**c-Axis Penetration Depth in Bi$_2$Sr$_2$CaCu$_2$O$_8 + \delta$**

**Single Crystals Measured by ac-Susceptibility and Cavity Perturbation Technique**


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Abstract—The $c$-axis penetration depth $\Delta_{c}$ in Bi$_2$Sr$_2$CaCu$_2$O$_8 + \delta$ (BSCCO) single crystals as a function of temperature has been determined using two techniques, namely, measurements of the $ac$-susceptibility at a frequency of 100 kHz and the surface impedance at 9.4 GHz. Both techniques yield an almost linear function $\Delta_{c}(T) \propto T$ in the temperature range $T < 0.5T_c$. Electrodynamic analysis of the impedance anisotropy has allowed us to estimate $\lambda_{c}(0) = 50 \mu$m in BSCCO crystals overdoped with oxygen ($T_c = 84$ K) and $\lambda_{c}(0) = 150 \mu$m at the optimal doping level ($T_c = 90$ K). © 2000 MAIK “Nauka/Interperiodica”.

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Cuprate high-temperature superconductors (HTS) are layered anisotropic materials. Therefore the electrodynamic problem of the magnetic field penetration depth in HTS in the low-field limit is characterized by two length parameters, namely, $\lambda_{ab}$ controlled by screening currents running in the CuO$_2$ planes (in-plane penetration depth) and $\lambda_{c}$ due to currents running in the direction perpendicular to these planes (out-of-plane or $c$-axis penetration depth). The temperature dependence of the penetration depth in HTS is largely determined by the superconductivity mechanism. It is known (see, e.g., [1] and references therein) that $\Delta_{ab}(T) \propto T$ in the range $T < T_c/3$ in high-quality HTS samples at the optimal level of doping, and this observation has found the most simple interpretation in the $d$-wave model of the high-frequency response in HTS [2]. Measurements of $\Delta_{c}(T)$ are quoted less frequently than those of $\Delta_{ab}(T)$. Most of such data published were derived from microwave measurements of the surface impedance of HTS crystals [3–11]. There is no consensus in literature about $\Delta_{c}(T)$ at low temperatures. Even in reports on low-temperature properties of high-quality YBCO crystals, which are the most studied objects, one can find both linear, $\Delta_{c}(T) \propto T$ [4, 9], and quadratic dependences [11] in the range $T < T_c/3$. In BSCCO materials, the shape of $\Delta_{c}(T)$ depends on the level of oxygen doping: in samples with maximal $T_c = 90$ K $\Delta_{c}(T) \propto T$ at low temperatures [7, 8]; at higher oxygen contents (overdoped samples) $T_c$ is lower and the linear function $\Delta_{c}(T)$ transforms to a quadratic one [8]. The common feature of all microwave experiments is that the change in the ratio $\Delta_{c}(T)/\Delta_{ab}(0)$ is especially large in BSCCO crystals, $\lambda_{c}(0) > 10 \mu$m and, according to some estimates, it ranges up to $\sim 500 \mu$m. The large spread of $\lambda_{c}(0)$ is caused by two factors, namely, the poor accuracy of the techniques used in determination of $\lambda_{c}(0)$ and effects of local and extended defects in tested samples, whose range is of the order of 1 mm and comparable to both $\lambda_{c}$ and total sample dimensions.

Recently, we suggested [12] a new technique for determination of $\lambda_{c}(0)$ based on the measurements of the surface barrier field $H_s(T) \propto 1/\lambda_{c}(T)$ at which Josephson vortices penetrate into the sample. The field $H_s$ corresponds to the onset of microwave absorption in the locked state of BSCCO single crystals. This paper suggests an alternative technique based on comparison between microwave measurements of BSCCO crystals aligned differently with respect to ac magnetic field and a numerical solution of the electrodynamic problem of the magnetic field distribution in an anisotropic plate at an arbitrary temperature. Moreover, since $\lambda_{c}(0)$ in BSCCO single crystals is relatively large, we managed to determine $\lambda_{c}(T)$ from the temperature dependences of $ac$-susceptibility and compare these measurements to results of microwave experiments.

