Temperature dependence of the critical current of a granular superconducting medium

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A nonuniform current distribution can arise in a network of Josephson and tunnel junctions dominated by the latter. The critical val ue of the average current density, i.e., the value at which the Josephson links are destroyed, will accordingly fall off with decrea sing temperature.

The temperature dependence of the maximum Josephson current $i_m(T)$ flowing through a tunnel barrier is described by the Ambegoakar–Baratoff formula[1]

$$i_m(T)=\frac{\pi}{4}\frac{2\Delta(T)}{er}\tanh\frac{\Delta(T)}{2T}, \tag{1}$$

where r is the resistance of the barrier above the temperature T_c, in the case of normal banks, and $\Delta(T)$ is the size of the superconducting gap. According to (1), the current $i_m(T)$ initially increases with decreasing temperature and then approaches the limiting value

$$i_m(0)=\frac{\pi}{4}\frac{2\Delta(0)}{er}, \quad i_m(T_c/2)\approx 0.9 i_m(0). \tag{2}$$

One might expect that in a granular superconductor, consisting of a medium with a large number of tunnel junctions, the critical current I_m in the medium would also vary with the temperature in accordance with (1). However, Lin *et al.*[2] have observed a maximum on the plot of $I_m(T)$ for a Pb–Ba–Bi–O metal oxide superconductor. A similar maximum was later observed in the high-T_c superconductor K–Ba–Bi–O (Ref. 3) and also in a high-resistivity metastable Zn–Sb alloy (Ref. 4). In the two latter cases, there was a quasireentrant superconducting transition.[5] In other words, the resistance never vanished below T_c, and a "quasi-Josephson" region on the current–voltage characteristic corresponded to a comparatively low but nonzero resistance.

Is the observed maximum a reflection of the properties of an individual junction or a manifestation of the properties of a network of such junctions?

Expression (1) presupposes a tunneling of electrons and Cooper pairs through the volume separating the superconductors. However, the mechanism for the conduction through this volume may be more complicated. Electron states (localized or delocalized) in this volume may be involved in the conduction. We know of one, admittedly extremely specific, example[6] in which the function $i_m(T)$ has a maximum: in the tunneling of pairs through a degenerate semiconductor with a degeneracy temperature on the order of the superconducting transition temperature T_c. On the other hand, the curves of $i_m(T)$ found in Refs. 7 and 8—in a study of an isolated separation volume with resonant levels and a hopping conductivity—did not have a maximum.

The suggestion in Ref. 1 of a purely tunneling mechanism for the flow of electrons and Cooper pairs across a junction means that r would have no temperature dependence above T_c, since the electron tunneling probability would be independent of the temperature. In all the experiments mentioned above, the superconducting transition was preceded by an increase in r with decreasing T. It seems natural to substitute a temperature-dependent value of r into (1). This value can be determined either by extrapolation or by using a magnetic field to disrupt the superconductivity of the banks of the junction. An increase in r with decreasing T may lead, against a background $\Delta(T) \approx \text{const}$, to a decrease in i_m. We also know of an experiment,[9] in which a maximum was observed on $i_m(T)$ for single Sn–SnO–Sn film junctions. We might point out that SnO is an example of a "nonideal" insulator, to which the theory of Ref. 1 may not apply. However, the conditions formulated in Ref. 6 definitely do not hold in this material, and there are no other grounds for substituting a T-dependent function for r into (1).

Putting aside the hypothetical possibility of a positive derivative $\partial i_m / \partial T$ for an isolated junction, we would like to discuss the role played by an ensemble of junctions. Our purpose here is to show that, even for an ordinary dependence of the type in (1) for the current $i_m(T)$ through isolated junctions, the critical current $I_m(T)$ through a random network of junctions, some of which are in a Josephson state, may have a maximum. We understand I_m here as the current at which the resistance of the medium begins to increase after a threshold is reached. This increase may begin from a nonzero value.

We consider a set of superconducting grains between which there are tunnel junctions. We assume that some small fraction $\alpha \ll 1$ of these junctions, with normal resistances r_1, are in a Josephson state (j), while the others have a normal resistance $r \gg r_1$, and a one-particle tunneling (t) current is flowing through them. The probable reason why there is no j-current through most of the junctions is that the value of r is greater than the critical value $r_{cr} = (\pi/2)(\hbar/e^2)$ (Refs. 10–12). It may turn out that r_{cr} should be compared with the function $r(T)$, and that the j-state is disrupted in some of the junctions with decreasing T; then α would decrease. This is a second possible explanation for a decrease in I_m. However, we will assume that α is independent of T.

Figure 1 shows the known j and t characteristics of a junction with a normal resistance r. The former is described by

$$i = [i_m^2 + (U/r)^2]^{1/2}, \tag{3}$$

while the latter has a resistance in the lower region given in order of magnitude by[13]

$$r_t = r(T/\Delta)\exp(\Delta/T). \tag{4}$$

We assume that the average current across the junctions is small, so we are in regions (4) on the t-characteristics. However, the j-junctions, with a zero resistance, will pass a far greater current. The effect is to increase the current through the z_1 t-junctions directly adjacent to the j-junction under consideration, on each side. They will be moved from the lower part of the t-characteristic to the steeply rising part of this characteristic. The resistance of each t-junction will thereby be reduced. The overall decrease in the resis-

