Topological Oscillations of the Magnetoconductance in Disordered GaAs Layers

S. S. Murzin,[1,4] A. G. M. Jansen,[2,4] and I. Claus[3,4]

[1]Institute of Solid State Physics RAS, 142432 Chernogolovka, Moscow District, Russia
[2]Service de Physique Statistique, Magnétisme, et Supraconductivité, Département de Recherche Fondamentale sur la Matière Condensée, CEA-Grenoble, 38054 Grenoble CEDEX 9, France
[3]Center for Nonlinear Phenomena and Complex Systems, Faculté des Sciences, Université Libre de Bruxelles, Campus Plaine, Code Postal 231, B-1050 Brussels, Belgium
[4]Grenoble High Magnetic Field Laboratory, Max-Planck-Institut für Festkörperforschung and Centre National de la Recherche Scientifique, BP 166, F-38042, Grenoble CEDEX 9, France

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Oscillatory variations of the diagonal (G_{xx}) and Hall (G_{xy}) magnetoconductances are discussed in view of topological scaling effects giving rise to the quantum Hall effect. They occur in a field range without oscillations of the density of states due to Landau quantization, and are, therefore, totally different from the Shubnikov–de Haas oscillations. Such oscillations are experimentally observed in disordered GaAs layers in the extreme quantum limit of applied magnetic field with a good description by the unified scaling theory of the integer and fractional quantum Hall effect.

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The integer quantum Hall effect (QHE) is usually observed at high magnetic fields, $\omega_c\tau \gg 1$ ($\omega_c = eB/m$ is the cyclotron frequency, τ is the transport relaxation time), and its appearance develops from the Shubnikov–de Haas oscillations based on the Landau quantization of the two-dimensional (2D) electron system. However, the scaling treatment of the integer QHE [1] predicts the existence of the QHE without the Landau quantization of the electron spectrum. It could exist even at low magnetic field $\omega_c\tau \ll 1$ [2] in the absence of magnetoquantum oscillations of the density of states. The QHE at low magnetic fields $\omega_c\tau \ll 1$ has not been observed thus far, probably because extremely low temperatures are required [3]. In addition, the QHE could exist in a layer whose thickness d is much larger than the electron transport mean-free path l, i.e., $d \gg l$, in the extreme quantum limit (EQL) of applied magnetic field, where only the lowest Landau level is occupied. Such a layer has a three-dimensional (3D) "bare" (nonrenormalized) electron spectrum without oscillations of the density of states in the EQL. In this situation, the QHE has been observed in heavily Si-doped n-type GaAs layers [4,5].

Here, we address the problem of the arising of the QHE in the absence of magnetoquantum oscillations of the density of states. In this case, the variation with temperature of the diagonal conductance per square (G_{xx}) and Hall conductance (G_{xy}) is due to diffusive interference effects (below G_{xx} and G_{xy} are taken in units e^2/h), which in a scaling approach can be described by the renormalization-group equations. For comparison, according to the conventional theory, the temperature dependence of the Shubnikov–de Haas oscillations preceding the QHE is due to thermal broadening of the Fermi distribution [6]. At the moment, two theories give explicit expressions for the renormalization-group equations. The first theory has been derived for both integer and fractional QHE and for any value of G_{xx} [7]. It is based on the assumption that a certain symmetry group unifies the structure of the integer and fractional quantum Hall states [7–9]. This so-called unified scaling (US) theory describes well the shape of the scaling flow diagram depicting the coupled evolution of G_{xx} and G_{xy} for decreasing temperatures in heavily Si-doped n-type GaAs layers with different thicknesses for a wide range of G_{xx} values [10]. The second theory has been developed in the "dilute instanton gas" approximation (DIGA), first for noninteracting [11] and then for interacting electrons [12]. Both theories are developed for a totally spin-polarized electron system. For $2\pi G_{xx} \gg 1$, they predict an oscillating topological term in the scaling β function with the same periodicity. However, they differ in predictions on the oscillation amplitude. The oscillating topological term in the β function should lead to oscillations in the magnetic-field dependence of G_{xx} and G_{xy} which are not related to oscillations in the density of states such as, e.g., for the case of the Shubnikov–de Haas oscillations.

