Topological Oscillations of the Magnetococonductance in Disordered GaAs Layers

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Oscillatory variations of the diagonal ($G_{xx}$) and Hall ($G_{xy}$) magnetococonductances are discussed in view of topological scaling effects giving rise to the quantum Hall effect. They occur in a field range without oscillations of the density of states due to Landau quantization, and are, therefore, totally different from the Shubnikov–de Haas oscillations. Such oscillations are experimentally observed in disordered GaAs layers in the extreme quantum limit of applied magnetic field with a good description by the unified scaling theory of the integer and fractional quantum Hall effect.

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The integer quantum Hall effect (QHE) is usually observed at high magnetic fields, $\omega_c \tau \gg 1$ ($\omega_c = eB/m$ is the cyclotron frequency, $\tau$ is the transport relaxation time), and its appearance develops from the Shubnikov–de Haas oscillations based on the Landau quantization of the two-dimensional (2D) electron system. However, the scaling treatment of the integer QHE [1] predicts the existence of the QHE without the Landau quantization of the electron spectrum. It could exist even at low magnetic field $\omega_c \tau \ll 1$ [2] in the absence of magneto-quantum oscillations of the density of states. The QHE at low magnetic fields $\omega_c \tau \ll 1$ has not been observed thus far, probably because extremely low temperatures are required [3]. In addition, the QHE could exist in a layer whose thickness $d$ is much larger than the electron transport mean-free path $l$, i.e., $d \gg l$, in the extreme quantum limit (EQL) of applied magnetic field, where only the lowest Landau level is occupied. Such a layer has a three-dimensional (3D) “bare” (nonrenormalized) electron spectrum without oscillations of the density of states in the EQL. In this situation, the QHE has been observed in heavily Si-doped $n$-type GaAs layers [4,5].

Here, we address the problem of the arising of the QHE in the absence of magnetoquantum oscillations of the density of states. In this case, the variation with temperature of the diagonal conductance per square ($G_{xx}$) and Hall conductance ($G_{xy}$) is due to diffusive interference effects (below $G_{xx}$ and $G_{xy}$ are taken in units $e^2/h$), which in a scaling approach can be described by the renormalization-group equations. For comparison, according to the conventional theory, the temperature dependence of the Shubnikov–de Haas oscillations preceding the QHE is due to thermal broadening of the Fermi distribution [6]. At the moment, two theories give explicit expressions for the renormalization-group equations. The first theory has been derived for both integer and fractional QHE and for any value of $G_{xx}$ [7]. It is based on the assumption that a certain symmetry group unifies the structure of the integer and fractional quantum Hall states [7–9]. This so-called unified scaling (US) theory describes well the shape of the scaling flow diagram depicting the coupled evolution of $G_{xx}$ and $G_{xy}$ for decreasing temperatures in heavily Si-doped $n$-type GaAs layers with different thicknesses for a wide range of $G_{xy}$ values [10]. The second theory has been developed in the “dilute instanton gas” approximation (DIGA), first for noninteracting [11] and then for interacting electrons [12]. Both theories are developed for a totally spin-polarized electron system. For $2\pi\alpha G_{xx} \gg 1$, they predict an oscillating topological term in the scaling $\beta$ function with the same periodicity. However, they differ in predictions on the oscillation amplitude. The oscillating topological term in the $\beta$ function should lead to oscillations in the magnetic-field dependence of $G_{xx}$ and $G_{xy}$, which are not related to oscillations in the density of states such as, e.g., for the case of the Shubnikov–de Haas oscillations.