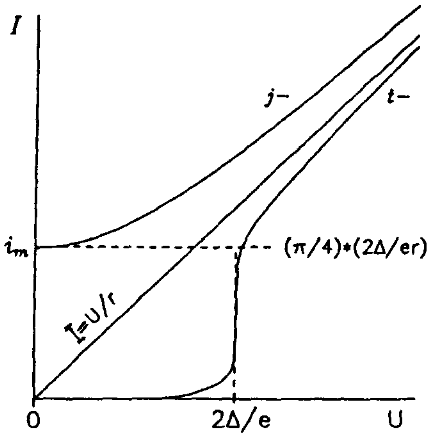

FIG. 1. Josephson (j) and tunneling (t) characteristics of a junction between two identical superconductors having a normal resistance r.

tance of this region will lead to an increase in the current through it and will thus reduce the resistance of the second layer of $z_2 \approx z_1^2$ junctions, and so forth, until the current across the j-junction reaches a value on the order of i_m. Each of the z_1 t-junctions of the first layer will carry a current i_m/z_1 and will have a resistance rz_1. The total resistance of the layer will be r. The same comments apply to the succeeding layers. The resultant resistance of each layer will be on the order of r, until the average current across the junctions in the given layer falls below $2\Delta/er_t$.

Regions with characteristic resistances $r \ll r_t$ thus form around j-junctions in a medium characterized by exponentially large junction resistances r_t. Let us model the average resistance of a medium by means of a parallel connection of one resistance r and ν resistances r_t. The current I through each packet is related to the current i through the resistance r by

$$I = i\frac{\nu r + r_t}{r_t} = i\left[1 + \nu\frac{\Delta}{T}\exp\left(-\frac{\Delta}{T}\right)\right]. \tag{5}$$

When i reaches the critical value i_m, and, correspondingly, when

$$I = I_m = i_m\left[1 + \nu\frac{\Delta}{T}\exp\left(-\frac{\Delta}{T}\right)\right], \tag{6}$$

the nonuniform current structure begins to break up, and the effective resistance of the sample rises rapidly. Substituting the $\Delta(T)$ function from the BCS theory into (6), using the experimental value $T_c = 6.8$ K, and using the function $i_m(T)$ from (1), we find an expression with a single adjustable parameter ν. We have compared this expression with an experimental curve from Ref. 4 (see Fig. 2, where we have $\nu = 250$).

In this discussion a j-junction acts as a seed around which a low-resistance region forms. It is apparently important that the seed be specifically a j-junction. In a network consisting exclusively of t-junctions, a nonuniform current distribution will lead to an increase in the dissipation, which must be offset by the absence of dissipation at the j-junction. There is the further possibility that an isolated j-junction will not be

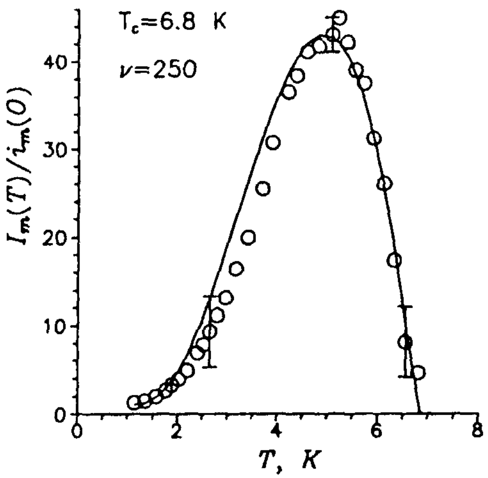

FIG. 2. Comparison of function (6) with experimental data from Ref. 4 on the alloy $Zn_{41}Sb_{59}$ in a metastable state with an effective resistivity on the order of 10 Ω·cm.

sufficient—that only a cluster of several j-junctions will be adequate as a seed. This circumstance might explain the large value of ν which we had to substitute in Eq. (6) in making the comparison with experiment (Fig. 2).

In the experiment described in Ref. 4, at temperatures on the order of $0.9T_c$, at which we have $r_t \approx r$, the total resistance of the sample decreases by a factor of almost 2 from its normal value. The fact that this decrease stops when the measurement current increases comparatively slightly (see Fig. 1 in Ref. 4) confirms that it is caused by specifically j-currents. In accordance with the magnitude of the current decrease, we adopt $\alpha \approx 0.1$ as an estimate of the relative number of j-junctions (an infinite superconducting cluster arises at $\alpha \approx 0.15$, while at far smaller values of α the effective-medium approximation[14] yields $\Delta R/R \approx 1-3\alpha$). In a simple cubic lattice, in which each junction makes contact with ten others, the number of clusters of a single j-junction, normalized to the total number of junctions, is $\alpha(1-\alpha)^{10} \approx 0.035$. In other words, corresponding to one such cluster are $\nu_1 \approx 30$ junctions of the original lattice. For clusters of two and three j-junctions, the corresponding numbers are[15]

$$\nu_2 = [5\alpha^2(1-\alpha)^{14}]^{-1} \approx 90 \quad \text{and} \quad \nu_3 \approx 210,$$

i.e., on the same order of magnitude as ν in Fig. 2.

In summary, a nonuniform current distribution can arise in a nonlinear medium consisting of j- and t-junctions. The nonuniformity intensifies as the temperature is lowered. The effect is to reduce the critical value of the average current, at which the superconducting current across the j-junctions is disrupted; the effective resistance of the medium will begin to rise rapidly.

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