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Menisci masses and weights in Stepanov (EFG) technique: ribbon, rod, tube

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Abstract

The present paper deals with the integration of the famous Laplace equation resulting in the expressions of full mass and weight of the liquid menisci in the system "melt–crystal" during the crystal growth process by Stepanov (EFG) technique. The problem for the cases of growing ribbon, cylindrical rod, cylindrical tube, rod of arbitrary cross-section and tube of arbitrary inner and outer cross-sections is solved. The problem for a hydrostatic approximation of the menisci of growing crystals is solved and it is shown via an analysis of the full energy functional, that the hydrodynamic factor is too small to be considered in the automated crystal growth systems. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Automated control systems (ACS) of the crystal growth processes from the melt are developing now for the needs of crystal production and scientific research. The weight sensors of crystal and computers are widely used now in ACS. The problem of exact expressions of program weight and mass, which weight sensors produce during the crystal process, exists now in some papers [1,2]. In the paper [2], an expression for the weight of the meniscus contacted with crystal and shaper of arbitrary shape is produced as

$$P_m = \rho_L g h S(t) + \sigma_{LG} \Gamma(t) \cos\varepsilon + \rho_L g h_{eff} S(t) + K_R V_0 + \int_S p(h,\ V_0)\,\mathrm{d}s.$$

Here σ_{LG} is the surface tension at the liquidus–gas interface, ρ_L the density of the melt, g the acceleration due to gravity, h the meniscus height, $S(t)$ the crystal cross-section area, $\Gamma(t)$ its perimeter, ε the growth angle, h_{eff} the height of the die edges above the melt pool, K_R is a factor of shaper resistance to melt flow and $p(h,\ V_0)$ the hydrodynamic pressure under crystallization front. But this expression is not written correctly. First of all, there is no term related to the contact of

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the meniscus with the shaper. Secondly, the hydrostatic pressure is related to the cross-section area of the crystal instead of that of the shaper. Finally, the last two terms corresponding to the hydrodynamic factor should be small enough to make sensitive influence to the weight sensor. The present paper deals with the proof of the first two statements for the various cross-sections of the shaper and crystal in the Stepanov (EFG) technique. An estimation of the influence of hydrodynamics in the Stepanov (EFG) technique is made via the analysis of the partial energies in the free energy functional.

The expressions for program weight and mass of menisci are determined via the integration of the famous Laplace equation. In its turn, Laplace equation is a result of the minimization of the functional of the full energy of the liquid meniscus. According to Ref. [3], an expression of the full energy functional is expressed as

$$I = \sigma \int_S \mathrm{d}s + \rho_L g \int_V y \, \mathrm{d}V \to \min. \tag{1}$$

Here σ is the surface tension at the liquidus–gas interface, ρ_L the density of the melt, S the surface of the liquid meniscus, V the volume of meniscus melt, and y the vertical coordinate of the point of the meniscus surface (Fig. 1). The first part of Eq. (1) is the surface free energy of the meniscus and the second part of this functional is the potential energy of the meniscus in the gravitation field. Eq. (1) describes the meniscus melt in a hydrostatic approximation. Minimization of this functional under the condition of the constant volume of the meniscus results in the well-known Laplace equation [3]

$$\sigma\left(\pm\frac{1}{R_1} \pm \frac{1}{R_2}\right) = \rho_L g(y + H_d). \tag{2}$$

Here, R_1 and R_2 are the main curvature radii at the point of the meniscus surface, and H_d is the constant defining external pressure in the meniscus. External pressure may be produced by the different levels of the meniscus basis and melt surface in the crucible. The constant H_d is usually negative in the real crystal growth processes by the Stepanov (EFG) technique when the melt level is lower than the meniscus basis, as

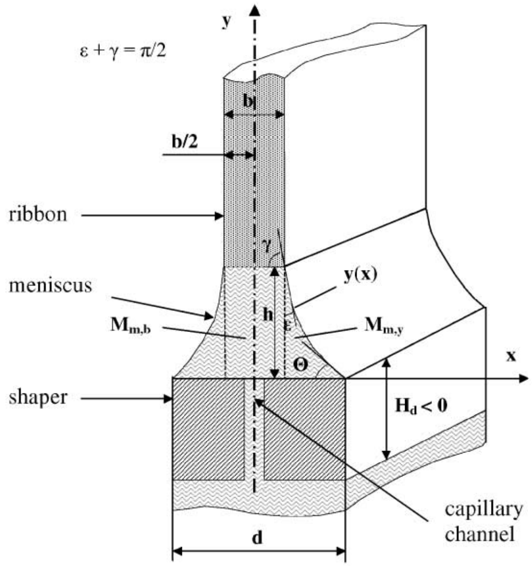

Fig. 1. Scheme of growing a crystal ribbon by the Stepanov (EFG) technique.

shown in Fig. 1; although, in general, it may be zero, as in the Czochralski technique, or positive. The signs before curvatures in Eq. (2) depend upon the directions of normal radii of curvature [3].