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1 This article was submitted by the authors in English.
Single crystals of BSCCO were grown by the floating-zone method [13] and shaped as rectangular platelets. This paper presents measurements of two BSCCO samples with various levels of oxygen doping. The first sample (#1), characterized by a higher critical temperature, $T_c \approx 90$ K (optimally doped), has dimensions $a \times b \times c = 1.5 \times 1.5 \times 0.1$ mm$^3$ ($a = b$). The second (#2, $a \times b \times c = 0.8 \times 1.8 \times 0.03$ mm$^3$) is slightly overdoped ($T_c = 84$ K).

When measuring the $ac$-susceptibility $\chi = \chi' - i \chi''$, we placed a sample inside one of two identical induction coils. The coils were connected to one another, and the out-of-phase and in-phase components of the imbalance signal were measured at a frequency of $10^5$ Hz. These components are proportional to the real and imaginary parts of the sample magnetic moment $M$, respectively: $M = \chi \nu H_0$, where $\nu$ is the sample volume and $H_0$ is the ac magnetic field amplitude, which was within 0.1 $\text{Oe}$ in our experiments.

Figure 1 shows temperature dependences $\chi(T)/|\chi(0)|$ in sample #1 for three different sample alignments with respect to the ac magnetic field: the transverse (T) orientation, $H_0 \parallel c$, (the inset on the left of Fig. 1), when the screening current flows in the $a$-$b$-plane (full circles); in the longitudinal (L) orientation, $H_0 \perp c$, (the inset on the right of Fig. 1, $H_0 \perp c$ is parallel to the $b$-edge of the crystal), when currents running in the directions of both CuO$_2$ planes and the $c$-axis are present (up triangles); in the L-orientation, $H_0 \perp c$, whose difference from the previous configuration is that the sample is turned around the $c$-axis through $90^\circ$ (down triangles). Figure 1 clearly shows that at $T < T_c \chi_{ab}(T)$ is notably smaller in the T-orientation than $\chi_{ab}^\perp(T)$ in the L-orientation (the subscripts of $\chi$ denote the direction of the screening current). The coincidence of $\chi_{ab}^\perp(T)$ curves at $H_0 \perp c$ and the small width of the superconducting transition at $H_0 \parallel c$ ($\Delta T_c < 1$ K) indicate that the quality of tested sample #1 is fairly high. This is supported by precision measurements of surface impedance $Z_s(T) = R_s(T) + iX_s(T)$ of sample #1 at frequency $f = 9.4$ GHz in the T-orientation, which are plotted in Fig. 2. The measurement technique was described in detail elsewhere [1]. It applies to both surface impedance components $R_s(T)$ and $X_s(T)$:

