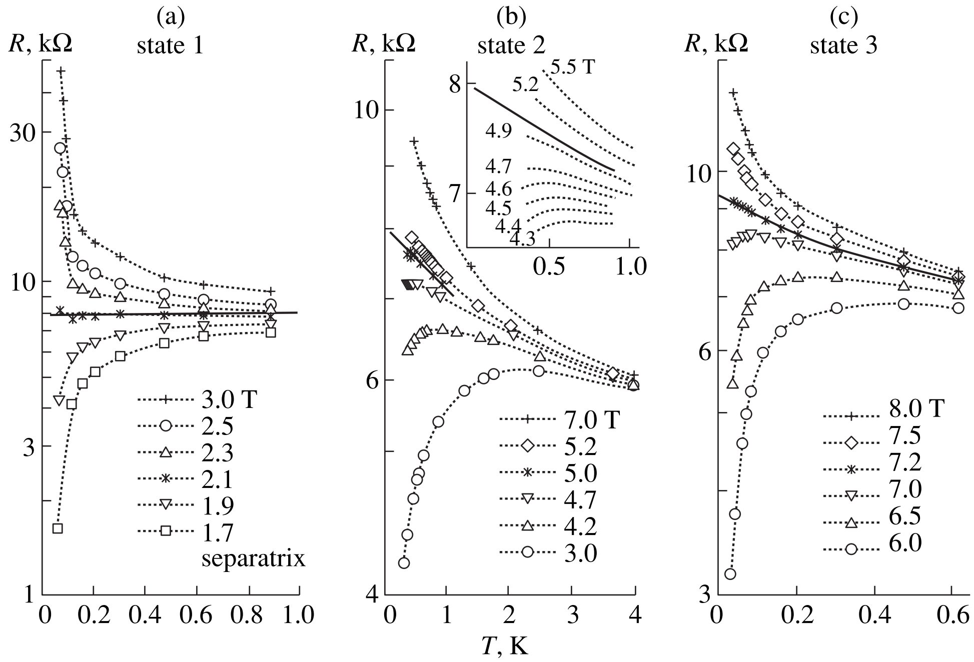

Fig. 1. Temperature-dependent resistances for the states studied at different magnetic fields. The separatrices $R_c(T)$ are shown by solid lines. For state 2, a close-up view of the critical region is displayed in the inset.

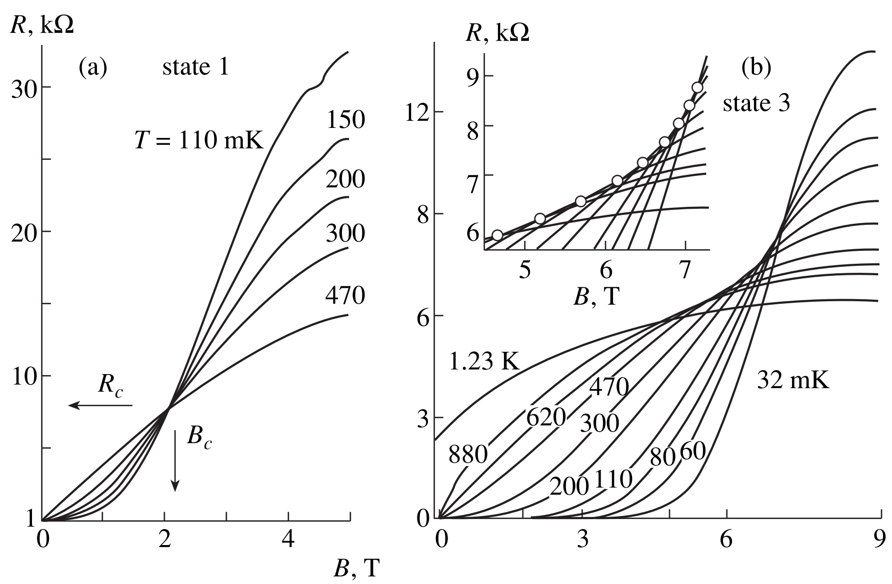

Fig. 2. Isotherms in the (B, R) plane for states (a) 1 and (b) 3. The curve intersection region for state 3 is blown up in the inset. The circles mark the crossing points of the isotherms with neighboring temperatures.

obtained in [7, 8], where the authors claimed observation of the field-induced 2D SIT for the states close to the zero-field SIT. This fact counts in favor of the 2D SIT scenario for the deeper film states of the superconducting phase as well.

