

# Role of nonlocality and Landau damping in the dynamics of a Quantum dot coupled to surface plasmons.

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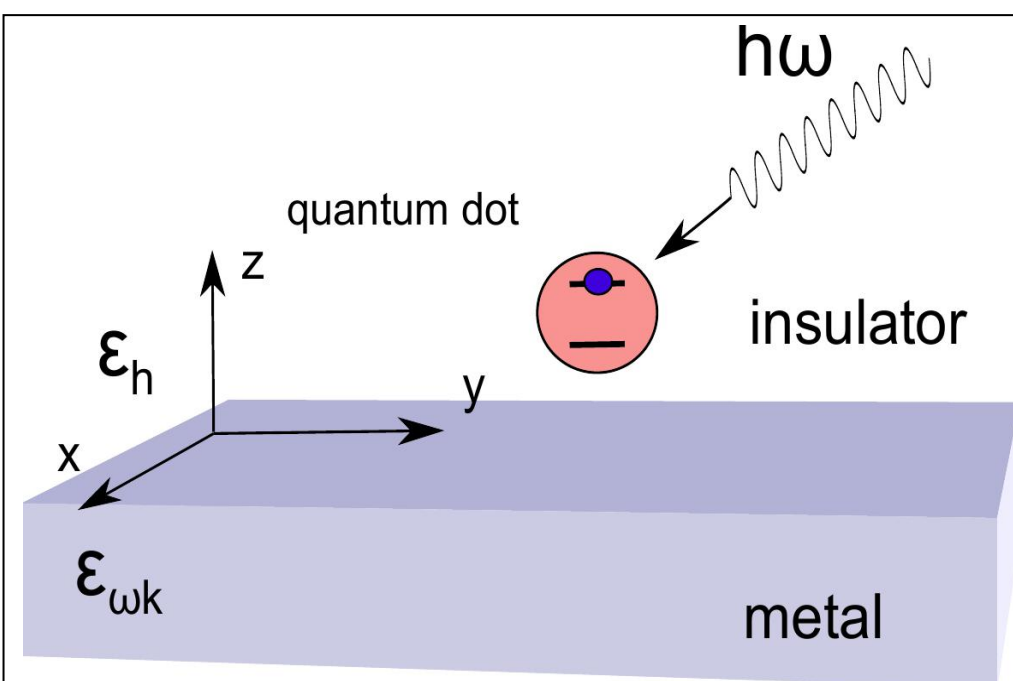


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Metal slab (thick), and **quantum dot** (quantum scatterer with well separate levels). Exciton in the dot is created by external laser pulse.

## Non-local electromagnetic response

Delocalized carriers in the metal → spatially non-local response

Bulk dielectric susceptibility →

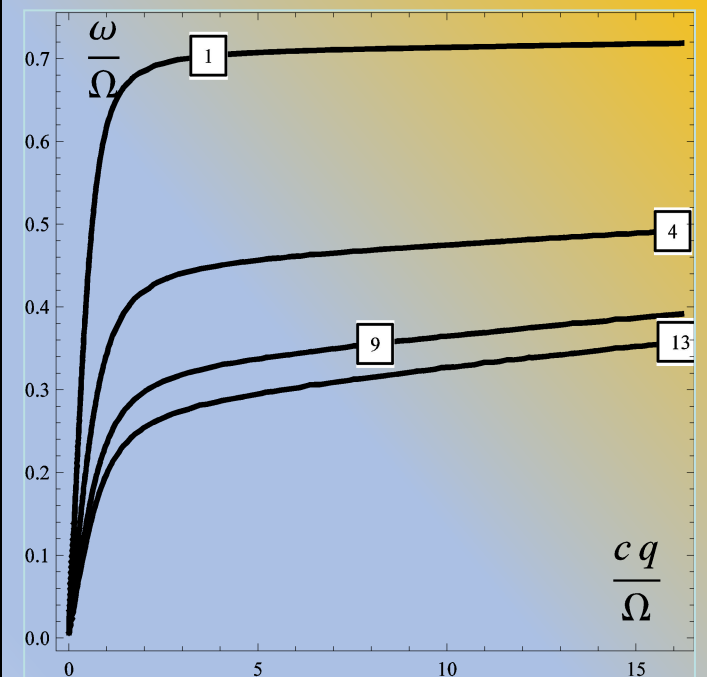
Lindhard function

$$\varepsilon(\omega, k) = 1 + \frac{3\Omega_p^2}{(kv_F)^2} \left[ 1 - \frac{\omega}{2kv_F} \ln \frac{\omega + kv_F}{\omega - kv_F} \right]$$