In the presented work, we derive explicit expressions for the topological oscillations of the Hall conductivity $G_{xy}$ for both theories, and compare them with experiment for thick ($d \gg l$) disordered heavily Si-doped GaAs layers with rather large $G_{xx}$ and $G_{xy}$ compared to unity. The layers studied before in Refs. [4,5] have a 3D bare electron spectrum. However, below 4 K the characteristic diffusion lengths, $L_\varphi = (D_\varphi \tau_\varphi)^{1/2}$ and $L_\tau = (D_\tau \tau)^{1/2}$ for coherent diffusive transport increase to values larger than $d$, and the system becomes 2D for coherent diffusive phenomena ($D_\varphi$ is the diffusion coefficient of electrons along the magnetic field, $\tau_\varphi$ is the phase breaking time).
The US theory describes the renormalization-group flow of the conductances by the equation [7]

\[ s - s_0 = -\ln(f/f_0), \]  

as found by substituting \( G_{xx}^T \) [Eq. (7)] for \( G_{xx}^1 \) in Eq. (6). This dependence is totally different from the exponential variation with temperature of the Shubnikov–de Haas oscillations.

In the dilute instanton gas approximation for the case of interacting electrons [12],

\[ \frac{dG_{xx}}{d\ln L} = -\frac{\lambda}{\pi} - D_1 G_{xx}^1 e^{-2\pi G_{xx}^1} \cos(2\pi G_{xy}), \]  

\[ \frac{dG_{xy}}{d\ln L} = -D_1 G_{xx}^1 e^{-2\pi G_{xy}} \sin(2\pi G_{xy}). \]

Here \( L = (\hbar D_{xx}/k BT)^{1/2} \) and \( D_1 = 64\pi/e = 74.0. \) Solving the quotient of these equations by ignoring terms of order \( \exp(-4\pi G_{xx}) \), one obtains

\[ G_{xy} = G_{xy}^0 - \frac{\pi D_1}{\lambda} [F(G_{xx}^T) - F(G_{xy}^T)] \sin(2\pi G_{xy}). \]

Both theories have been developed for a totally spin-polarized electron system. However, in a real system electrons can have two different spin projections. For the case of noninteracting electrons, the electrons can be described in terms of two independent, totally spin-polarized systems in the absence of spin-flip scattering. This approach remains valid for interacting electrons as well, if the triplet part of the constant of interaction is much smaller than the singlet one [13,14], because only the interaction between electrons with the same spin leads to a renormalization of the conductance in this case. For the small spin splitting in strongly disordered GaAs, the conductances of the electron systems with different spin projection (\( G_{ij}^1 \) and \( G_{ij}^0 \)) are approximately equal to half the measured conductance, i.e., \( G_{ij}^1 = G_{ij}^0 = G_{ij}/2 \). It allows us to compare quantitatively the experimental results with the theories. For large spin splitting, this is impossible, because \( G_{ij}^1 \) and \( G_{ij}^0 \) are different, and only the sum \( G_{ij}^1 + G_{ij}^0 \) can be measured.

The investigated heavily Si-doped \( n \)-type GaAs layers sandwiched between undoped GaAs were prepared by molecular-beam epitaxy. The nominal thickness \( d \) equals 100 nm for the layers 2, 3, 6, and 140 nm for layer 7. The Si-donor bulk concentration \( n \) equals 1.8, 2.5, 1.6, and \( 1.6 \times 10^{17} \text{ cm}^{-3} \) for samples 2, 3, 6, and 7 as derived from the period of the Shubnikov–de Haas oscillations at \( B < 5 \text{ T} \). The mobilities of the samples at \( T = 4.2 \text{ K} \) are 2400, 2500, 2600, and 2600 cm²/V s, and the electron densities per square \( N_s \) as derived from the slope of the Hall resistance \( R_{xy} \) in weak magnetic fields (0.5–3 T) at \( T = 4.2 \text{ K} \) are 1.26, 2, 2.08, and \( 2.86 \times 10^{12} \text{ cm}^{-2} \) for samples 2, 3, 6, and 7, respectively. For all samples, the electron transport mean-free path \( l \) is around 30 nm at zero magnetic field. The detailed structure of the samples is described in Ref. [4].