In the presented work, we derive explicit expressions for the topological oscillations of the Hall conductivity G_{xy} for both theories, and compare them with experiment for thick ($d \gg l$) disordered heavily Si-doped GaAs layers with rather large G_{xx} and G_{xy} compared to unity. The layers studied before in Refs. [4,5] have a 3D bare electron spectrum. However, below 4 K the characteristic diffusion lengths, $L_\varphi = (D_{zz}\tau_\varphi)^{1/2}$ and $L_T = (D_{zz}\hbar/k_BT)^{1/2}$, for coherent diffusive transport increase to values larger than d, and the system becomes 2D for coherent diffusive phenomena (D_{zz} is the diffusion coefficient of electrons along the magnetic field, τ_φ is the phase breaking time).

The US theory describes the renormalization-group flow of the conductances by the equation [7]

$$s - s_0 = -\ln(f/f_0), \quad (1)$$

for a real parameter s monotonically depending on temperature, where $G \equiv G_{xy} + iG_{xx}$, $f_0 = f(s_0)$, and

$$f = -\frac{(\sum_{n=-\infty}^{\infty} q^{n^2})^4 (\sum_{n=-\infty}^{\infty} (-1)^n q^{n^2})^4}{(2\sum_{n=0}^{\infty} q^{(n+1/2)^2})^8}, \quad (2)$$

with $q = \exp(i\pi G)$. For $|q|^2 = \exp(-2\pi G_{xx}) \ll 1$, the function $f = -1/(256q^2) + 3/32 + O(q^2)$ and Eq. (1) is reduced to

$$s - s_0 \approx i2\pi(G - G^0) + 24(e^{i2\pi G} - e^{i2\pi G^0}). \quad (3)$$

In the first-order approximation, by ignoring the last oscillating term in Eq. (3), this equation has the solution

$$G_{xx}^1 = G_{xx}^0 - (s - s_0)/2\pi, \qquad G_{xy}^1 = G_{xy}^0. \quad (4)$$

In the second-order approximation, the solution looks like

$$G_{xx} = G_{xx}^1 + \frac{12}{\pi}[e^{-2\pi G_{xx}^1} - e^{-2\pi G_{xx}^0}]\cos(2\pi G_{xy}^0), \quad (5)$$

$$G_{xy} = G_{xy}^0 - \frac{12}{\pi}[e^{-2\pi G_{xx}^1} - e^{-2\pi G_{xx}^0}]\sin(2\pi G_{xy}^0). \quad (6)$$

This is a solution of Eq. (3) for fixed s. However, for our experiment we are interested in the solution for fixed temperature T. In the first-order approximation, it should coincide with the result of the first-order perturbation theory for the electron-electron interaction in coherent diffusive transport leading to logarithmic temperature-dependent corrections in the diagonal conductance,

$$G_{xx}^T = G_{xx}^0 + \lambda/2\pi \ln(T/T_0), \quad (7)$$

without any temperature dependence in the Hall conductance [13]. Therefore, $s = -\lambda \ln(T)$ in this approximation. For a totally spin-polarized electron system $\lambda = 1$ [14].

In second order, s will oscillate as a function of G_{xy}^0 at fixed temperature T and will give an additional oscillating term in Eq. (5), but the relation between s and T is unknown and the amplitude of the G_{xx} oscillations cannot be found. In this respect, we note that the last term in Eq. (5) shows maxima at integer G_{xy}^0 as opposed to the expected minima for the integer QHE. The difference between G_{xx}^1 and G_{xx}^T can be ignored in the exponents of Eq. (6). Therefore the Hall conductivity G_{xy} oscillates as a function of the bare Hall conductance G_{xy}^0 and, hence, as a function of the magnetic field B, with amplitude

$$A_{xy}^{\mathrm{US}} = \frac{12}{\pi}[e^{-2\pi G_{xx}^T} - e^{-2\pi G_{xx}^0}] = \frac{12}{\pi} e^{-2\pi G_{xx}^0}[(T_0/T)^\lambda - 1], \quad (8)$$

as found by substituting G_{xx}^T [Eq. (7)] for G_{xx}^1 in Eq. (6). This dependence is totally different from the exponential variation with temperature of the Shubnikov–de Haas oscillations.