$$\varepsilon(\omega, k) = 1 - \frac{\Omega_p^2}{\omega^2}, \quad \text{Im} \varepsilon = \frac{3\pi\Omega_p^2\omega}{2(kv_F)^3} \sim \frac{\Omega_p^2\omega r_D^3}{v_F^3}$$

$k \rightarrow 0$   $kv_F > \omega$

N. W. Ashcroft and N. D. Mermin, *Solid State Physics*, (Thomson, Toronto, 1976)



Dispersion of the Surface Plasmon at the boundary of metal with dielectric media. Numbers in the boxes are dielectric constants of the host media.

## Plasmon's quantization:

In general, plasmon has arbitrary amplitude, for quantization we need to make it such that total energy of SP  $W = \hbar\omega$ .

## Hamiltonian of the system

$$H = \hbar \left[ \frac{1}{2} \Omega_{ex} (\sigma_z + 1) + \sum_k \omega_k b_k^\dagger b_k \right] + \hbar \sum_k g_k \sigma_- b_k^\dagger + g_k^* \sigma_+ b_k$$

$\Omega_{ex}$  is the frequency of the exciton transition,  $\omega_k$  is the frequency of the surface plasmon, and  $g_k$  are the coupling constants.

Quantum state  $|\Psi\rangle = a(t)|\Psi_e\rangle + \sum_k c_k(t)|\Psi_k\rangle$

Dynamic  $i\partial_t a(t) = \Omega_{ex} a(t) + \sum_k g_k c_k(t)$ ,

equations  $i\partial_t c_k(t) = \omega_k c_k(t) + g_k^* a(t)$

Laplace transform solution

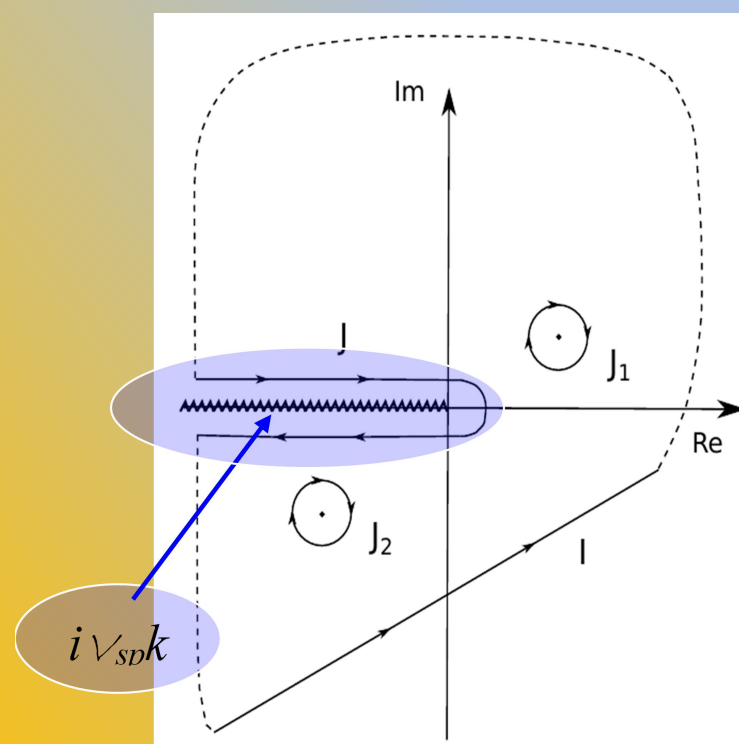
$$a(s) = \frac{1}{s + i\Omega_{ex} + K(s)}$$

$$K(s) = \sum_k \frac{|g_k|^2}{s + i\omega_k} \approx \int_0^\infty \frac{k^2 e^{-2kd} dk}{s + i\Omega_{sp} + \Gamma + i\nu_{sp}k}$$