Complicated menisci are not considered in the Stepanov (EFG) technique. Thus, the vertical coordinate y describes a profile curve of the meniscus. Eq. (2) may be rewritten as

$$\rho_L g \, y(x) = \sigma\left(\pm\frac{1}{R_1(x)} \pm \frac{1}{R_2(x)}\right) - \rho_L g H_d. \tag{3}$$

Various techniques of integrating the meniscus profile curve $y(x)$ result in the square of the figure under this meniscus curve or volume of the three-dimensional meniscus figure, which may be described via this profile curve. The volume of the meniscus figure, multiplied by the density of the melt, defines the mass of the meniscus. The mass of the meniscus, multiplied by the earth acceleration g, defines the weight of the meniscus. In our case, when the meniscus profile curve $y(x)$ exists obviously in Laplace equation, Eq. (3), it is a the best condition for integrating.

Five cases of different menisci are considered in this paper:

- The meniscus contacted with the growing crystal ribbon. The part of the meniscus in contact with the wide side of ribbon is considered. The thickness of the ribbon is supposed to be much smaller than its wide side.
- The meniscus contacted with the growing crystal cylindrical rod.
- The meniscus contacted with the growing crystal tube.
- The meniscus contacted with the growing crystal rod of arbitrary cross-section.
- The meniscus contacted with the growing crystal tube of arbitrary outer and inner cross-sections.

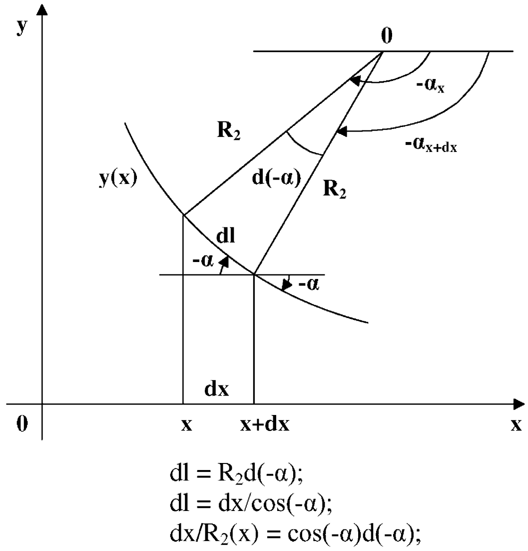

Fig. 2. Scheme of the integration of the second main curvature.

2. Case of growing crystal ribbon

In the process of growing ribbon by the Stepanov (EFG) technique, only the meniscus contacted with the wide side of the ribbon is considered. In this case, one main curvature $1/R_1(x)$ is equal to zero (Fig. 1). Therefore, Eq. (3) results in

$$\rho_L g\, y(x) = -\sigma \frac{1}{R_2(x)} - \rho_L g H_d. \quad (4)$$

Here "–" sign is used because curvature normal (Fig. 2) is directed into the melt of the meniscus [3,4].

Weight $P_{m,y}$ of the part of the meniscus located under the surface defined by the profile curve $y(x)$ should be written according to Eq. (4) as

$$\begin{aligned} P_{m,y} &= g M_{m,y} = \rho_L g c \int_{b/2}^{d/2} y(x)\,dx \\ &= -\sigma c \int_{b/2}^{d/2} \frac{dx}{R_2(x)} - \rho_L g H_d c \int_{b/2}^{d/2} dx. \quad (5) \end{aligned}$$

Here, $M_{m,y}$ is the mass of the part of the meniscus located under the surface defined by the profile curve $y(x)$ (Fig. 1), b the thickness of the crystal ribbon, c the width of the ribbon, and d the thickness of the shaper. According to Eq. (5), the mass $M_{m,y}$ may be written as

$$M_{m,y} = -\frac{\sigma c}{g} \int_{b/2}^{d/2} \frac{dx}{R_2(x)} - \rho_L H_d c \int_{b/2}^{d/2} dx. \quad (6)$$

The mass $M_{m,b}$ of the part of the meniscus located under the planar interface boundary (Fig. 1) is written as

$$M_{m,b} = \rho_L h c \frac{b}{2} - \rho_L H_d c \frac{b}{2}. \quad (7)$$

Here h is the height of the meniscus with planar interface boundary. Full mass M_m of the meniscus should be written as

$$M_m = 2(M_{m,b} + M_{m,y}). \quad (8)$$

The most interesting thing in Eq. (6) is the first part, namely, the integral of curvature. As shown and described in Fig. 2, Eq. (6) may be written as

$$M_{m,y} = -\frac{\sigma c}{g} \sin\alpha \Big|_{\gamma}^{\theta} - \rho_L H_d c \left(\frac{d}{2} - \frac{b}{2}\right). \quad (9)$$

