

# Width of the Zero-Field Superconducting Resistive Transition in the Vicinity of the Localization Threshold<sup>1</sup>

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Resistive superconducting zero-field transition in amorphous In-O films in the states in the vicinity of the insulator–superconductor transition is analyzed in terms of two characteristic temperatures: the upper  $T_{c0}$ , where the finite amplitude of the order parameter is established, and the lower  $T_c$ , where the phase ordering takes place. It follows from the magnetoresistance measurements that the resistance in between,  $T_c < T < T_{c0}$ , cannot be ascribed to the dissipation by thermally dissociated vortex pairs. So, it is not a Kosterlitz–Thouless–Berezinskii transition that occurs at  $T_c$ . © 2001 MAIK “Nauka/Interperiodica”.

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The resistive superconducting ( $s$ ) transition in bulk conventional superconductors is very narrow. The reduced width  $t \equiv |T - T_{c0}|/T_{c0}$  of the region with strong fluctuations around transition temperature  $T_{c0}$  is  $t \propto (T_{c0}/\epsilon_F)^4$  in the clean limit and  $t \propto (T_{c0}/\epsilon_F)(k_F l)^{-3}$  in the dirty limit, with the product of the Fermi wave vector and mean free path  $k_F l > 1$ . It is different in 2D, where free magnetic vortices serve as thermal fluctuations. Broad  $s$  transitions in films were explained by existence of the temperature range where current dissipation is due to these fluctuations [1]. Transition starts at temperature  $T_{c0}$ , when Cooper pairs appear in the electronic spectrum. Below  $T_{c0}$ , the resistance remains finite because of free vortices. They appear with probability  $\mu(T)$  while inbinding vortex–antivortex pairs (magnetic loops). When an external magnetic field is zero, the system of thermal fluctuations contains equal numbers  $N_+(T) = N_-(T) = N(T)$  of free vortices of opposite signs. Each vortex lives independently until it annihilates after collision with a vortex of opposite sign. The annihilation probability  $\tau^{-1}(T)$ , together with probability  $\mu(T)$ , determines through dynamic equilibrium the concentration  $N(T)$ :

$$N^2(T) = a(\mu\tau)^{-1}, \quad a = \text{const.} \quad (1)$$

Assuming that there is no pinning, the resistance  $R$  is proportional to the total concentration  $2N$  of the vortices:

$$R = 2\pi\xi_c^2(2N)R_n, \quad (2)$$

with  $\xi_c$  being the effective radius of the vortex core and  $R_n$  being the resistance in the normal state. The finite

resistance exists until the Kosterlitz–Thouless–Berezinskii (KTB) transition [2] inside the vortex system takes place at some temperature  $T_c$ . Below  $T_c$ , practically all vortices are bound into loops and  $N = 0$ . Since loops do not dissipate energy, the resistance vanishes at  $T_c$ .

This scheme with two characteristic temperatures was very carefully checked several times with different materials. In particular, Hebard *et al.* [3], in experiments with amorphous InO<sub>x</sub> films with  $T_{c0} \approx 2.5$  K and  $T_c \approx 1.8$  K, have confirmed the transport characteristics predicted by the theory.