- Incomplete gamma function

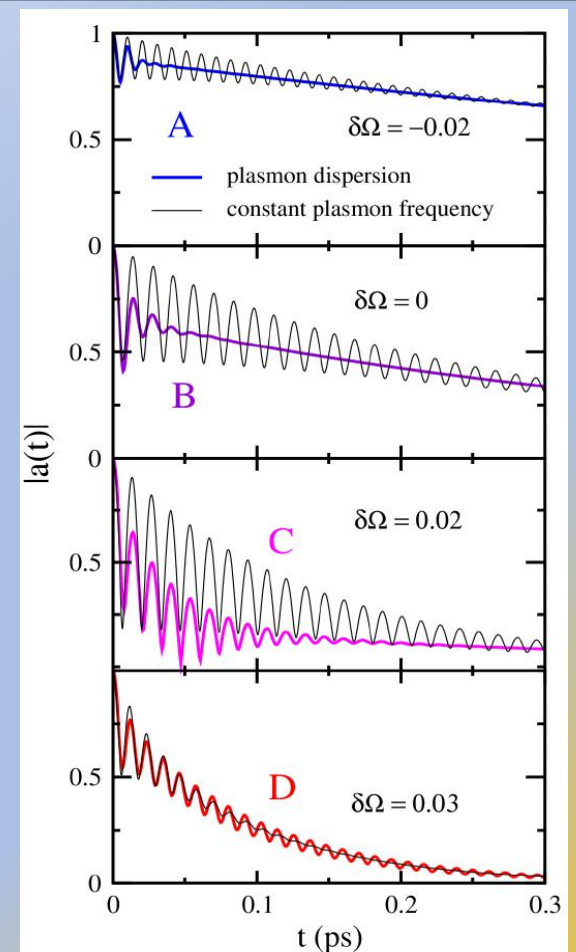
Linear spectrum generates branch cut -- without linear spectrum only poles exist, no branch-cut

## Inverse Laplace transform contour:



$$|a(t)| = |a_1 e^{-i\Omega_R t - \gamma_1 t} + a_2 e^{-\gamma_2 t}| \text{ or }$$

$$|a(t)| = |a_1 e^{-i\Omega_R t - \gamma_1 t} + a_2 t^{-3}|$$

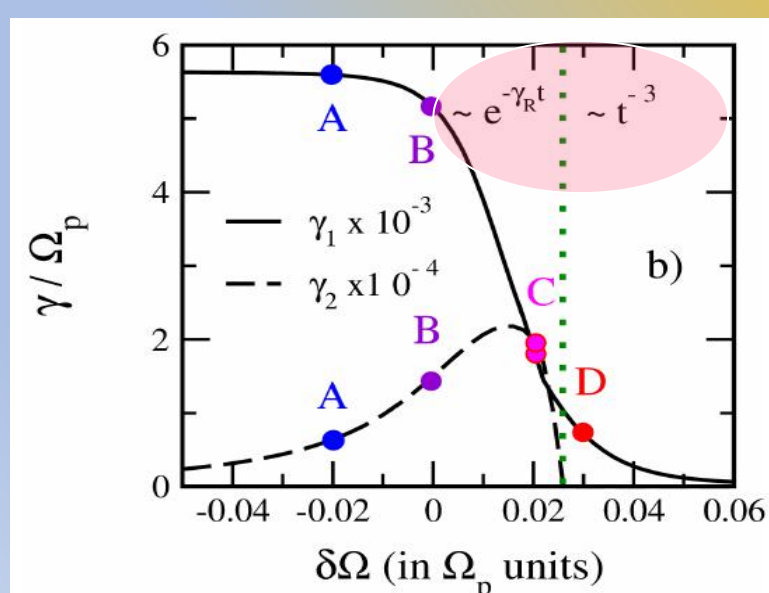


## Two type relaxations

- 1) fast relaxation of the Rabi oscillations ~0.1ps
- 2) slow monotonous relaxation ~100 ps