$$R_s = \Gamma_s \Delta(1/Q), \quad X_s = -2\Gamma_s \delta f/f.$$  

Here, $\Gamma_s = \omega \mu_0 \int_V H_0^2 dV/\int_S H_s^2 dS$ is the sample geometrical factor ($\omega = 2\pi f$; $\mu_0 = 4\pi \times 10^{-7}$ H/m, $V$ is the volume of the cavity, $H_0$ is the magnetic field generated in the cavity, $S$ is the total sample surface area, and $H_s$ is the tangential component of the microwave magnetic field on the sample surface); $\Delta(1/Q)$ is the difference between the values $1/Q$ of the cavity with the sample inside and empty cavity; and $\delta f$ is the frequency shift relative to that which would be measured for a sample with perfect screening, i.e., no penetration of the microwave fields. In the experiment, we measure the difference $\delta f(T)$ between resonant frequency shifts with temperature of the loaded and empty cavity, which is equal to $\delta f(T) = \delta f(T) + f_0$, where $f_0$ is a constant. The constant $f_0$ includes both the perfect-conductor shift and the uncontrolled contribution caused by opening and closing the cavity. In HTS single crystals, the constant $f_0$ can be directly derived from measurements of the surface impedance in the normal state; in particular, in the T-orientation, $f_0$ can be derived from the condition that the real and imaginary parts of the impedance should be equal above $T_c$ (normal skin-effect). In Fig. 2, $R_s(T) = X_s(T)$ at $T > T_c$ and its temperature dependence is adequately described by the expression $2R_s^2(T)/\omega \mu_0 = \rho(T) = \rho_0 + bT$ with $\rho_0 = 13$ $\mu\Omega$ cm and $b = 0.3$ $\mu\Omega$ cm/K. Given $R_s(T_c) = \sqrt{\omega \mu_0 \rho(T_c)/2} \approx 0.12$ $\Omega$, we obtain the resistivity $\rho(T_c) = 40$ $\mu\Omega$ cm. The insets to Fig. 2 show $R_s(T)$ and $\lambda(T) = X_s(T)/\omega \mu_0$ for $T < 0.7 T_c$ plotted on a linear scale. The extrapolation of the low-temperature sections of these curves to $T = 0$ yields estimates of $\chi_{ab}(0) = 2600$ $\text{Å}$ and the residual surface resistance $R_{res} \approx 0.5$ $m\Omega$. $R_{res}$ is due to various defects in the surface layer of the superconductor and it is generally accepted that the lower the $R_{res}$, the better the sample quality. The above mentioned

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$^2$ We note that $\delta f(T)$ includes the frequency shift due to the sample thermal expansion, which is essential for $T > 0.7 T_c$ in the T-orientation [1].
parameters of sample \#1 indicate that its quality is fairly high. In the T-orientation, linear functions \( \hat{R}_c(T) \) and \( \Delta \hat{\lambda}_{ab}(T) \) in the low-temperature range were previously observed in optimally doped BSCCO crystals at a frequency of about 10 GHz [7, 8, 14]. In the slightly overdoped sample \#2 we also observed \( \Delta \lambda_{ab}(T) \), \( \Delta \hat{R}_c(T) \propto T \) at low temperature, moreover, the measurement \( R_{es}\times 120 \mu\Omega \) is, to the best of our knowledge, the lowest value ever obtained in BSCCO single crystals.

In both superconducting and normal states of HTS, the relation between electric field and current density is local: \( j = \hat{\sigma} E \), where the conductivity \( \hat{\sigma} \) is a tensor characterized by components \( \sigma_{ab} \) and \( \sigma_c \). In the normal state, ac field penetrates in the direction of the c-axis through the skin depth \( \delta_{ab} = \sqrt{2/\omega \mu_0 \sigma_{ab}} \) and in the CuO\(_2\) plane through \( \delta_c = \sqrt{2/\omega \mu_0 \sigma_c} \). In the superconducting state, all parameters \( \delta_{ab}, \delta_c, \sigma_{ab} = \sigma_{ab}^{\prime} - i \sigma_{ab}^{\prime\prime}, \) and \( \sigma_c = \sigma_c^{\prime} - i \sigma_c^{\prime\prime} \) are complex. In the temperature range \( T < T_c \), if \( \sigma^{\prime} \ll \sigma^{\prime\prime} \), the field penetration depths are given by the formulas \( \lambda_{ab} = \sqrt{1/\omega \mu_0 \sigma_{ab}}, \lambda_c = \sqrt{1/\omega \mu_0 \sigma_c} \). In the close neighborhood of \( T_c \), if \( \sigma^{\prime} \gg \sigma^{\prime\prime} \), the decay of magnetic field in the superconductor is characterized by the functions \( \Re(\delta_{ab}) \) and \( \Re(\delta_c) \), which turn to \( \delta_{ab} \) and \( \delta_c \), respectively, at \( T \geq T_c \).