Here γ and θ are the angles between the tangent and horizontal line at the points of the contact of the meniscus and the crystal or shaper, respectively, as shown in Fig. 1. The angle ε is the growth angle. An isotropic approximation of crystal growth is considered and the growth angle is a constant value depending on the physical properties of the material. Therefore, $\varepsilon + \gamma = \pi/2$. Simplification of the Eq. (9) results in

$$M_{m,y} = \frac{\sigma c}{g}(\cos\varepsilon - \sin\theta) - \rho_L H_d c\left(\frac{d}{2} - \frac{b}{2}\right). \quad (10)$$

Using expressions (7), (8), and (10), the full meniscus mass can be written as

$$M_m = \rho_L hbc + \frac{2\sigma c}{g}(\cos\varepsilon - \sin\theta) - \rho_L H_d cd. \quad (11)$$

Taking into account the definition of capillary constant a [3] as $\alpha = \sqrt{2\sigma/\rho_L g}$, Eq. (11) may be rewritten as

$$M_m = \rho_L hbc + \rho_L ca^2(\cos\varepsilon - \sin\theta) - \rho_L H_d cd. \quad (12)$$

Eq. (12) represents the expression for the full mass of the meniscus of the wide sides of the growing crystal ribbon.

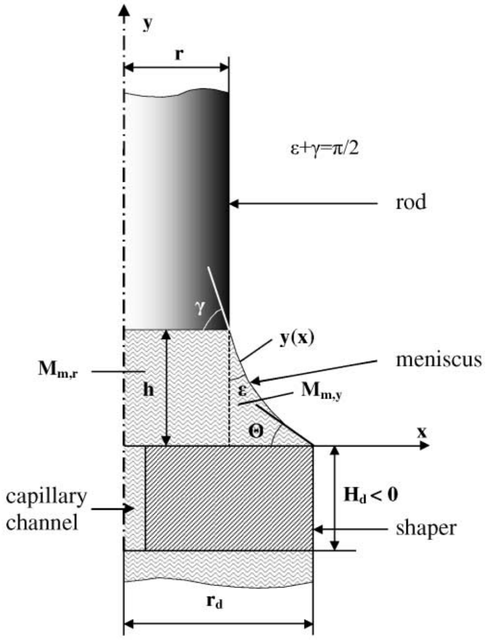

Fig. 3. Scheme of growing the cylindrical crystal rod by the Stepanov (EFG) technique.

3. Case of growing crystal cylindrical rod

In the case of the growing crystal cylindrical rod, both main curvatures are involved in Laplace equation, Eq. (3) (Figs. 3 and 4). The radii of the main curvatures have different signs, because they have different directions relative to the meniscus surface (Figs. 2 and 4). Thus, Laplace equation should be written as

$$\rho_L g\, y(x) = \sigma\left(\frac{1}{R_1(x)} - \frac{1}{R_2(x)}\right) - \rho_L g H_d. \quad (13)$$

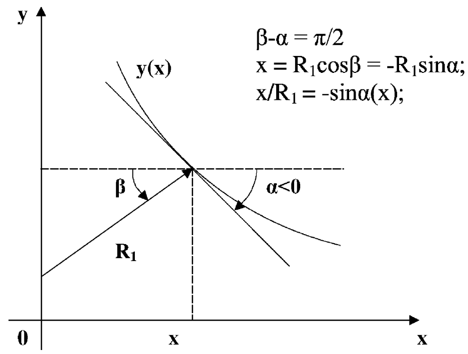

Fig. 4. Scheme of the calculation of the first main curvature.

Using the cylindrical coordinate system weight $P_{m,y}$ of the part of the meniscus located under the surface defined by the profile curve, $y(x)$ should be written according to Eq. (13) as

$$\begin{aligned} P_{m,y} &= gM_{m,y} \\ &= 2\pi\rho_L g\int_r^{r_d} x\, y(x)\,dx \\ &= 2\pi\sigma\int_r^{r_d}\frac{x\,dx}{R_1(x)} - 2\pi\sigma\int_r^{r_d}\frac{x\,dx}{R_2(x)} \\ &\quad - 2\pi\rho_L g H_d\int_r^{r_d} x\,dx. \end{aligned} \quad (14)$$