Knowing B_c and the scaling exponent, we can replot the experimental data as a function of scaling variable u (Fig. 5). As seen from Figs. 5a and 5b, the data for state 1 collapse onto a single curve, whereas for state 3 we obtain a set of similar curves shifted along the vertical axis. Subtracting the linear temperature term $R_c\alpha T$ (where α is a factor) formally from $R(T, B)$, we reveal the 2D scaling behavior for state 3 (Fig. 5c). Note that the procedure of dividing the experimental data in

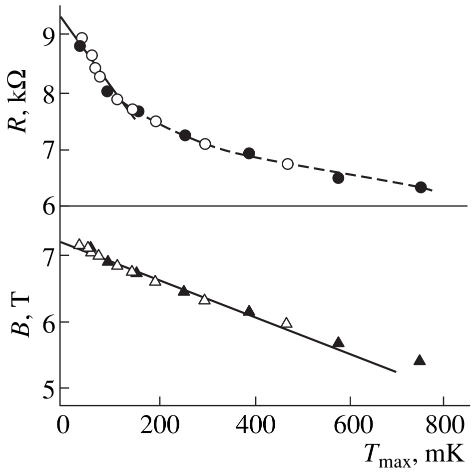

Fig. 3. $R(T_{max})$ and $B(T_{max})$ dependences, as determined from the data in Fig. 1c (open symbols) and Fig. 2b (filled symbols). The values of R_c and B_c are obtained by linear extrapolations (solid lines). The dotted line is a guide to the eye.

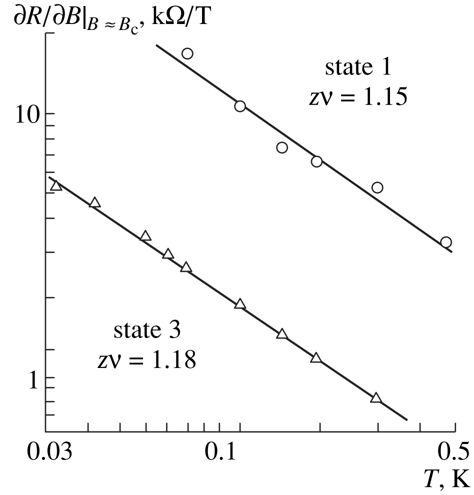

Fig. 4. Behavior of $\partial R/\partial B$ with temperature for states 1 and 3. The values of exponent $z\nu$ are indicated.