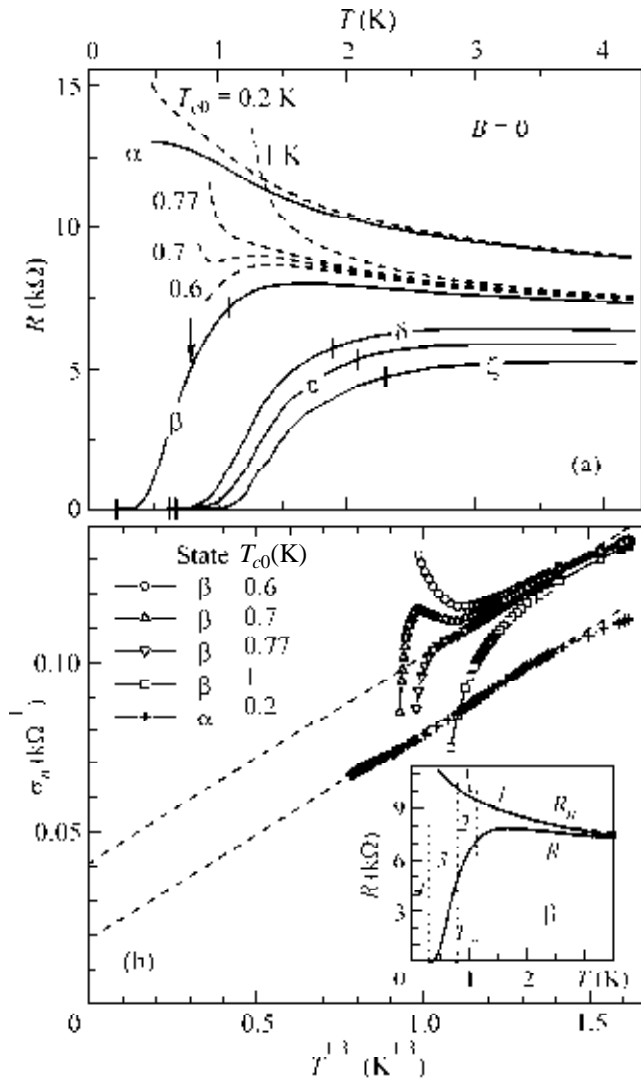
In the framework of the BCS theory applied to 2D, both regions controlled by fluctuations, below and above  $T_{c0}$ , are narrow differing only by a numerical factor [4]

$$(T_{c0} - T_c)/T_{c0} \approx 3(T - T_{c0})/T_{c0} \approx 3Gi \ll 1, \quad (3)$$

with Ginzburg parameter  $Gi$  being usually small,  $Gi \ll 1$ . When the disorder is strong so that the mean free path  $l$  reaches its minimum value of  $k_F^{-1}$ , the  $Gi$  increases and becomes on the order of unity, and the 2D KTB-transition temperature  $T_c$  is suppressed compared to  $T_{c0}$ , so that the region  $T_c < T < T_{c0}$  widens [4]. The vicinity of the superconductor–insulator ( $s$ – $i$ ) transition is just such a region. It is tempting to describe the main part of the broad resistive  $s$  transitions in terms of vortex-induced dissipation in this case too. However, experiments with granular Pb films [5] demonstrated that the scheme was not universal: the width of the zero-magnetic-field  $s$  transition for the states in the vicinity of the  $s$ – $i$  transition was controlled not by thermally activated free vortices.

The width of the fluctuation region  $t$  above  $T_{c0}$  increases along with  $k_F l$  approaching unity: strong disorder makes the fluctuation region wide. This happens not only in 2D [6] but in 3D as well [7, 8], so that specific properties of “short” vortices 2D are not of deci-

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**Fig. 1.** (a) Zero-field resistive *s* transitions for several states of an amorphous In-O film. Bars frame the transition region ( $0.9R_{\max} - 10^{-3}R_{\max}$ ) on the  $R(T)$  curves. Dashed curves—one for state  $\alpha$  and several, with different values of parameter  $T_{c0}$ , for state  $\beta$ —show virtual normal state resistance  $R_n^*(T)$  obtained after subtraction of the paraconductivity term calculated in accordance with Eq. (5). The selected value of  $T_{c0}$  for state  $\beta$  is shown by the arrow. (b) The dashed curves from panel (a) presented as  $\sigma_n^* = (R_n^*)^{-1}$  vs.  $T^{1/3}$  (form appropriate for a 3D non-Drude metal in critical region near the metal-insulator transition). Linear extrapolation cuts off the tail from the region of strong fluctuations and transforms  $\sigma_n^*$  into  $\sigma_n$ . Inset: Functions  $R$ ,  $R_n$ ,  $R_n^*$  (dashed line) for state  $T_c$ . About four temperature regions, see text.

sive importance here. Recent approaches for 3D [9] also distinguish between fluctuations of the amplitude of the order parameter and those which destroy long-range phase coherence. In such interpretation, the mean amplitude becomes finite in the vicinity of  $T_{c0}$  and the long-range phase coherence establishes itself at  $T_c <$

$T_{c0}$ . This problem is yet not well understood. Recently Valles *et al.* [10] concluded from tunneling measurements on ultrathin *s* films near the *s-i* transition that fluctuations in the amplitude of the superconducting order parameter dominated below  $T_{c0}$ .