Here, $M_{m,y}$ is the mass of the part of the meniscus located under the surface defined by the profile curve $y(x)$ (Fig. 3), r the radius of the crystal rod, and r_d the radius of the shaper. According to Eq. (14) and expressions in Fig. 4, the partial mass $M_{m,y}$ may be written as

$$\begin{aligned} M_{m,y} = &-\frac{2\pi\sigma}{g}\int_r^{r_d}\sin\alpha(x)\,dx \\ &-\frac{2\pi\sigma c}{g}\int_r^{r_d}\frac{x\,dx}{R_2(x)} - 2\pi\rho_L H_d\int_r^{r_d} x\,dx. \end{aligned} \quad (15)$$

The mass $M_{m,r}$ of the part of the meniscus located under the planar interface boundary (Fig. 1) is written as

$$M_{m,r} = \pi\rho_L h r^2 - \pi\rho_L H_d r^2. \quad (16)$$

Here h is the height of the meniscus with a planar interface boundary. The expression M_m for the full mass of the meniscus should be written as

$$M_m = M_{m,r} + M_{m,y}. \quad (17)$$

Now it is better to designate the first integral in Eq. (15) as J_1 and the second integral in the same expression as J_2. After this substitution, Eq. (15) may be written as

$$M_{m,y} = -\frac{2\pi\sigma}{g}(J_1 + J_2) - 2\pi\rho_L H_d \int_r^{r_d} x\,dx. \quad (18)$$

Here, $J_1 = \int_r^{r_d} \sin\alpha(x)\,dx$ and $J_2 = \int_r^{r_d} x\,dx/R_2(x)$. Integral J_2 should be calculated in parts. Using the well-known expression for integrating by parts $\int u\,dv = uv - \int v\,du$ [5] and designating $u = x$, $dv = dx/R_2(x)$ the expression for v can be written as $v = \int_r^{r_d} dx/R_2(x)$. The last integral is already known from Fig. 2 and Eq. (9). Thus, integral J_2 is written as

$$J_2 = x\sin\alpha(x)\big|_r^{r_d} - J_1. \quad (19)$$

Eq. (19) results in the sum $J_1 + J_2 = x\sin\alpha(x)|_r^{r_d}$ and Eq. (18) should be simplified as

$$M_{m,y} = -\frac{2\pi\sigma}{g}x\sin\alpha(x)|_r^{r_d} - 2\pi\rho_L H_d\frac{x^2}{2}|_r^{r_d}. \quad (20)$$

After further simplification, Eq. (20) becomes

$$M_{m,y} = \frac{2\pi\sigma}{g}(\mathrm{r}\cos\varepsilon - r_d\sin\theta) - \pi\rho_L H_d(r_d^2 - r^2). \quad (21)$$

According to Eqs. (16), (17) and (21), the expression for the full mass of the meniscus may be written as

$$M_m = \pi\rho_L h r^2 + \frac{2\pi\sigma}{g}(\mathrm{r}\cos\varepsilon - r_d\sin\theta) - \pi\rho_L H_d r_d^2. \quad (22)$$

Taking into account the definition of the capillary constant, the expression for the full mass of the meniscus may be rewritten as

$$M_m = \pi\rho_L h r^2 + \pi\rho_L a^2(\mathrm{r}\cos\varepsilon - r_d\sin\theta) - \pi\rho_L H_d r_d^2. \quad (23)$$

4. Case of growing crystal tube

In the case of the growing crystal tube, the meniscus consists of three parts (Fig. 5): part $M_{m,r}$ of meniscus located under the interface boundary, part $M_{m,y,1}$ of the meniscus located under the surface defined by the outer meniscus curve $y_1(x)$ and part $M_{m,y,2}$ of the meniscus located under the surface defined by the inner meniscus curve $y_2(x)$. Thus, the expression for the full mass M_m of the meniscus is written as

$$M_m = M_{m,r} + M_{m,y,1} + M_{m,y,2}. \quad (24)$$

A planar interface boundary is considered in this paper. Generally, menisci heights h_1 and h_2 at the

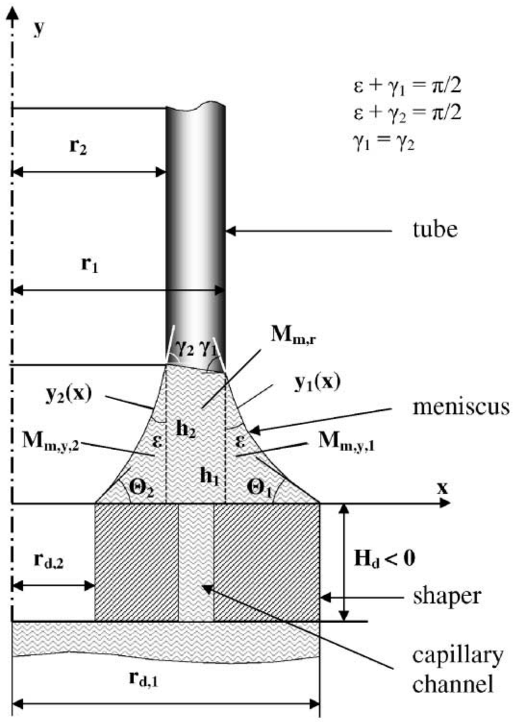

Fig. 5. Scheme of growing the crystal tube by the Stepanov (EFG) technique.