## Power law relaxation

When the branch-cut contribution is important (single pole), the time asymptote may change from exponential to a power law. Consequence: much longer Rabi oscillations



## Summary

### Non-locality in the electro-magnetic response:

1. is important in systems of finite size
2. changes the frequency dispersion (spectrum) of surface plasmons
3. for a flat surface leads to the two-velocity frequency dispersion (velocities are very different)
4. modifies the damping rate of surface plasmons (Landau damping)
5. leads to notable changes in the dynamics of a nearby quantum system (two time relaxation pattern, change in the long-time asymptotic, etc.)

## References

1. Larkin, I.A., Stockman, M.I., Achermann, M., Klimov, V.I.; Dipolar emitters at nanoscale proximity of metal surfaces: Giant enhancement of relaxation in microscopic theory Phys. Rev. B 69, 121403(R), (2004).
2. A. Vagov, I. A. Larkin, M. D. Croitoru, V. M. Axt "Role of nonlocality and Landau damping in the dynamics of a quantum dot coupled to surface plasmons"; Phys. Rev. B 93, 195414, (2016)
3. I. A. Larkin, K. Keil, A. Vagov, M. D. Croitoru, and V. M. Axt Superanomalous Skin Effect for Surface Plasmon Polaritons Phys. Rev. Lett. 119, 176801 (2017)

## Outline

1. Theory with the non-local electro-magnetic response
  - a) Lindhard response
  - b) finite size corrections
2. Surface plasmon modes
  - a) electromagnetic problem
  - b) frequency dispersion and non-locality
  - c) Landau damping
3. Dynamics of a quantum dot induced by surface plasmons
  - a) effective quantum Hamiltonian
  - b) dynamical patterns & non-locality

## Theoretical description for dot-metal coupling

There are three theory components – dot, material carriers, el.-m. fields

1. Dot – quantum system with few (two) levels
2. Charge carriers in materials – material equations, for example Boltzmann kinetic equation
3. Maxwell equations for the fields

2. + 3. Maxwell equations for the field in the matter

**surface plasmons** – the modes localized near the metal-insulator surface

## Non-local electromagnetic response

Delocalized carriers in the metal → spatially non-local response

Bulk dielectric susceptibility → Lindhard function

$\varepsilon(\omega, k) = 1 + \frac{3\Omega_p^2}{(kv_F)^2} \left[ 1 - \frac{\omega}{2kv_F} \ln \frac{\omega + kv_F}{\omega - kv_F} \right]$ $\varepsilon(\omega, k) = 1 - \frac{\Omega_p^2}{\omega^2}, \quad \text{Im } \varepsilon = \frac{3\pi\Omega_p^2\omega}{2(kv_F)^3} \sim \frac{\Omega_p^2\omega r_D^3}{v_F^3}$ $k \rightarrow 0 \quad k v_F > \omega$	$i\mathbf{k} \cdot \mathbf{P}_k = e n_k; \quad \mathcal{J}(\mathbf{k}) = \frac{\mathbf{E}(\mathbf{k}) \cdot \nabla_{\mathbf{p}} f_0}{\mathbf{k} \cdot \mathbf{v} - \omega + i/\tau}$ $4\pi \mathbf{P}(\mathbf{k}) = (\varepsilon - 1) \cdot \mathbf{E}(\mathbf{k})$ $(\varepsilon_{k,\omega} - 1) = \frac{4\pi e^2}{k^2} \iiint_{\mathbf{p}} \frac{\mathbf{k} \cdot \nabla_{\mathbf{p}} f_0(\mathbf{p}) d^3 p}{\mathbf{k} \cdot \mathbf{v} - \omega + i/\tau}$
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Plasmon's quantization: In general, plasmon has arbitrary amplitude, for quantization we need to make it such that total energy of SP  $\hbar\omega_k$

$$\hbar\omega_k = \iiint_V w dV; \quad w = \frac{1}{16\pi} \left( \frac{\partial}{\partial \omega} (\omega \varepsilon(\omega)) E^2 + H^2 \right)$$