In the dilute instanton gas approximation for the case of interacting electrons [12],

$$\frac{dG_{xx}}{d\ln L} = -\frac{\lambda}{\pi} - D_1 G_{xx}^2 e^{-2\pi G_{xx}} \cos(2\pi G_{xy}), \quad (9)$$

$$\frac{dG_{xy}}{d\ln L} = -D_1 G_{xx}^2 e^{-2\pi G_{xx}} \sin(2\pi G_{xy}). \quad (10)$$

Here $L \approx (\hbar D_{xx}/k_B T)^{1/2}$ and $D_1 = 64\pi/e \approx 74.0$. Solving the quotient of these equations by ignoring terms of order $\exp(-4\pi G_{xx})$, one obtains

$$G_{xy} = G_{xy}^0 - \frac{\pi D_1}{\lambda}[F(G_{xx}^T) - F(G_{xx}^0)]\sin(2\pi G_{xy}^0), \quad (11)$$

where $F(x) = 1/4\pi^3 (2\pi^2 x^2 + 2\pi x + 1)\exp(-2\pi x)$.

Both theories have been developed for a totally spin-polarized electron system. However, in a real system electrons can have two different spin projections. For the case of noninteracting electrons, the electrons can be described in terms of two independent, totally spin-polarized systems in the absence of spin-flip scattering. This approach remains valid for interacting electrons as well, if the triplet part of the constant of interaction is much smaller than the singlet one [13,14], because only the interaction between electrons with the same spin leads to a renormalization of the conductance in this case. For the small spin splitting in strongly disordered GaAs, the conductances of the electron systems with different spin projection ($G_{ij}^{\uparrow}$ and $G_{ij}^{\downarrow}$) are approximately equal to half the measured conductance, i.e., $G_{ij}^{\uparrow} \approx G_{ij}^{\downarrow} \approx G_{ij}/2$. It allows us to compare quantitatively the experimental results with the theories. For large spin splitting, this is impossible, because $G_{ij}^{\uparrow}$ and $G_{ij}^{\downarrow}$ are different, and only the sum $G_{ij}^{\uparrow} + G_{ij}^{\downarrow}$ can be measured.

The investigated heavily Si-doped n-type GaAs layers sandwiched between undoped GaAs were prepared by molecular-beam epitaxy. The nominal thickness d equals 100 nm for the layers 2, 3, 6, and 140 nm for layer 7. The Si-donor bulk concentration n equals 1.8, 2.5, 1.6, and 1.6×10^{17} cm^{-3} for samples 2, 3, 6, and 7 as derived from the period of the Shubnikov–de Haas oscillations at $B <$ 5 T. The mobilities of the samples at $T = 4.2$ K are 2400, 2500, 2600, and 2600 cm^2/V s, and the electron densities per square N_s as derived from the slope of the Hall resistance R_{xy} in weak magnetic fields (0.5–3 T) at $T =$ 4.2 K are 1.26, 2, 2.08, and 2.86×10^{12} cm^{-2} for samples 2, 3, 6, and 7, respectively. For all samples, the electron transport mean-free path l is around 30 nm at zero magnetic field. The detailed structure of the samples is described in Ref. [4].

In Fig. 1, the magnetotransport data of the diagonal (R_{xx}, per square) and Hall (R_{xy}) resistance (both given in

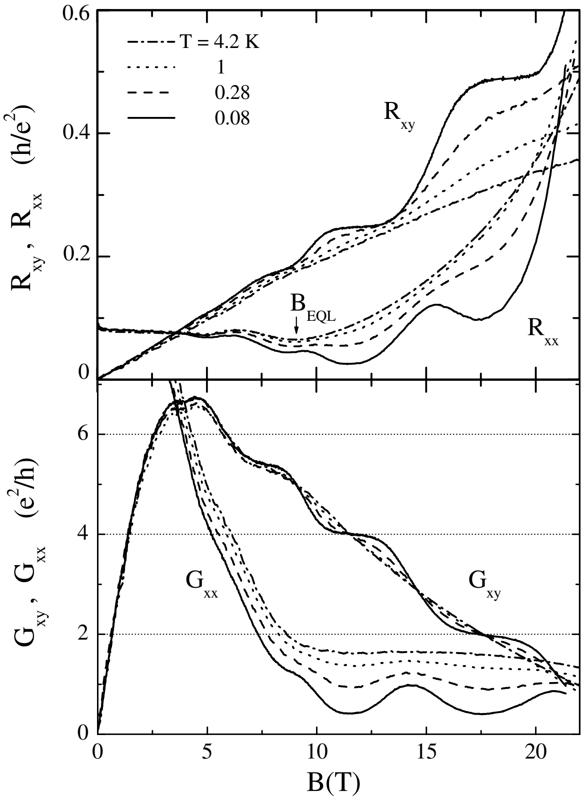

FIG. 1. Magnetic field dependence of the diagonal (R_{xx}, per square) and Hall (R_{xy}) resistance and of the diagonal (G_{xx}) and Hall (G_{xy}) conductance for sample 2 in a magnetic field perpendicular to the heavily doped GaAs layer (thickness 100 nm) at different temperatures. The arrow indicates the field $B_{\rm EQL}$ of the extreme quantum limit.