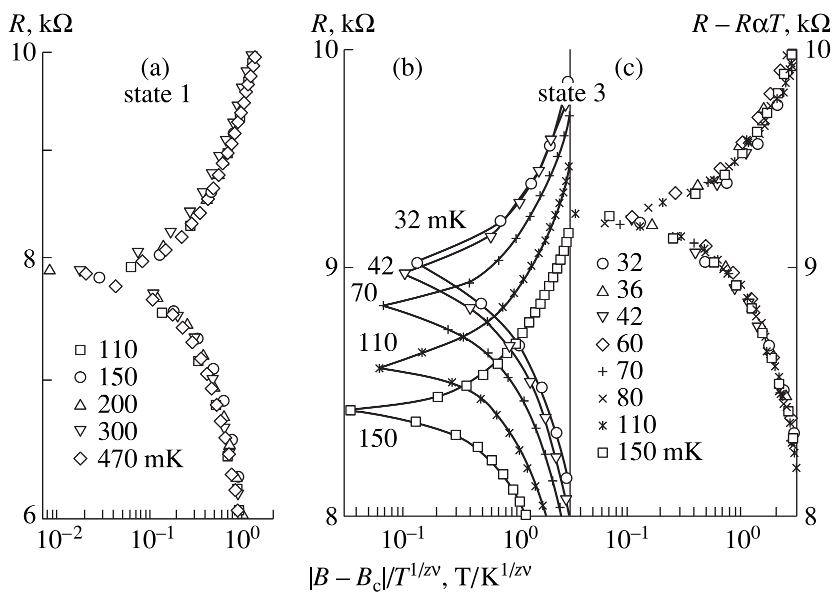

Fig. 5. Scaling plots for (a) state 1 and (b) for state 3 without and (c) with the linear temperature term.

Fig. 5b by $R_c(T)$, which corresponds to formula (2) for the 3D scaling, has not met with success.

Thus, we find that the 2D scaling holds for the states near the zero-field SIT, while the data for the deeper states of the superconducting phase are best described by relation (1) with an additive temperature-dependent correction $f(T)$:

$$R(T, B) \equiv R_c[r(u) + f(T)]. \quad (4)$$

To get a basis for the formal analysis of the experimental data, one should answer two questions: (i) whether our film is really 2D, and (ii) what is the physical origin for the temperature dependence of $R_c(T)$. In the first case, we need to compare film thickness h with the characteristic lengths. These are the coherence length $\xi_{sc} = c\hbar/2eB_{c2}l$ (where l is the mean free path in the normal state) in the superconducting state and the dephasing length $L_\phi(T) \simeq \hbar^2/m\xi_{sc}T$ [1, 2] that restricts the diverging correlation length ξ in the vicinity of quantum SIT. Knowing the film resistance $R \approx 5$ kΩ in the normal state at $T \approx 4$ K and assuming that we deal with an amorphous 3D metal in which the mean free path is normally close to the lowest possible value $l \approx 1/k_F$, we estimate the length $l \approx 8$ Å. If we roughly estimate the field B_{c2} at $B_c = 7.2$ T, as was found for state 3, we get the upper limit for $\xi_{sc} \sim 500$ Å and the value $L_\phi \sim 400$ Å at $T = 0.5$ K. This confirms the 2D scenario of quantum SIT, although in the normal state the film turns out to be 3D.

As to the temperature-dependent $R_c(T)$, the conductivity of the film near B_c at finite temperatures should include a contribution from the localized normal electrons, in addition to the conductivity caused by the diffusion of the fluctuation-induced Cooper pairs [3, 8]. It is the normal electron conductivity that explains the nonuniversality of the critical resistance [8] and the additional term in (4). We write this term in the general form, because the linear extrapolation used is likely to break in the vicinity of $T = 0$.

Thus, all the experimental observations can be reconciled with the 2D scaling scenario. Curiously, the same scaling behavior was established for a parallel magnetic field [18]. Although not in favor of the 2D concept, this fact can also indicate that the restrictions imposed by the theory [1] are too severe.

It is worth mentioning an alternative way of constructing the $f(T)$ term in (4): introduction of the temperature-dependent field $B_c(T)$ defined through the constancy of R_c. Formally, both ways are equivalent and correspond to shifting the isotherms in Fig. 2 either along the R-axis or along the B-axis, so that a common crossing point is attained in the vicinity of the transition. Unlike the normal behavior of the critical fields in superconductors, the $B_c(T)$ thus defined increases with temperature. This can be interpreted in terms of the temperature-induced boson delocalization.

In summary, in the experiments with a-In–O films with different oxygen contents, a change of the field-driven 2D SIT scenario was observed as the film state departed from the zero-field SIT. For the deep film states in the superconducting phase, the temperature-dependent contribution to the film resistance emerges, in addition to the universal function of a scaling variable that describes the conductivity of fluctuation-induced Cooper pairs. This contribution can be attributed to the conductivity of normal electrons.

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