Hamiltonian of the system: dot + SP

$$H = \hbar \left[ \frac{1}{2} \Omega_{ex} (\sigma_z + 1) + \sum_{\vec{k}} \omega_k b_k^+ b_k \right] + \hbar \sum_{\vec{k}} g_k \sigma_- b_k^+ + g_k^* \sigma_+ b_k$$



Where  $\Omega_{ex}$  is the frequency of the exciton transition,  $\omega_q$  is the frequency of the surface plasmon with the wave vector  $q$ ,  $c^+$  and  $c$  are the creation and annihilation operators of electron in the quantum dot in conduction band,  $d^+$  and  $d$  are creation and annihilation operators of the electron in valence band, and  $b_q^+$  and  $b_q$  are creation and annihilation operators of the SP.  $H_I$  is the interaction Hamiltonian of the two-level dipole with SP and  $g_q$  are the coupling constants. To solve Schrödinger equation, we apply Laplace transform:

$$\text{Quantum state } |\Psi\rangle = a(t)|\Psi_e\rangle + \sum c_k(t)|\Psi_k\rangle$$

$$\text{Dynamic } i\partial_t a(t) = \Omega_{ex} a(t) + \sum g_k c_k(t),$$

$$\text{equations } i\partial_t c_k(t) = \omega_k c_k(t) + g_k^* a(t)$$

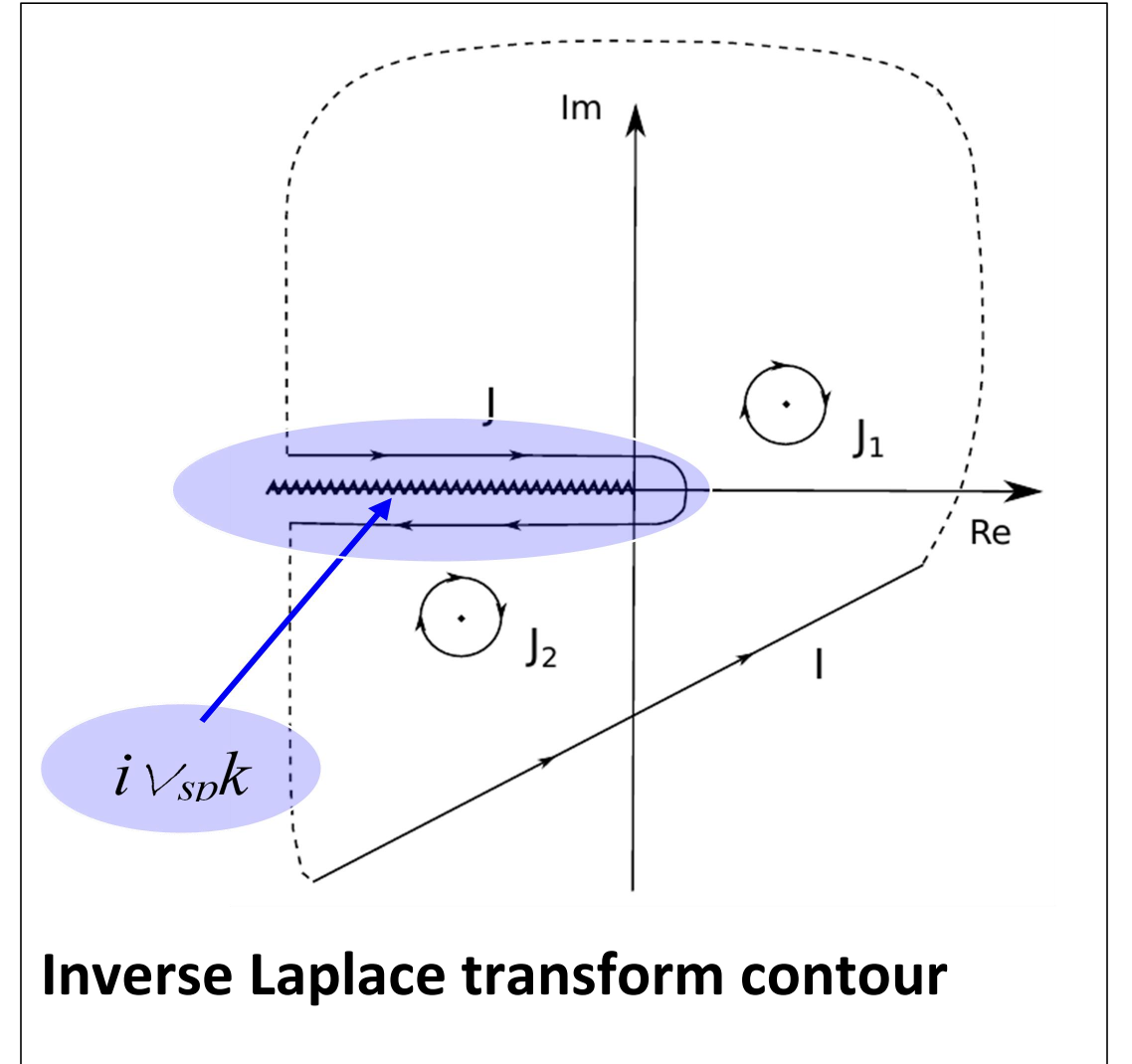
$$a(s) = \frac{1}{s + i\Omega_{ex} + K(s)} \quad \text{Laplace}$$