In the L-orientation of BSCCO single crystals at \( T < 0.9T_c \), the penetration depth is smaller than characteristic sample dimensions. If we neglect the anisotropy in the \( ab\)-plane and the contribution from ac-faces (see the inset to Fig. 1), which is a factor ~ \( c/b \) smaller than that of the \( ab\)-surfaces, the effective impedance \( Z_{c^{\prime\prime}}^{ab+c} \) in the L-orientation can be expressed in terms of \( Z_{c^{\prime\prime}}^{ab} \) and \( Z_{c^{\prime\prime}} \) averaged over the surface area [1, 5] (the superscripts of \( Z_s \) denote the direction of the screening current). Thus, given measurements of \( \Delta \hat{\lambda}_{ab}(T) = \Delta \hat{\lambda}_{ab}^{ab}(T)/\omega \mu_0 \) in the T-orientation and of the effective value \( \Delta \hat{\lambda}_{ab+c}(T) = \Delta \hat{\lambda}_{ab+c}^{ab+c}(T)/\omega \mu_0 \) in the L-orientation, we obtain

\[
\Delta \hat{\lambda}_c = [(a + c)\Delta \hat{\lambda}_{ab+c} - a\Delta \hat{\lambda}_{ab}]/c.
\] (2)

This technique for determination of \( \Delta \hat{\lambda}_c(T) \) was used in microwave experiments [3–9] at low temperatures, \( T < T_c \). Even so, this cannot be applied to the range of higher temperatures because the size effect plays an important role. Really, at \( T > 0.9T_c \), the lengths \( \lambda_c \) and \( \delta_c \) are comparable to the sample dimensions. In order to analyze our measurements in both superconducting and normal states of BSCCO, we used formulas [15] for field distributions in an anisotropic long strip (\( b \gg a, c \)) in the L-orientation. These formulas neglect the effect of \( ac\)-faces of the crystal, but take account of the size effect. In addition, in a sample shaped as a long strip, there is a simple relation between its surface impedance components and \( ac\)-susceptibility, which is expressed in terms of parameter \( \mu \) introduced in [15]:

\[
\Delta(1/Q) - 2i\delta f/F = i\gamma \mu V/V, \quad \chi = -1 + \mu, \quad (3)
\]

where \( \gamma = V H^2_0/I \int_V H^2 dV \) = 10.6 is a constant characterizing our cavity [1]. At an arbitrary temperature, the complex parameter \( \mu = \mu^{\prime} - i \mu^{\prime\prime} \) is controlled by the components \( \sigma_{ab}(T) \) and \( \sigma_c(T) \) of the conductivity tensor:

\[
\mu = \frac{8}{\pi^2} \sum_{n}\frac{1}{n^2} \left[ \frac{\tan(\alpha_n)}{\alpha_n} + \frac{\tan(\beta_n)}{\beta_n} \right],
\] (4)

where the sum is performed over odd integers \( n > 0 \), and

\[
\alpha_n^2 = -\frac{a^2}{\delta_{ab}^2} \left( i + \frac{\pi^2 \delta_{ab}^2}{4 \ c^2} n^2 \right),
\]

\[
\beta_n^2 = -\frac{c^2}{\delta_c^2} \left( i + \frac{\pi^2 \delta_c^2}{4 \ a^2} n^2 \right).
\]

In the superconducting state at \( T < 0.9T_c \), we find that \( \lambda_{ab} \ll c \) and \( \lambda_c \ll a \). In this case, we derive from (4) a simple expression for the real part of \( \mu \):

\[
\mu^{\prime} = 1 + \chi^{\prime} = \frac{2\lambda_c}{a} + \frac{2\lambda_{ab}}{c}.
\] (5)