In [5], the vortex-determined-dissipation scheme was questioned for granular material. Here, we study the same problem for amorphous films, where the disorder is supposed to be on the atomic scale. Our amorphous InO<sub>x</sub> films were 200 Å thick. They were certainly 2D from the viewpoint of vortex electrodynamics since the magnetic penetration depth  $\lambda \approx 1000$  Å. The 2D character of our films becomes not so obvious when the thickness  $d$  is to be compared with some other lengths: various estimates give for the superconducting coherence length  $\xi_0$  a value in the range 100–500 Å, and the magnetic length  $l_B = (\hbar c/eB)^{-1/2}$  is 200 Å at  $B = 1$  T. The films are 3D in the normal state, since the thickness  $d$  is certainly larger than  $l \approx k_F^{-1}$ .

The properties of the film are determined by the oxygen concentration  $x$  [11]. The starting value of  $x$  can be changed to some extent by thermal treatment. This affects the carrier concentration and the position of the state in the *s-i*-phase diagram [11–13]. We remain in the region where the carrier density  $n$ , judging from Hall effect measurements, is in the range  $(2-4) \times 10^{21}$  cm<sup>-3</sup> and the parameter  $k_F l$  is in the range 0.2–0.3 [11]. Hence, in terms introduced by Emery and Kivelson [9], we deal with a “bad” (non-Drude) metal, where the transport phenomena are not described by Boltzmann theory.

In the experiments, resistive *s* transition  $R(T, B)$  is measured. Below, data for several states of one of the films are demonstrated. Results for other films are similar. The aspect ratio of the film is close to one: its resistance  $R$  serves, within 10% accuracy, as resistance per square. The measurements were made for six states of the film, labeled  $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$ , with *s* properties gradually increasing along this row. Figure 1a shows functions  $R(T)$  in zero magnetic field for five of these states. In state  $\alpha$ , the *s* transition, if it exists, starts somewhere below  $T \leq 0.4$  K. For all the others, two conditional temperatures  $T_{c0}$  and  $T_c$  are marked by bars. They may be considered as the onset and the end of the transition. The upper is positioned at the level  $R \approx 0.9R_{\max}$ , where  $R_{\max}$  is the value of the maximum in the  $R(T)$  curve. The lower is at the level

$$R \approx 10^{-3} R_{\max}, \tag{4}$$

which roughly corresponds to the usual position of the KTB transition [3]. The problem is in the factors that control the shape of the *s* transition in between the marks.

Under the standard approach in conventional superconductors,  $T_{c0}$  is determined from experimentally measured  $R(T)$  with the help of the expression for the

paraconductivity  $\sigma_{fl}$  due to superconducting fluctuations [14]. In 2D,

$$\sigma = \sigma_n + \sigma_{fl} = \frac{e^2}{\hbar} \left[ g + \frac{T_{c0}}{16(T - T_{c0})} \right], \quad (5)$$

$$\sigma = R^{-1}, \quad \sigma_n = R_n^{-1}.$$

For films far from the localization threshold, the dimensionless sheet conductance is  $g \gg 1$  and the correction to  $R$  from  $\sigma_{fl}$  soon becomes negligible when  $T$  increases above  $T_{c0}$ . For our films,  $g$  is on the order of unity. Hence, the contribution  $\sigma_{fl}$  really affects the temperature dependence  $R(T)$  above  $T_{c0}$ .

The term  $\sigma_{fl}$  in Eq. (5) contains  $T_{c0}$  as the only parameter which we have to choose. Equation (5) is valid only until the correction is small:  $\sigma_{fl} \ll \sigma_n$ . Hence, even with right value of parameter  $T_{c0}$ , we will get from Eq. (5) function  $R_n(T)$  which falsely tends to infinity near  $T_{c0}$ . To emphasize this, we will mark these functions by an asterisk, as  $R_n^*$ .