units of h/e^2), and of the diagonal (G_{xx}) and Hall (G_{xy}) conductance, are plotted for sample 2. At 4.2 K, the magnetoresistance shows the typical behavior of bulk material with weak Shubnikov–de Haas oscillations for increasing field B and a strong monotonous upturn in the extreme quantum limit (EQL) where only the lowest Landau level is occupied. At lower temperatures, R_{xy}, R_{xx}, G_{xy}, and G_{xx} start to oscillate. Minima of G_{xx} and of $|\partial G_{xy}/\partial B|$ arise at magnetic fields where G_{xy} at 4 K attains even-integer values, in accordance with both theories mentioned above. These oscillatory structures develop into the QHE at the lowest temperatures where R_{xy} and G_{xy} reveal remarkable steps near the values $R_{xy} = 1/2$ and $1/4$, and $G_{xx} = 2$, and 4. In the corresponding fields pronounced minima are observed in R_{xx} and G_{xx}. Note that, contrary to the QHE structures, the amplitude of the weak Shubnikov–de Haas oscillations below the EQL does not depend on temperature because the thermal damping factor $2\pi^2 k_B T/[\hbar\omega_c \sinh(2\pi^2 k_B T/\hbar\omega_c)] = 0.994$ is close to 1 for $B = 5$ T at $T = 1$ K. Similar but less pronounced structures are observed for the other samples investigated. Moreover, for samples 3 and 7, additional minima of G_{xx} and of $|\partial G_{xy}/\partial B|$ are observed, at fields where $G_{xy} = 6$ at $T = 4$ K.

The size quantization could result in oscillatory structures in the magnetotransport data in the EQL in a pure layer with ballistic motion across the layer when $l/d \gg 1$. In our case, however, $l/d \approx 0.2 \div 0.3$ in zero magnetic field, where the ratio even decreases in the EQL for the mean-free path along the field. The 3D character of the bare electron spectrum of the samples has been confirmed in experiments in a tilted magnetic field [5]. Note that the absence of oscillations at $T = 4.2$ K cannot be explained by temperature broadening of the oscillatory structures, because disorder broadening dominates largely with $\hbar/\tau \gg k_B T$(for our samples $\hbar/\tau k_B > 80$ K).

In Fig. 2, we plot the residual variation $\Delta G_{xx}(T) = G_{xx}(T) - G_{xx}^0$ as a function of G_{xy}^0 for sample 6 at different temperatures, $\Delta G_{xy} = G_{xy}(T) - G_{xy}^0$ at $T = 0.46$ K for sample 6, and ΔG_{xx} at $T = 0.1$ K for the thickest sample 7. Here G_{ij}^0 is the conductance at $T = 4.2$ K taken as the bare conductance (see below). Both ΔG_{xx} and ΔG_{xy} oscillate with comparable amplitudes under the same conditions of applied field and temperature. The minima of ΔG_{xx} are at even-integer values of G_{xy}^0 (slightly shifted in the case of a superimposed smooth variation of ΔG_{xx}) and the minima of ΔG_{xy} are shifted on $+0.5$ unit in the G_{xy}^0 scale, in accordance with theory [1,2,7,11,12].

The smoothly varying part of G_{xx}, by ignoring the oscillatory part, decreases for decreasing temperature while that of G_{xy} does not change. The temperature