One can easily check up in the range of low temperatures, the change in \( \Delta \hat{\lambda}_c(T) \) prescribed by (5) is identical to (2). Figure 3 shows measurements of \( \Delta \hat{\lambda}_c(T) \) in sample...
The low-temperature section of the curve of frequency measurements taken in the T- and L-orientations to numerical calculations by (3) and (4), which take into account the size effect in the high-frequency response of an anisotropic crystal. The procedure of comparison for sample #1 is illustrated by Fig. 4. Unlike the case of the T-orientation, the measured temperature dependence of $\Delta(1/Q)$ in the L-orientation deviates from $-2\Delta f/f$ owing to the size effect. Using the measurements of $R_s = \sqrt{\sigma_0 \mu_0 / 2\sigma_{ab}}$ at $T > T_c$ in the L-orientation (Fig. 2) for determination of $\sigma_{ab}(T)$, alongside the data on $\Delta(1/Q)$ in the L-orientation (open circles in Fig. 4), from (3) and (4) we obtain the curve of $\rho_s(T) = 1/\sigma_s(T)$ shown in the right-hand inset to Fig. 4. Further, using the functions $\sigma_s(T)$ and $\sigma_{ab}(T)$, we calculate $-2\Delta f/f$ versus temperature for $T > T_c$, which is plotted by the solid line in Fig. 4. This line is approximately parallel to the experimental curve of $-2\Delta f/f$ in the L-orientation (open circles in Fig. 4). The difference $-2(\delta f - \Delta f)/f$ yields the additive constant $f_0$. Given $f_0$ and $\Delta f(T)$ measured in the range $T < T_c$, we also obtain $\delta f(T)$ in the superconducting state in the L-orientation. As a result, with due account of $\lambda_{ab}(T)$ (the inset to Fig. 2), we derive from (3) and (5) $\lambda_s(0)$, which equals approximately $150 \, \mu m$ in sample #1. A similar procedure performed with sample #2 yields $\lambda_s(0) = 50 \, \mu m$, which is in agreement with our measurements of overdoped BSCCO obtained using a different technique [12]. We also estimated $\lambda_s(0)$ on the base of absolute measurements of the susceptibility $\chi_s(0)$ from (5), and we obtained $\lambda_s(0) = 210 \, \mu m$ for sample #1 and $\lambda_s(0) = 70 \, \mu m$ for sample #2. These results are in reasonable agreement with our microwave measurements if we take into

**Fig. 3.** Temperature dependences $\Delta\lambda_c$ in samples #1 (circles) and #2 (squares) at $T < 0.9 T_c$. Open symbols plot low-frequency measurements, full symbols show microwave data. The inset shows low temperature sections of the $\Delta\lambda_c$ curves in sample #1.

**Fig. 4.** Temperature dependences of $\Delta(1/Q)$ (open squares) and $-2\Delta f/f$ (open circles) in the L-orientation of sample #1 at $T > T_c$. Solid line shows the function $-2\Delta f/f$ deriving from (3). Left-hand inset: $\Delta(1/Q)$ and $-2\Delta f/f$ as functions of temperature in the neighborhood of $T_c$. Right-hand inset: $\rho_s(T)$ in sample #1 (triangles).
consideration the fact that the accuracy of $\lambda_c$ measurements is rather poor and the error can be up to 30%.

In conclusion, we have used the $ac$-susceptibility and cavity perturbation techniques in studying anisotropic high frequency properties of BSCCO single crystals. We have observed almost linear dependences $\Delta\lambda_c(T) \propto T$, which are in fair agreement with both experimental [7, 8, 12] and theoretical [16] results by other researchers. We have also investigated a new technique for determination of $\lambda_c(0)$, which is a factor of three higher in the optimally doped BSCCO sample than in the overdoped crystal. The ratio between the slopes of curves of $\Delta\lambda_c(T)$ in the range $T \ll T_c$ is the same. These facts could be put down to dependences of $\lambda_c(0)$ and $\Delta\lambda_c(T)$ on the oxygen content in these samples. At the same time, we cannot rule out the influence of defects in the samples on $\lambda_c(0)$, even though their quality in the $ab$-plane is fairly high, according to our experiments. In order to draw ultimate conclusions concerning the nature of the transport properties along the $c$-axis in BSCCO single crystals, studies of more samples with various oxygen contents are needed.

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