Figure 1a presents functions  $R_n^*$  for state  $\beta$ , with different values of  $T_{c0}$  as a parameter in  $\sigma_{fl}$ . The curve  $R_n^*$  obtained with  $T_{c0} = 1$  K has improbably strong temperature dependence below 1.5 K, whereas the curves with  $T_{c0} = 0.6$  and 0.7 K contain maxima. Hence,  $T_{c0}$  should be in between.

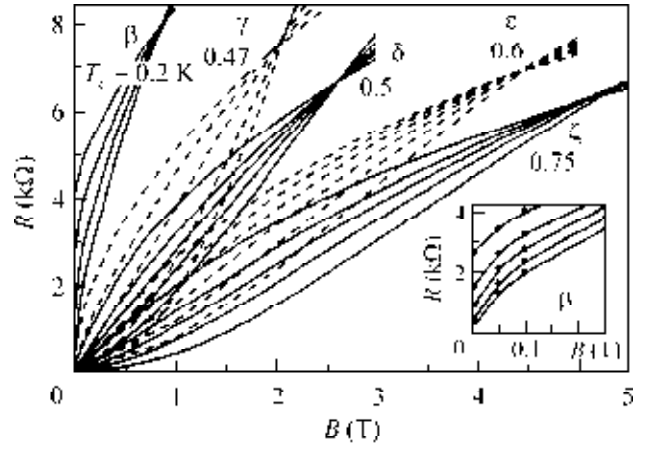
The specific choice of 0.77 K as  $T_{c0}$  is justified by the plot of  $\sigma_n$  vs.  $T^{1/3}$  (Fig. 1b). The representation

$$\sigma_n = u + vT^{1/3} \quad (6)$$

is usually used for “bad” 3D metals to distinguish by extrapolation to  $T = 0$  metals and insulators (see, for instance, [15]). The chosen value of  $T_{c0}$  gives the lowest left-edge value of the temperature interval where data follow Eq. (6). For state  $\beta$  with this  $T_{c0}$ , the extrapolated value of  $u = \sigma_n(0)$  is  $0.15e^2/\hbar$ . Note that, according to definition (5),  $\sigma_n$  is 2D conductivity,  $\sigma_n \equiv \sigma_n^{(2D)}$ , whereas 3D conductivity is  $\sigma_n^{(3D)}(0) = \sigma_n^{(2D)}(0)d = (0.15d)e^2/\hbar$ .

For state  $\alpha$ , the contribution from  $s$  fluctuations is clearly seen in Fig. 1a as a tendency to decline at low temperatures. The procedures from Fig. 1 applied to this state give  $T_{c0} = 0.2$  K: this is the lowest value of parameter  $T_{c0}$  which brings the curve  $R_n^*(T)$  without maximum. According to Fig. 1b, the extrapolated value of  $\sigma_n(0)$  for state  $\alpha$  is twice as small as for state  $\beta$ . One more such step should bring the system to the localization threshold. We know from [11] that this would result in a zero-field  $s$ - $i$  transition.

Returning to state  $\beta$ , the kink on the curve  $\sigma_n(T^{1/3})$  in Fig. 1b reveals the point where fluctuations become so strong that Eq. (5) fails. As is shown in the inset, the



**Fig. 2.** Magnetoresistance isotherms for a sequence of temperature values for several states of the In-O film. The temperatures in each bunch are downward from 0.85 K with step 0.1 K. About the labeled values of  $T_c$ , see text. Inset: denser set of isotherms for state  $\beta$ , down from 0.58 K with step 0.05 K.

temperature axes breaks out into four regions. In the right one, paraconductivity exists. In region 2, strong superconducting fluctuations prevail. At the opposite end, region 4 is superconducting. Our next task is to study region 3 and to check whether it is the vortex dissipation that controls the resistance in this region, i.e., in the lower part of the transition.