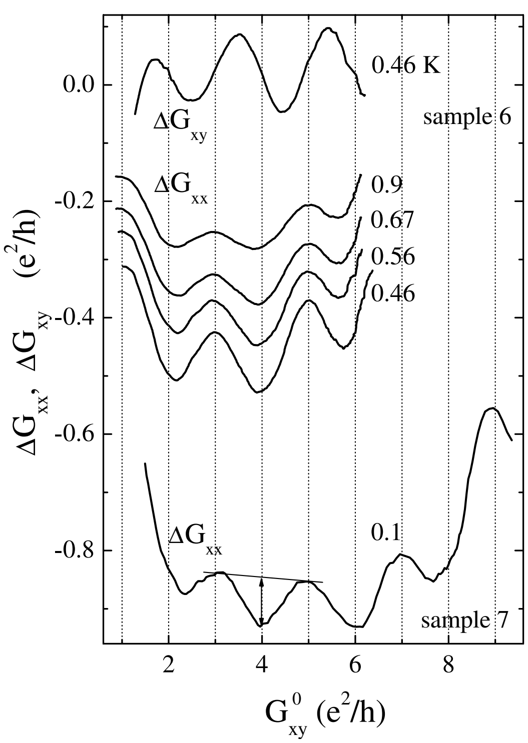

FIG. 2. Residual variation for the diagonal ΔG_{xx} and for Hall conductance ΔG_{xy} after subtraction of the 4.2-K values at different temperatures, for samples 6 and 7. Numbers near curves indicate temperatures in K.

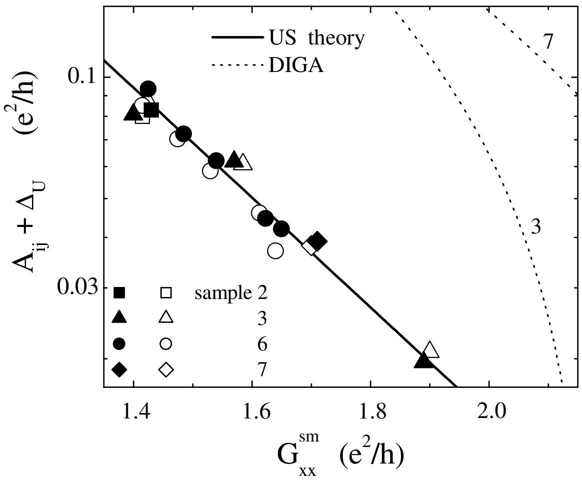

FIG. 3. Amplitude A_{ij} of the topological oscillation of the Hall (solid symbols) and diagonal (open symbols) conductance plus $\Delta_U \equiv 24/\pi \exp(-\pi G_{xx}^0)$ as a function of the smooth part $G_{xx}^{\rm sm}$ of the diagonal conductance for four samples. The full line shows the dependence $24/\pi \exp(-\pi G_{xx}^{\rm sm})$ following from the unified scaling theory. The dotted lines show the result of the "dilute instanton gas" approximation theory for samples 3 and 7.

dependence of the smooth part $G_{xx}^{\rm sm}$ of the diagonal conductance, taken as the midpoint value of the arrow in Fig. 2, is well described by the first-order electron-electron-interaction correction [Eq. (7)] with $\lambda = 1.9$ for samples 2, 3, and 6, and $\lambda = 2$ for sample 7 in the temperature range from 0.15 to 1 K followed by a saturation around 4.2 K. These values are close to the theoretical upper limit $\lambda = 2$ for a system with two spins [13,14], corresponding to a negligibly small triplet part of the electron-electron interaction. The choice of the 4.2 K value for the bare conductance G_{xx}^0 agrees with the saturation of $G_{xx}^{\rm sm}$ around $T = 4.2$ K.

The amplitudes A_{ij} of the oscillations of G_{xx} and G_{xy} conductances are very similar as shown in Fig. 3, where the sum $A_{ij} + \Delta_U$ is plotted as a function of the smooth part of the diagonal conductance $G_{xx}^{\rm sm}$ for all our samples with $\Delta_U = 24/\pi \exp(-\pi G_{xx}^0)$. The values of $\Delta_U =$ 0.044, 0.009, 0.02, and 0.002 for samples 2, 3, 6, and 7, respectively, are smaller than the corresponding values of A_{ij}. The experimental data are rather well described by the result of the US theory for A_{xy} [Eq. (8)] applied to the total conductance of two independent electron systems of opposite spin. Although showing a very similar dependence, A_{xx} cannot be deduced in frame of this theory. The DIGA theory predicts much larger amplitudes than experimentally observed, as shown by the dotted lines in Fig. 3 for $A_{xy}^{\rm DIGA} + \Delta_U$ according DIGA theory for samples 3 and 7.

In summary, due to topological scaling effects, oscillations of the diagonal and Hall magnetoconductances can exist when there are no oscillations of the density of states due to Landau quantization. The oscillations observed in the extreme quantum limit of the applied magnetic field in disordered GaAs layers, with thickness larger than the electron transport mean-free path, fall into this category. The oscillations of G_{xy} are quantitatively well described by the unified scaling theory for the integer and fractional quantum Hall effect [7]. Their amplitude is much smaller than the dilute instanton gas approximation [12] predicts.

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