outer and inner sides of the tube are different (Fig. 5) because radii R_1 of the main curvatures at different sides of the tube have different directions relative to the volume of the meniscus: at the outer surface of the meniscus, radius R_1 is directed outside of the meniscus volume and at the inner surface, it is directed inside of the meniscus volume. But growth angle ε must be the same on both sides of the growing tube under the consideration of isotropic crystal growth [6]. The partial mass functional $M_{m,r}$ should be written as

$$M_{m,r} = \pi\rho_L \frac{h_1 + h_2}{2}(r_1^2 - r_2^2) - \pi\rho_L H_d(r_1^2 - r_2^2). \tag{25}$$

Here, r_1 and r_2 are the outer and inner radii of the tube, respectively. The outer meniscus defined by the curve $y_1(x)$ should be described as for the case of the growing cylindrical rod and the expression for partial mass $M_{m,y,1}$ is written as

$$\begin{aligned} M_{m,y,1} = &\frac{2\pi\sigma}{g}\left(r_1 \cos\varepsilon - r_{d,1}\sin\theta_1\right) \\ &- \pi\rho_L H_d(r_{d,1}^2 - r_1^2). \end{aligned} \tag{26}$$

Here, $r_{d,1}$ is the outer radius of the shaper, and θ_1 the angle of the contact of the outer part of the meniscus with the shaper (Fig. 5).

As mentioned above, the radius R_1 of the first main curvature of the part of the meniscus defined by the profile curve $y_2(x)$ has another sign in comparison with the case of the growing cylindrical rod. Therefore, the inner meniscus is described by the Laplace equation as

$$\rho_L g\, y(x) = \sigma\left(-\frac{1}{R_1(x)} - \frac{1}{R_2(x)}\right) - \rho_L g H_d. \tag{27}$$

The expression for the partial mass $M_{m,y,2}$ according to Eq. (27) may be written as

$$\begin{aligned} M_{m,y,2} = &-\frac{2\pi\sigma}{g}\int_{r_{d,2}}^{r_2} \frac{x\,dx}{R_1(x)} \\ &- \frac{2\pi\sigma}{g}\int_{r_2}^{r_{d,2}} \frac{x\,dx}{R_2(x)} - 2\pi\rho_L H_d \int_{r_{d,2}}^{r_2} x\,dx. \end{aligned} \tag{28}$$

The lower and upper limits in Eq. (28) in the second integral changed their places because of the other direction of integration of the second main curvature for the inner contour in comparison with the outer contour (in a double-contour system of the cross-section of the tube). The integrals in Eq. (28) are calculated as in the case of the cylindrical rod and thus the expression for the partial mass $M_{m,y,2}$ may be written as

$$M_{m,y,2} = -\frac{2\pi\sigma}{g}x\sin\alpha(x)\Big|_{r_2}^{r_{d,2}} + 2\pi\rho_L H_d \frac{x^2}{2}\Big|_{r_2}^{r_{d,2}}. \tag{29}$$

After simplification, Eq. (29) should be written as

$$\begin{aligned} M_{m,y,2} = &\frac{2\pi\sigma}{g}\left(r_2\cos\varepsilon - r_{d,2}\sin\theta_2\right) \\ &+ 2\pi\rho_L H_d(r_{d,2}^2 - r_2^2). \end{aligned} \tag{30}$$

Here, $r_{d,2}$ is the inner radius of the shaper, and θ_2 the angle of the contact of the inner part of the meniscus with the shaper (Fig. 5). Following Eqs. (25), (26) and (30), the expression for the full mass M_m can be written as

$$\begin{aligned} M_m = &\pi\rho_L\frac{h_1 + h_2}{2}\left(r_1^2 - r_2^2\right) + \frac{2\pi\sigma}{g}((r_1 + r_2)\cos\varepsilon \\ &- r_{d,1}\sin\theta_1 - r_{d,2}\sin\theta_2) \\ &- \pi\rho_L H_d(r_{d,1}^2 - r_{d,2}^2). \end{aligned} \tag{31}$$

Taking into account the definition of capillary constant a, the expression for the full mass M_m may be written as

$$\begin{aligned} M_m = &\pi\rho_L\frac{h_1 + h_2}{2}\left(r_1^2 - r_2^2\right) + \pi\rho_L a^2((r_1 + r_2)\cos\varepsilon \\ &- r_{d,1}\sin\theta_1 - r_{d,2}\sin\theta_2) \\ &- \pi\rho_L H_d(r_{d,1}^2 - r_{d,2}^2). \end{aligned} \tag{32}$$