Let us turn to isotherms  $R(B)$  in Fig. 2. All the states studied are situated on the  $s$  side of the phase diagram [12] of the  $s$ - $i$  transition. Certain critical field values of  $B_c$  induce  $s$ - $i$  transition in these states and bring the sample in the intermediate position between the superconductor and the insulator [16]. The resistance at this field,  $R(T, B_c) = R_c$ , should not depend on temperature at all [16] or may have only a weak temperature dependence [13]. Hence, the isotherms  $R(B, T = \text{const})$  cross in the vicinity of  $B_c$ . At low fields, all the isotherms from the vicinity of  $T_c$  approach the origin, possibly ending at one of the axes near the origin. Hence, the bunches in Fig. 2 have specific shape of lenses.

Inside each lens, one can more or less confidently select some mean isotherm which separates those with different signs of the second derivative  $\partial^2 R / \partial B^2$  inside the interval  $0 - B_c$ . Corresponding temperatures of these separating isotherms are written near the bunches. For the left bunches  $\gamma$  and  $\delta$ , the separating isotherms turn out to be straight lines in the aforementioned interval with the slope  $\partial R / \partial B \approx R_c / B_c$ . For the states  $\epsilon$  and  $\zeta$  situated deeper in the  $s$  region, the lenses are slightly deformed and the separating isotherms remain straight only below 2–3 T.

The temperatures of the separating isotherms practically coincide with the values of  $T_c$  determined by criterion (4). To some extent, this justifies the choice of the coefficient in criterion (4). On the other hand, we get a

more convenient tool to determine the temperature where the  $s$  transition becomes complete: by finding the isotherm  $R_{T_c}(B)$  which is a linear function going through the origin.

The isotherms with  $T > T_c$  cross the ordinate at finite  $R(0) \neq 0$ . It is clear from the specific shape of the low-field part of the lenses that they have nonzero slope at  $B = 0$  (detailed demonstration can be found in the inset in Fig. 2):

$$(\partial R / \partial B)_0 > 0. \quad (7)$$

This linear increase in  $R$  exists only with the field perpendicular to the film. The field directed along the film, which does not bring the vortices from outside into the film, results in zero field derivative  $(\partial R / \partial B)_0 = 0$ , i.e., does not affect dissipation in the linear approximation. This can be seen from the previously published data on In-O films (Fig. 2 in [17]), which compare isotherms for two almost similar states of the film but with different directions of the applied magnetic field.

In the model of thermally excited free vortices, the normal applied field increases the density of vortices of the corresponding sign:  $\Delta N_+ \propto B$ . But this leads, due to the recombination processes, to the lessening of the density of vortices of opposite sign. When

$$\Delta R \equiv (R(B) - R(0)) \ll R(0), \text{ i.e., } \Delta N_+ \ll N, \quad (8)$$

it follows from dynamic equilibrium equation (1) that  $(N + \Delta N_+)(N + \Delta N_-) = N^2$ , i.e., that in the linear approximation the density changes cancel each other:  $\Delta N_- = -\Delta N_+$ . This qualitative inference illustrates the result calculated by Minnhagen [18] long ago: the free vortex density did not change until relation (8) was valid.

Hence, in the framework of the free vortex model, one should expect the resistance change under condition (8) to be  $\Delta R = O(B^2)$ . Experimental observation that  $\Delta R \propto B$  when  $\Delta R \ll R(0)$  means that the zero-field resistance  $R$  at this temperature is determined not by vortices from thermally dissociated pairs.

Summarizing, we have described the upper part of the resistive  $s$  transition of a “bad” (non-Drude) metal, amorphous In-O film, by the usual expression [14] for 2D paraconductivity but failed to describe the lower part in terms of an  $s$  material with thermally excited vortices. Being in line with the idea of two characteristic temperatures, our analysis does not confirm existence of the KTB transition in the vicinity of the lower one. According to Emery and Kivelson [9], separation of onset temperature  $T_{c0}$ , where the amplitude of the order parameter is established, and of the phase ordering temperature  $T_c < T_{c0}$  happens also in 3D non-Drude

metals. Here, the phase-order-breaking thermal fluctuations are certainly not vortices. Our experiment seems to be closer to this model.

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