In Fig. 1, the magnetotransport data of the diagonal (\( R_{xx} \), per square) and Hall (\( R_{xy} \)) resistance (both given in
The size quantization could result in oscillatory structures in the magnetotransport data in the EQL in a pure layer with ballistic motion across the layer when $l/d \gg 1$. In our case, however, $l/d = 0.2 \div 0.3$ in zero magnetic field, where the ratio even decreases in the EQL for the mean-free path along the field. The 3D character of the bare electron spectrum of the samples has been confirmed in experiments in a tilted magnetic field [5]. Note that the absence of oscillations at $T = 4.2$ K cannot be explained by temperature broadening of the oscillatory structures, because disorder broadening dominates largely with $h/\tau \gg k_B T$ (for our samples $h/\tau k_B > 80$ K).

In Fig. 2, we plot the residual variation $\Delta G_{xx}(T) = G_{xx}(T) - G_{xx}^0$ as a function of $G_{xx}^0$, for sample 6 at different temperatures, $\Delta G_{xy} = G_{xy}(T) - G_{xy}^0$ at $T = 0.46$ K for sample 6, and $\Delta G_{xx}$ at $T = 0.1$ K for the thickest sample 7. Here $G_{xx}^0$ is the conductance at $T = 4.2$ K taken as the bare conductance (see below). Both $\Delta G_{xx}$ and $\Delta G_{xy}$ oscillate with comparable amplitudes under the same conditions of applied field and temperature. The minima of $\Delta G_{xx}$ are at even-integer values of $G_{xx}^0$ (slightly shifted in the case of a superimposed smooth variation of $\Delta G_{xx}$) and the minima of $\Delta G_{xy}$ are shifted on +0.5 unit in the $G_{xx}^0$ scale, in accordance with theory [1,2,7,11,12].

The smoothly varying part of $G_{xx}$, by ignoring the oscillatory part, decreases for decreasing temperature while that of $G_{xy}$ does not change. The temperature
dependence of the smooth part $G_{xx}^{\text{sm}}$ of the diagonal conductance, taken as the midpoint value of the arrow in Fig. 2, is well described by the first-order electron-electron-interaction correction [Eq. (7)] with $\lambda = 1.9$ for samples 2, 3, and 6, and $\lambda = 2$ for sample 7 in the temperature range from 0.15 to 1 K followed by a saturation around 4.2 K. These values are close to the theoretical upper limit $\lambda = 2$ for a system with two spins [13,14], corresponding to a negligibly small triplet part of the electron-electron interaction. The choice of the 4.2 K value for the bare conductance $G_{xx}^0$ agrees with the saturation of $G_{xx}^{\text{sm}}$ around $T = 4.2$ K.

The amplitudes $A_{ij}$ of the oscillations of $G_{xx}$ and $G_{xy}$ conductances are very similar as shown in Fig. 3, where the sum $A_{ij} + \Delta_U$ is plotted as a function of the smooth part of the diagonal conductance $G_{xx}^{\text{sm}}$ for all our samples with $\Delta_U = 24/\pi \exp(-\pi G_{xx}^{\text{sm}})$. The values of $\Delta_U = 0.044, 0.009, 0.02, \text{and} 0.002$ for samples 2, 3, 6, and 7, respectively, are smaller than the corresponding values of $A_{ij}$. The experimental data are rather well described by the result of the US theory for $A_{xy}$ [Eq. (8)] applied to the total conductance of two independent electron systems of opposite spin. Although showing a very similar dependence, $A_{xx}$ cannot be deduced in frame of this theory. The DIGA theory predicts much larger amplitudes than experimentally observed, as shown by the dotted lines in Fig. 3 for $A_{xy}^{\text{DIGA}} + \Delta_U$ according DIGA theory for samples 3 and 7.

In summary, due to topological scaling effects, oscillations of the diagonal and Hall magnetoconductances can exist when there are no oscillations of the density of states due to Landau quantization. The oscillations observed in the extreme quantum limit of the applied magnetic field in disordered GaAs layers, with thickness larger than the electron transport mean-free path, fall into this category. The oscillations of $G_{xy}$ are quantitatively well described by the unified scaling theory for the integer and fractional quantum Hall effect [7]. Their amplitude is much smaller than the dilute instanton gas approximation [12] predicts.

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