5. Case of growing crystal rod of arbitrary cross-section

A simplified scheme of the growing crystal rod of arbitrary cross-section is shown in Fig. 6. In this case the meniscus consists of two parts: the part of the meniscus located under the planar interface boundary and the part of the meniscus located under the complicated surface $y(x, \varphi)$ contacted with the crystal rod and the shaper. Here, φ is the azimuthal angle in a cylindrical coordinate system. The expression for the full mass M_m is the sum of

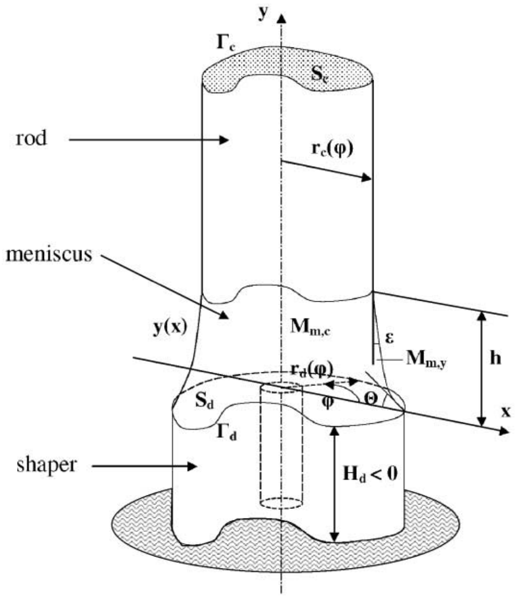

Fig. 6. Scheme of growing the crystal rod of arbitrary cross-section.

two corresponding partial masses $M_{m,c}$ and $M_{m,y}$ (Fig. 6) written as

$$M_m = M_{m,c} + M_{m,y}. \tag{33}$$

The first partial mass functional $M_{m,c}$ defining the part of the meniscus under the interface boundary may be written as

$$M_{m,c} = \rho_L h S_c - \rho_L H_d S_c. \tag{34}$$

Here, S_c is the square of the crystal cross-section. Relating to the part of meniscus located under the surface $y(x, \varphi)$ its radii R_i of the main curvatures and the local height y of this part of the meniscus do not depend obviously on the azimuthal angle φ, because they only depend on the shapes of contors $r_c(\varphi)$ and $r_d(\varphi)$ of the crystal and the shaper, respectively. It is possible in this case, because hydrostatic menisci are considered, which are in good agreement with the conditions of the real processes of crystal growth from the melt by the Stepanov (EFG) and Czochralski technique [4]. Following this statement, a shape of the cross-section of the crystal is similar to the form of the shaper. It is useful to introduce a relative coefficient $k = r_c(\phi)/r_d(\phi)$, $k > 0$. Thus, the local height $y(x)$ and radii $R_i(x)$ depend obviously only on the radial coordinate x (Fig. 6) and it is possible to integrate Laplace Eq. (13) written for the case of growing crystal rod. Weight $P_{m,y}$ of the part of the meniscus located under the surface defined by the function $y(x)$ of the local meniscus height should be written according to Eq. (13) as

$$P_{m,y} = gM_{m,y} = \rho_L g \int_0^{2\pi} \int_{r_c(\phi)}^{r_d(\phi)} x\, y(x)\, dx\, d\phi. \tag{35}$$

Partial mass $M_{m,y}$ should be written according to Eq. (13) and Eq. (35) as

$$\begin{aligned} M_{m,y} = {} & \frac{\sigma}{g} \int_0^{2\pi} \int_{kr_d(\phi)}^{r_d(\phi)} \frac{x}{R_1(x)}\, dx\, d\phi \\ & - \frac{\sigma}{g} \int_0^{2\pi} \int_{kr_d(\phi)}^{r_d(\phi)} \frac{x}{R_2(x)} \\ & - \rho_L H_d \int_0^{2\pi} \int_{kr_d(\phi)}^{r_d(\phi)} x\, dx\, d\phi. \end{aligned} \tag{36}$$

Eq. (36) may be written as

$$M_{m,y} = -\frac{\sigma}{g}(J_1 + J_2) - \rho_L H_d J_3. \tag{37}$$

Here, integral J_1 is equal to

$$\begin{aligned} J_1 & = -\int_0^{2\pi} \int_{kr_d(\phi)}^{r_d(\phi)} \frac{x}{R_1(x)}\, dx\, d\phi \\ & = \int_0^{2\pi} \int_{kr_d(\phi)}^{r_d(\phi)} \sin\alpha(x)\, dx\, d\phi. \end{aligned} \tag{38}$$