transform solution

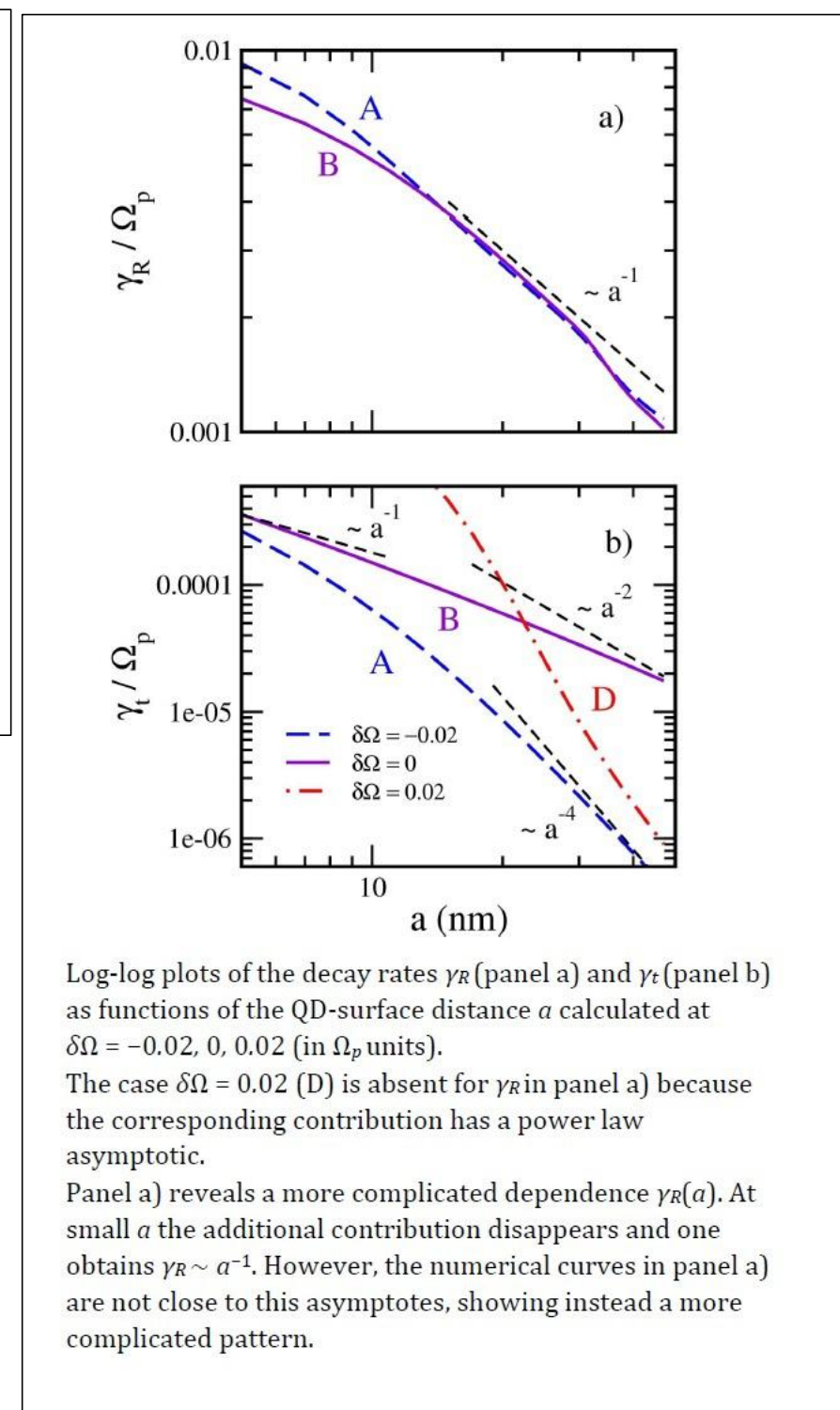
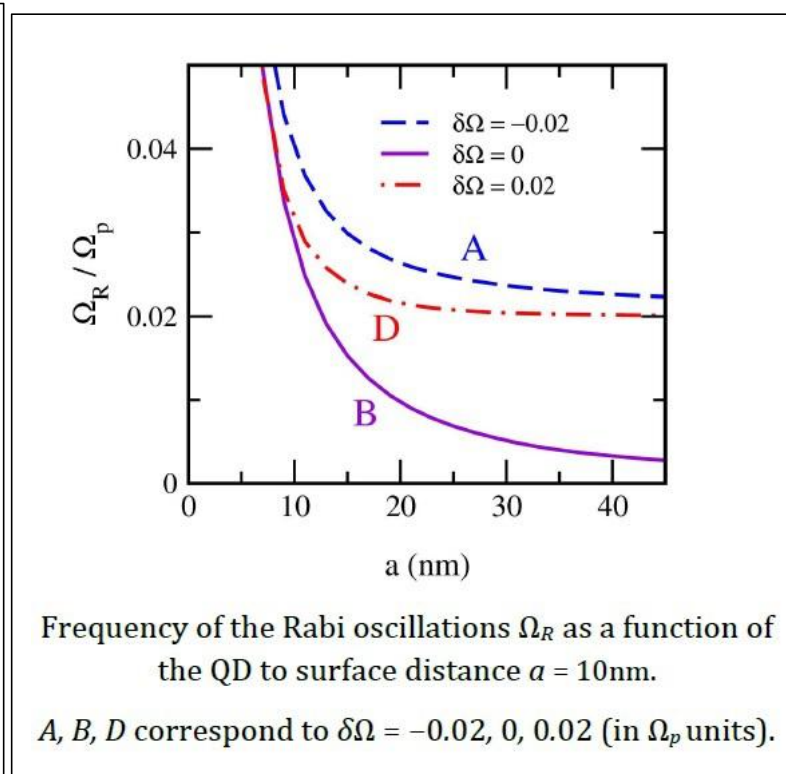
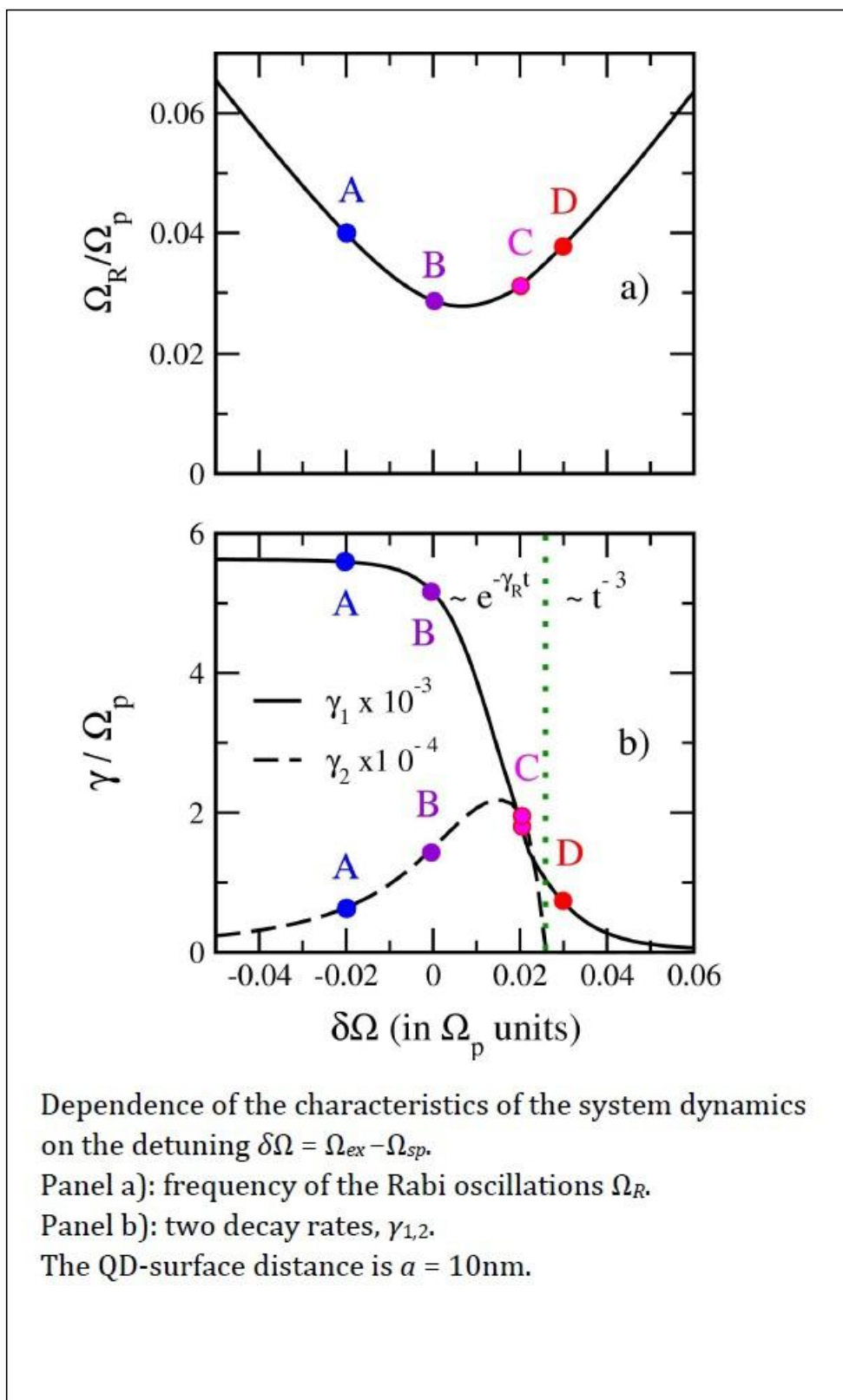
$$K(s) = \sum_k \frac{|g_k|^2}{s + i\omega_k} \approx \int_0^\infty \frac{g^2 k^2 e^{-2kd} dk}{s + i\Omega_{sp} + \Gamma + i v_{sp} k}$$

$K(s)$  - Incomplet gamma function

With the cut from  $-\infty$  to zero.



Linear spectrum generates brunch cut: without linear spectrum only poles exist, no branch-cut. As a result  $|a(t)| = |a_1 e^{-i\Omega_R t - \gamma_1 t} + a_2 e^{-\gamma_2 t}|$  when contribution from two poles dominates, or  $|a(t)| = |a_1 e^{-i\Omega_R t - \gamma_1 t} + a_2 t^{-3}|$ . First term is the Rabi oscillations that have fast relaxation  $\sim 0.1$ ps, whereas at  $\Omega_{ex} > \Omega_{sp} + g^2$  the branch-cut contribution is important (single pole), the time asymptote may change from exponential to a power law. At this regime system has much longer Rabi oscillations.



## Summary

Non-locality in the electro-magnetic response:

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