Integral J_3 is equal to

$$\begin{aligned} J_3 & = \int_0^{2\pi} \int_{kr_d(\phi)}^{r_d(\phi)} x\, dx\, d\phi \\ & = (1 - k^2) \int_0^{2\pi} \frac{r_d^2(\phi)}{2}\, d\phi \\ & = (1 - k^2) S_d = S_d - S_c. \end{aligned} \tag{39}$$

Here, $S_d = \int_0^{2\pi} r_d^2(\varphi)/2\, d\varphi$ and $S_c = k^2 S_d$ are the definitions of the areas of the cross-sections of the shaper and crystal, respectively. According to the process of integration described in the case of the ribbon integral, J_2 may be written as

$$J_2 = \int_0^{2\pi} x \sin \alpha(x) \Big|_{kr_d(\phi)}^{r_d(\phi)} \, d\phi - J_1. \tag{40}$$

According to Eq. (37)–Eq. (40) the partial mass functional $M_{m,y}$ should be written as

$$M_{m,y} = \frac{\sigma}{g}(\Gamma_c \cos \varepsilon - \Gamma_d \sin \theta) - \rho_L H_d (S_d - S_c). \tag{41}$$

Here, $\Gamma_d = \int_0^{2\pi} r_d(\phi)\, d\phi$ and $\Gamma_c = k\Gamma_d$ are the definitions of the lengths of the contours of the shaper and crystal, respectively. Angle ε is the growth angle and θ is the contact angle between meniscus and shaper (Fig. 6). According to Eqs. (33), (34) and (41) the expression for the full mass M_m should be written as

$$M_m = \rho_L h S_c + \frac{\sigma}{g}(\Gamma_c \cos \varepsilon - \Gamma_d \sin \theta) - \rho_L H_d S_d. \tag{42}$$

Taking into account the definition of the capillary constant, the expression for the full mass may be written as

$$M_m = \rho_L h S_c + \frac{1}{2}\rho_L a^2 (\Gamma_c \cos \varepsilon - \Gamma_d \sin \theta) - \rho_L H_d S_d. \tag{43}$$

6. Case of growing crystal tube of arbitrary outer and inner cross-sections

An example of the cross-section of the crystal tube of arbitrary outer and inner contours is schematically shown in Fig. 7. The shape of the cross-section of the shaper is described in the cylindrical coordinate system with functions $r_{d,1}(\phi)$ and $r_{d,2}(\phi)$ of outer and inner contours of the shaper cross-section, respectively. The shape of the cross-section of the growing crystal is similar to the shape of the shaper cross-section and described by the functions $r_{c,1}(\phi)$ and $r_{c,2}(\phi)$, respectively. The cross-sections of the shaper and crystal have their own lengths Γ and squares S. The shaper has the length $\Gamma_{d,1}$ of its outer contour, the length $\Gamma_{d,2}$ of its inner contour and the square S_d of the cross-section (Fig. 7). The crystal has the length $\Gamma_{c,1}$ of its outer contour, the length $\Gamma_{c,2}$ of its inner contour and the square S_d of the cross-section. (Fig. 7). The outer and inner meniscus profiles may have different meniscus heights h_1 and h_2 at their contours, respectively, under the consideration of the planar interface boundary.

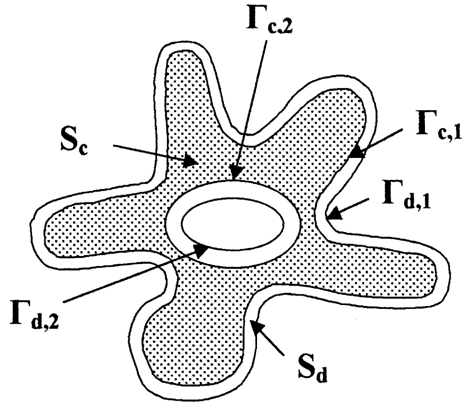

Fig. 7. Scheme of the cross-sections of the shaper and crystal in the case of the crystal tube with arbitrary outer and inner contours grown by the Stepanov (EFG) technique.

The expression for the full mass M_m in the case described should be written as

$$M_m = \rho_L \frac{h_1 + h_2}{2} S_c + \frac{\sigma}{g}\big((\Gamma_{c,1} + \Gamma_{c,2})\cos \varepsilon - \Gamma_{d,1} \sin \theta_1 - \Gamma_{d,2} \sin \theta_2\big) - \rho_L H_d S_d. \tag{44}$$

Taking into account the definition of the capillary constant, this expression may be written as

$$M_m = \rho_L \frac{h_1 + h_2}{2} S_c + \frac{1}{2}\rho_L a^2 \big((\Gamma_{c,1} + \Gamma_{c,2})\cos \varepsilon - \Gamma_{d,1} \sin \theta_1 - \Gamma_{d,2} \sin \theta_2\big) - \rho_L H_d S_d. \tag{45}$$

7. Numerical analysis and conclusions

Taking into account the external pressure factor $H_d = 0$ and contact angle $\theta = 0$ between the meniscus and free surface of the melt in the crucible the well-known expression [7] of the full mass of the meniscus for the cylindrical rod grown by the Czochralski technique can be produced as

$$M_{\mathrm{m}} = \pi\rho_{\mathrm{L}}hr^2 + \frac{2\pi\sigma}{g} r\cos\varepsilon.$$

The authors of papers [1,2] mechanically add two terms related to the hydrodynamic factor to the expression of full mass. But it is not clear what equation of non-Laplace type was considered for the meniscus surface. To obtain additional terms in the expression of the full mass it is necessary to consider another, more general, full energy functional instead of that used in this paper, Eq. (1). Then, a new full energy functional must be minimized and a new equation for the meniscus profile curve distinguishing from the Laplace equation must be produced. After these two steps, a new equation for the profile curve must be integrated and correct additional terms can be produced. But this is not necessary as the hydrodynamic factor is too small. An analysis of the influence of the hydrodynamic factor is made in the book [4] and it was shown that it is possible to make hydrostatic approximations in the consideration of the meniscus profile curves and to use the Laplace equation up to crystal growth rates of $0.1 - 1\,\mathrm{m/s}$. These large values of rates are impossible in the Stepanov (EFG) technique.

Using Eq. (1) it is possible now to make comparison of the surface free energy of the meniscus, its potential energy and hydrodynamic factor (e.g. kinematic energy) for the meniscus of concrete dimensions. The growth of a cylindrical crystal with radius $r_{\mathrm{c}} = 5\,\mathrm{mm}$, height $h = 0.1\,\mathrm{mm}$ of the meniscus, crystal growth rate $\upsilon = 1\,\mathrm{mm/min}$ and distance $H_{\mathrm{d}} = -20\,\mathrm{mm}$ between the working edge of the shaper and the surface of the melt in the crucible is considered.

First, integral part I_1 of the full energy functional (1) (surface free energy) is approximately equal: $I_1 = 2\pi r_{\mathrm{c}}h\sigma = \pi r_{\mathrm{c}}h\rho_{\mathrm{L}}ga^2$. Here, the shape of the surface of the meniscus is represented by the cylindrical surface when $r_{\mathrm{c}} \approx r_{\mathrm{d}}$. Second, the integral part I_2 of the functional (1) (potential energy) is equal: $I_2 = \pi r_{\mathrm{c}}^2 h\rho_{\mathrm{L}}g\left(h/2 + H_{\mathrm{d}}\right)$. It is possible to evaluate kinetic energy I_3 of the meniscus considering the movement of the whole meniscus with the crystallization rate υ. In this consideration, kinetic energy is equal: $I_3 = m\upsilon^2/2 = \pi r_{\mathrm{c}}^2 h\rho_{\mathrm{L}}\upsilon^2/2$. For the case of the growing crystal described above, the values of the surface, potential and kinetic energies are equal: $I_1 = 10^{-6}\,\mathrm{J}$, $I_2 = -6.28\times10^{-6}\,\mathrm{J}$, $I_3 = 5\times10^{-15}\,\mathrm{J}$. As shown, the kinetic energy is too small in comparison with the other two energies. It is interesting to find the formula for the relations between these energies. The relation

$$\left|\frac{I_2}{I_1}\right| = \frac{\rho_{\mathrm{L}}gr_{\mathrm{c}}\left|\ h/2 + H_{\mathrm{d}}\right|}{2\sigma} \approx 6.28,$$

is equal to the characteristic undimensioned Bond number $B_0 = \rho_{\mathrm{L}}gL^2/\sigma$ [4]. Here L is the characteristic linear dimension of the system "melt–meniscus–crystal". The relation $I_3/I_1 = \rho_{\mathrm{L}}\upsilon^2 r_{\mathrm{c}}/4\sigma \approx 5\times10^{-9} \ll 1$ defines the undimensioned Weber number $\mathrm{We} = \rho_{\mathrm{L}}\upsilon^2 L/\sigma$ [4]. The relation

$$\left|\frac{I_3}{I_2}\right| = \frac{\upsilon^2}{2g\left|\ h/2 + H_{\mathrm{d}}\right|} \approx 7\times10^{-9} \ll 1,$$

defines the undimensioned Froude number $F = \upsilon^2/gL$ [3].

The authors of papers [1,2] also do not take into account the contact of the meniscus with the edges of the shaper in their expressions of full meniscus mass. They also use the area of the crystal cross-section instead of the cross-section of the shaper in places of the mass expressions where the external pressure factor H_{d} appears.

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